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ELEMENTS OF ARITHMETIC

HOPKINS AND UNDERWOOD

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THE ELEMENTS OF ARITHMETIC

IN THEORY AND PRACTICE

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ELEMENTS OF ARITHMETIC

IN THEORY AND PRACTICE

BY

JOHN W. HOPKINS
SUPERINTENDENT OF SCHOOLS, GALVESTON, TEXAS

AND

P. H. UNDERWOOD
TEACHER OF MATHEMATICS, BALL HIGH SCHOOL, GALVESTON, TEXAS

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"Perhaps no single subject of elementary instruction has suffered so much from lack of scholarship on the part of those who teach it as mathematics. Arithmetic is universally taught in schools, but almost invariably as the art of mechanical computation only. The true significance and the symbolism of the processes employed are concealed from pupil and teacher alike. This is the inevitable result of the teacher's lack of mathematical scholarship.

"The subtlety, delicacy, and accuracy of mathematical processes have the highest educational value, both direct and indirect. To treat them as mechanical routine, not susceptible of explanation or illumination from a higher point of view, is to destroy in large measure the value of mathematics as an educational instrument, and to aid in arresting the mental development of the pupil."

NICHOLAS MURRAY BUTLER, Ph.D., LL.D.,
President of Columbia University.

"The chief thing is that every teacher be active, and growing scientifically and pedagogically; that he add to his mathematical attainments year by year, and that he study to improve in the art of teaching."

J. W. A. YOUNG, Ph.D.,
Assistant Professor of the Pedagogy of Mathematics in the University of Chicago.

"Since it appears that arithmetic is taught for these two general reasons (culture and utility), a question arises as to their relative importance. But this it is impossible to answer. We lack a unit of measure. Laisant remarks that it is like asking which is the more important, eating or sleeping; the loss of either is fatal. The teacher who recognizes in the subject only its applications to trade would better give up teaching; the one who sees in it only an exercise in logic will also fail; but the greatest failure comes from seeing in the subject neither utility nor logic, as is the case with the teacher who blindly follows the old style, traditional textbook."

DAVID EUGENE SMITH, Ph.D.,
Professor of Mathematics, Teachers' College, Columbia University.

PREFACE

In the preparation of the "Elements of Arithmetic" the authors have aimed to give, in the first place, a simple, clear, and rational development of the principles of arithmetic, and, in the second place, to illustrate these principles by an abundance and a variety of examples in simple computations of a character that may be of interest to the average individual in the everyday life of our time. Arithmetic is generally taught with two ends in view, namely, as an exercise to induce thought and as the art of ciphering. We have endeavored to keep both these ends prominent.

This is not a book of verbal rules. We have in a few instances, however, taken the liberty to state rules after a thorough exposition of the reasons underlying them. We are believers in the maxim of Sir Isaac Newton, "Exempla magis prosunt quam precepta." We have shown evidence of this belief in giving in this book solutions of almost every variety of problem. These solutions are in mathematical form and are for pupils to think over and to imitate.

The first half of this book contains no difficult examples. It is believed that a child of average ability can readily be trained to solve any problem in the first nine chapters. The mind, as Dryden long ago pointed out, can take in but a single idea at a time. Children cannot do two things at one time. They should not be expected, for instance, to grapple with problems requiring for their solutions much ability to reason, until they have attained some proficiency in rapid and accurate work in the four simple rules. On the other hand, in problems requiring some ability in arithmetical analysis, complexity in the mechanical computations should not be a characteristic feature. As a rule, throughout the book the examples illustrating each topic are at first very simple, and, in all examples requiring for their solution close reasoning, the mechanical work is so simple that, in almost all instances, it can be performed mentally.

It may not be out of place to say that all the most difficult examples in Compound Quantities, Longitude and Time, Percentage, Interest, Bank Discount, Taxes, Duties, Insurance, and in the first set of Miscellaneous Examples have been assigned to the pupils of two classes reviewing arithmetic during the past three months under the tutorship of one of the authors.

The authors are not theorists, they have no hobbies to ride, they are believers in the Socratic method of teaching, and they think that the prime requisite in the teaching of arithmetic is a teacher possessing a knowledge of arithmetic and some ability to teach the subject.

Long experience in the class room and observation of the work done by other teachers have convinced us that there are weak spots in the teaching of arithmetic.

Our Decimal System of Notation, so essential as a groundwork in arithmetic, is rarely presented from a correct or a logical point of view; and as a result the pupils' knowledge of notation is hazy and inexact. This is our apology for giving more space to this topic than is given in any other elementary text-book with which we are acquainted.

We have given also more space to Compound Quantities than is usual in text-books. We have here aimed at simplicity, fullness, and depth. If we have gone deep into the matter, we have at the same time taken pains to throw light upon the subject. Teachers who lay stress upon the ratio idea of number are compelled, if they know what ratio means, to give space to denominate quantities. While we are not of the ratio school, we have tried to illustrate the ratio idea, at least, as fully as the most ardent ratio pedagogue.

Attention is invited to our treatment of Percentage. If our point of view is right, the majority of text-books are at fault in presenting this topic. We are decidedly of the opinion that upwards of ninety per cent of the authors of text-books on arithmetic have never seriously considered what rate and rate per cent mean.

Nearly every elementary text-book on arithmetic gives space to the Metric System. In almost all books the Metric System is treated so as to make the system uninteresting, especially to the young learner. The purpose of treating the Metric System should not be the conversion of metric units into the units of some other system. The one chief aim of taking up the Metric System should be to acquaint the pupils with the Metric System itself, and thereby to let them see for themselves how much simpler, more rational, and more scientific this system is than any other system devised.

In the last few years a great deal has been written on the teaching of arithmetic. The recent reports of city superintendents, especially in the larger cities, are full of suggestions and of criticisms. One superintendent remarks, "There is a tendency on the part of some of our teachers to ignore the necessity of drill on very simple matters, and to be in a hurry to pass to considerations involving complexity. There should be a large amount of practice on simple phases of the work until we get something like expertness before we attempt complications." Looking over several recent reports of men of wide reputation, we are forcibly reminded of the words of Professor Chrystal in his presidential address before a section of the British Association : —

"Science in the hands of specialists soars higher and higher in the light of day, while educators and the educated are left more and more to wander in primeval darkness."

Modern writers on education whose views are entitled to profound respect recommend the early introduction of general symbols to represent number. We have not introduced literal notation so early as these writers recommend. We have, however, adhered closely to the mathematical form, and, consequently, one could use this book, substituting a symbol for an idea, without any further change in the text.

We have not introduced into this book the negative quantity. An early introduction of negative number gives the unskilled teacher an opportunity to make the first lessons in algebra a mere blind manipulation of symbols.

Nearly all the examples in this book are new. A few of the exercises were suggested by questions given at public examinations.

THE AUTHORS.

GALVESTON, TEXAS,
May 15, 1903.

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ELEMENTS OF ARITHMETIC

CHAPTER I

DEFINITIONS

1. **Arithmetic** is the science of numbers and the art of computation.

2. **Counting** is the first and fundamental operation in arithmetic. From the earliest times people used their fingers in counting. They used also pebbles, sticks, and notches on wood, for the same purpose. The word *calculate* is derived from a Latin word which means a pebble. The word *digit*, to be explained presently, is also derived from a Latin word which means a finger. When we count things and thereby find out an answer to the question, how many? we get a number.

3. **Number** is an answer to the question, *how many?*

4. Any one of the natural numbers, one, two, three, four, five, etc., is called an **integer**, **integral**, or **whole number**.

5. A **unit** is one thing or a group of things regarded as a single thing.

We may count by ones, by twos, by threes, or by any other number. Any one thing that we count by, or any standard that we measure by, is a unit. When eggs are bought by the dozen, the unit is a dozen eggs; when lumber is bought by the thousand feet, the unit is a thousand feet of lumber; when time is computed in hours, the unit is one hour; when distances are reckoned in feet, in yards, or in miles, the unit in each case is respectively a foot, a yard, or a mile.

6. A number thought of with reference to some specific unit is called a **concrete number.** The term *concrete* is used for purposes of explanation. Strictly speaking, there is nothing concrete about number. Four apples, six men, eight dollars, are examples of concrete number.

7. An **abstract number** is one thought of without reference to any particular unit. In case of an abstract number, the unit is not named. Examples: three, nine, fifty.

NUMERATION

8. Numeration is the art of expressing numbers in words.

The names for the first ten numbers are one, two, three, four, five, six, seven, eight, nine, ten. These names have no etymological connection with one another. The name *seven* does not indicate any relation to *six*. If the names of numbers were framed on the principle of absolute independence of one name of another name, the task of learning number names and number values would be laborious in the extreme.

The method of numeration adopted everywhere is quite simple. By combining a few number names, we are enabled to express in words all numbers.

After ten comes eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty. The remaining number names are thirty, forty, fifty, sixty, seventy, eighty, ninety, hundred, thousand, million, billion, trillion.

Eleven literally means one placed upon ten.

Twelve literally means two placed upon ten.

Thirteen means three and ten.

Fourteen means four and ten. Seventeen means seven and ten. Twenty means two tens. Twenty-one means two tens and one. Thirty means three tens. Thirty-four means three tens and four. Sixty means six tens. Eighty-seven means eight tens and seven. One hundred is ten tens. One thousand is ten hundreds. Ten-thousand is ten groups of a thousand each. One hundred-thousand is ten groups of ten-thousand each.

Ones, tens, hundreds, thousands, ten-thousands, are respectively called units of the first order, units of the second order, units of the third order, units of the fourth order, units of the fifth order. In this system of naming numbers, ten units of any order are equal to one unit of the next higher order. Obviously, ten is the basis of the system. On this account our number system is known as the **decimal system** (Latin *decem*, ten).

NOTATION

9. **Notation** is the art of expressing numbers by means of figures. Every number can be expressed by one or more of the following figures or symbols:

1	2	3	4	5	6
one	two	three	four	five	six
7	8	9	0		
seven	eight	nine	zero, naught, or cipher.		

The first nine of these figures are called **digits** or **significant figures.**

10. **Notation** of **tens** and **ones**:

In order to understand the principle upon which numbers greater than nine are expressed in figures, consider the case of a person counting upon his fingers. Suppose he makes a mark for the first ten he counts, another mark for the second ten he counts, a third mark for the third ten, and so on. Suppose the final result is as follows:

```
*
*  //////
*
*
```

The stars indicate the number of tens counted, and the strokes the number of single objects. Here, we have four tens and six, or forty-six. Suppose further that instead of the four stars we agree to write the digit four and instead of the six strokes we write the digit six, we then have the combination 46. By this agreement we can represent by two figures all integers larger than nine and less than one hundred. What does the combination 37 represent? Evidently three tens and seven ones, or thirty-seven. What does the combination 60 represent? It represents six tens and no ones, and is read sixty. In integral numbers of two figures the right-hand digit represents as many ones as its name indicates, and the left-hand figure as many tens as its name indicates. The place the right-hand figure occupies is called the units' place, and the place the figure to its left occupies is called the tens' place.

NOTATION OF HUNDREDS, TENS, ONES

11. Hundreds are expressed by writing the digit indicating the number of hundreds immediately to the left of the tens' place. Thus the number three hundred fifty-four is written 354. The digit 4 is written in the units' place, 5 in the tens' place, and 3 in the hundreds' place. What number is expressed by the combination 708? This means 7 hundreds, no tens, and 8 ones, and is read seven hundred eight. Similarly, the combination 930 means 9 hundreds, 3 tens, and no ones, and is read nine hundred thirty. The combination 110 means 1 hundred, 1 ten, no ones, and is read one hundred ten.

EXERCISE I

Express in words:

107, 209, 500, 523, 192, 181, 175, 163, 361, 316, 613, 300, 900, 784, 927, 737, 222, 555, 801, 624, 642, 690, 609, 906, 342, 234, 432, 999.

What is the value of a digit in the units' place?

What is the value of a digit in the tens' place?

What is the value of a digit in the hundreds' place?

Has the figure 0 ever any value? Of what use, then, is the figure 0? *Ans.* It serves to keep the digits in their proper places.

Upon what does the value of a digit depend?

Write in figures:

Three hundred twenty-four, two hundred eighty, four hundred four, three hundred three, seven hundred eight, three hundred twelve, eight hundred fourteen, six hundred eighteen, five hundred thirteen, seven hundred seventeen, nine hundred fifteen, eight hundred sixteen, two hundred eighty-four, six hundred ninety-eight, seven hundred, four hundred.

Nine hundred seven, six hundred sixty, four hundred thirty, two hundred forty, eight hundred eighty, five hundred eighty, five hundred ten, seven hundred eleven, three hundred nine, three hundred ninety, nine hundred forty, five hundred forty-four, seven hundred seventy-one, six hundred ninety-nine, four hundred eighty-three, nine hundred seven, two hundred nine, eight hundred.

What is the largest integer expressed by three figures? What is the smallest integer expressed by three figures? What is the smallest integer expressed by three digits? What is the largest integer, greater than

800 and beginning with 8, expressed by three digits? What is the largest integer beginning with 2, greater than 200, expressed by three figures and containing two digits?

NOTATION OF NUMBERS OF FOUR OR MORE FIGURES

12. Since ten groups of a hundred each make one thousand and, furthermore, since one thousand is a unit of the fourth order, it is natural to expect that thousands should be written in the fourth place, *i.e.* just to the left of hundreds. This is actually the case. Two thousand three hundred twelve is written 2312. What does the combination of figures 7082 mean? It means 7 thousands, no hundreds, 8 tens, 2 ones, and is read seven thousand eighty-two. What does the combination 2003 mean? It means 2 thousands, no hundreds, no tens, three ones, and is read two thousand three.

EXERCISE 2

Express in figures:

Four thousand, two hundred eighty-three; three thousand, three hundred twenty; five thousand, four hundred eight; four thousand, four; eight thousand, ten; six thousand, forty; eight thousand; nine thousand; one thousand, one; seven thousand, seventy.

13. The next place to the left of thousands is called the ten-thousands' place, and the one immediately to the left of the ten-thousands' is called the hundred-thousands' place. Thus the figures 652384 represent 6 hundred-thousands, 5 ten-thousands, 2 thousands, 3 hundreds, 8 tens, 4 ones. This combination is read six hundred fifty-two thousand, three hundred eighty-four. The place next to the hundred-thousands' place, and to the left, is called the millions' place.

This system of notation is called the **decimal system**; also the Arabic, on account of its having been brought into Europe by the Arabs. The system came originally from the Hindus.

READING INTEGERS

14. A number represented by three figures, for example, by the figures 456, is read four hundred fifty-six. **To read a number** expressed by more than three figures we proceed as follows: begin at the right, *i.e.* with units of the first order and mark off with commas the figures in groups of three. The three places in which units of the first order occur, constitute what is called the units' period; the next three places, the thousands' period; the next three places, the millions' period; the next three places, the billions' period, and so on as far as may be required. As an illustration, take the number represented by the figures 34728316. Marking off in periods, we have 34,728,316. Second step: beginning with the left-hand period, read the figures in each period as if they alone were the only figures under consideration, adding at the same time the name of the period they occupy, excepting that of units in the units' period. The above number is read: thirty-four million, seven hundred twenty-eight thousand, three hundred sixteen.

Read the number represented by 3026083. First mark off, and we have 3,026,083. The number is read three million, twenty-six thousand, eighty-three.

NOTE. Hereafter we shall designate combinations of figures as numbers; this amounts to making use of a figure of speech.

EXERCISE 3

Express in words:

1. 45289.
2. 90208.
3. 75307.
4. 392394.
5. 738211.
6. 328993.
7. 401012.
8. 300287.
9. 200020.
10. 910003.
11. 728000.
12. 400098.
13. 902023.
14. 630006.
15. 1000000.
16. 2227001.
17. 3456000.
18. 9287003.
19. 8307308.
20. 8000401.
21. 7000014.
22. 6100079.
23. 3927173.
24. 5009020.
25. 8000904.
26. 6203003.
27. 9000090.

WRITING INTEGERS

15. **To express numbers in figures** the following directions must be attended to:

1. Begin with the highest period and write it down as if it were a number consisting of one, two, or three figures, as the case may be, then write after it a comma.
2. Treat the next highest period in the same manner, and so on; a comma is written after each period excepting the units' period. Should any places be without digits, fill them in with ciphers.

Write in figures two million, three thousand ten, 2,003,010. Here we write down 2 in the millions' place, 3 in the thousands' place, and 1 in the tens' place. There are no hundreds in the units' period and there are no ten-thousands and hundred-thousands in the thousands' period. Hence we fill those places with zeros. Notice, the digits in each period other than the units' period are read in exactly the same way as the digits of the units' period.

EXERCISE 4

Write in figures:

1. Four thousand, eight hundred twenty-seven.
2. Nine thousand, seven hundred one.
3. Sixty-eight thousand, four hundred fifty-two.
4. Forty-seven thousand, three hundred eight.
5. Ninety thousand, six hundred four.
6. Eighty-seven thousand, one hundred one.
7. Twenty-two thousand, three hundred eleven.
8. Twelve thousand, fifteen.
9. Eighteen thousand, eighteen.
10. Fourteen thousand, thirty-four.
11. Thirteen thousand, five.
12. Ninety thousand, nine.
13. Fifty-four thousand, eleven.
14. Seventy-three thousand, one.
15. Six hundred four thousand, two hundred one.
16. One hundred sixty-three thousand, ten.
17. One hundred one thousand, three hundred.
18. One hundred thousand, seven.
19. Four hundred ten thousand, one hundred twenty-seven.
20. Five hundred four thousand, three hundred eight.
21. Five hundred thousand, eleven.
22. Six hundred thousand, seventeen.
23. Nine hundred ninety thousand, fifteen.
24. Two hundred one thousand, one.
25. Seventy-two thousand, four.

16. What is the largest number of five figures?

What is the smallest number of five figures?

What is the largest number of five figures one of which is a digit?

What is the smallest number of five digits? Answer like questions, substituting four for five and also six for five.

What is the largest number expressed by the digits 9, 4, 7, 8, 3?

What is the smallest? Write all the numbers of three digits expressed by the digits 1, 2, 3.

CHAPTER II

ADDITION

17. How many are 6 apples and 7 apples?

A child who reckons with the aid of his fingers would work this exercise as follows: moving a finger for each one counted, he would say 7, 8, 9, 10, 11, 12, 13, stopping when he has moved the second finger of the second hand. If we designate the apples by strokes, writing 6 strokes to represent the six apples, and 7 strokes to represent the 7 apples, and if furthermore we place those strokes close by and in the same line, we have then the following arrangement: */// ///, /// ////.* Counting from left to right we get as before 13. As a matter of fact, in combining two numbers into a single number we count forward from the first number. In the above instance we count from 6, advancing one for each individual the second number contains.

18. A boy walks on one day 4 miles, the next day 7 miles, and the third day 6 miles. How many miles did he walk altogether? Representing the miles by strokes, we have, *////, /// /// /, /// ///.* Counting onward from 4, we have 17 in all.

19. The result of counting two or more numbers is called their **sum.**

20. **Addition is the process of finding the sum of two or more numbers of the same kind.** It is a short method of counting.

21. The numbers to be added are called **addends.**

22. **Numbers of the same kind** are numbers which can be compared with each other as to largeness or smallness. For example, 5 dollars and 4 dollars are concrete quantities of the same kind; so, also, are 6 feet and 5 yards, although in this instance the units are of different denominations, one being a foot and the other a yard. Examples of number not of the same kind, 7 ducks, 4 miles; 3 inches, 2 acres. Such quantities cannot be added.

In the addition of abstract numbers, the addends are assumed to be of the same kind.

23. The **symbol for addition** is + and it is read plus.

The symbol = is read equal or equals, and denotes equality. Thus, $9 + 8 = 17$ is read 9 plus 8 equals 17.

1. Add: 7 feet, 6 feet; 9 yards, 8 yards; 5 horses, 9 horses; 17 dollars, 8 dollars.

2. Add: 5 ones, 11 ones; 16 ones, 8 ones; 6 tens, 9 tens; 8 hundreds, 5 hundreds; 9 thousands, 4 thousands; express the results in the decimal notation.

3. How many ones make ten? How many tens make a hundred? How many hundreds make a thousand?

4. Add: 7 dimes, 12 dimes; 4 dollars, 9 dollars; 6 ten-dollar bills, 5 ten-dollar bills.

5. What effect upon the sum has changing the order in which numbers are added? Explain your answer.

EXERCISE 5 (Oral)

Count to 100 in each of the following exercises:

1. By 2's from 0, 1.

2. By 3's from 0, 1, 2.

3. By 4's from 0, 1, 2, 3.

4. By 5's from 0, 1, 2, 3, 4.

5. By 6's from 0, 1, 2, 3, 4, 5.
6. By 7's from 0, 1, 2, 3, 4, 5, 6.
7. By 8's from 0, 1, 2, 3, 4, 5, 6, 7.
8. By 9's from 0, 1, 2, 3, 4, 5, 6, 7, 8.

Note. In every exercise in addition results only should be announced. Thus, in adding 3, 4, 7, 9, a pupil should say 7, 14, 23, and not 3 and 4 are 7, 7 and 7 are 14, 14 and 9 are 23.

EXERCISE 6 (Oral)

Add in succession 2, 3, 4, 5, 6, 7, 8, 9, to each of the following numbers:

1. 13, 23, 33, 27, 37, 47, 54, 56, 59, 64, 68, 73.
2. 15, 25, 19, 17, 23, 29, 35, 41, 47, 53, 58, 74.
3. 14, 21, 24, 28, 32, 36, 44, 49, 59, 69, 76, 78.
4. 18, 23, 28, 33, 36, 39, 43, 47, 76, 67, 81, 88.
5. 12, 19, 26, 37, 46, 55, 64, 71, 79, 87, 95, 97.
6. 5, 11, 17, 25, 29, 31, 39, 41, 46, 52, 59, 69.
7. 3, 7, 18, 22, 26, 37, 38, 45, 47, 58, 68, 98.
8. 2, 8, 13, 21, 27, 72, 93, 95, 97, 91, 39, 38.
9. Count by 6's, by 7's, by 8's, by 9's up to 130, beginning with 19.

Note. Exercises like the above are of great use in securing rapid and accurate work in addition. A teacher writes on the blackboard several numbers, and one pupil is called upon to add, say 5 to each number, another 6 to each number, and so on.

24. What is the sum of 38, 44, 79, 97? These numbers being too large to handle mentally, we break them up into parts, first adding the units of the first order, and then the tens. For convenience, we arrange the numbers so that units of the same order stand in the same vertical column.

$$\begin{array}{r}38\\44\\79\\\underline{97}\\258\end{array}$$

Adding 7, 9, 4, 8, we have 16, 20, 28. Now 28 ones equal 2 tens and 8 ones. We write the 8 ones in the column for ones, and we carry 2 tens to the second column. We have then (2, 9, 7, 4, 3) tens, *i.e.* 25 tens; and since 10 tens make 1 hundred, 25 tens make 2 hundreds and 5 tens. The final result is then 2 hundreds, 5 tens, 8 ones, or 258.

25. Since 10 is the basis of our system of notation, it is not at all necessary in an addition exercise to make specific reference to the name of the units in adding any column of figures. A pupil, before undertaking an exercise in addition, should see that ones are in the same column with ones, tens in the same column with tens, hundreds in the same column with hundreds, and so forth.

NOTE. Hereafter, we shall designate ones, or units of the first order, simply as units.

EXERCISE 7 (Written)

Add:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**
29	38	98	57	98	63	345	641	777
37	94	37	76	94	56	993	258	938
38	80	40	73	82	97	400	923	924
49	63	64	92	74	92	732	806	389

10.	**11.**	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**
386	894	268	4987	3245	7777	9288	5239
592	939	397	3829	6784	6283	5573	3976
937	528	728	9283	9293	7209	3096	2898
884	776	909	5478	3289	3908	9999	3929
			9376	9009	7207	8839	5684

18.	19.	20.	21.	22.	23.
59287	19299	59397	32777	327398	734445
65398	76543	34568	88888	234299	928997
97324	39286	52308	99319	328763	529396
70707	77596	32987	44397	456909	243384
38448	83938	76923	39328	123456	739928
93924	98923	83744	55319	227389	219288
59449	52334	98997	77389	928542	928997
				829887	384386

24.	25.	26.	27.	28.
392368	327983	729992	735892	4387449
734949	48728	847777	584999	9777543
394873	95493	318394	777888	6393449
449387	278384	738706	538992	8587606
561449	559999	888774	606445	9020838
666666	878447	929666	827394	6599998
738392	923987	123456	248296	7394294
329286	644772	790492	927888	8972876
547385	904709	987789	404549	7449475
	810777	494555	729447	6218298

29.	30.	31.	32.	33.
4792799	7998992	5998476	5879824	6876894
8399384	8409499	9208503	9473473	9219479
7207383	7294792	8392393	7777888	7328337
9476583	8514798	6555777	6456789	8474494
4728737	9998777	7918924	9875874	9318327
9219777	6666784	5729998	8294295	6438444
6444673	8542728	6329888	7318392	7473478
8299299	5293294	7774673	9299497	9888777
9218288	4445454	6428427	6713729	6666679
7666729	7778898	9444779	9873876	9218289

Add vertically and horizontally, and finally sum the vertical and the horizontal totals :

392	965	873	599	222
876	329	888	307	399
543	707	393	448	937
878	538	427	388	542
929	928	925	394	234
396	555	443	776	457
878	320	289	732	901
543	889	326	587	443

Why should the sum of the vertical and the horizontal totals be the same? Examples like the above have been repeatedly given in United States Civil Service examinations. They save both teacher and pupil the trouble of looking up answers.

26. To **verify** or **check** an exercise is to perform an operation to test the correctness of the work. The most practical way of checking an exercise in addition is to add the columns downward.

EXERCISE 8 (Written)

1. A man spends for board 288 dollars a year; for clothes, 68 dollars; for railroad fare, 96 dollars; for room rent, 125 dollars; for sundries, 83 dollars, and he saves 249 dollars. Find his yearly salary.

2. In a certain school the number of pupils enrolled was:

First grade	119	Fifth grade	94
Second grade	105	Sixth grade	78
Third grade	98	Seventh grade	73
Fourth grade	97	Eighth grade	68

Find the total enrollment.

3. Find the number of days in the first 9 months of this year (1903).

4. How many times does a clock strike in 24 hours?

5. A man owns land worth 5487 dollars, buildings worth 3912 dollars, merchandise valued at 2398 dollars, and has 682 dollars cash. How much is he worth?

6. The largest seven cities of Texas with their populations, as given by the census of 1900, are as follows:

San Antonio	53,321	Galveston	37,789
Houston	44,633	Fort Worth	26,688
Dallas	42,637	Austin	22,258
Waco	20,686		

Find the total population of these cities.

7. The number of bales of cotton received at all the United States ports on December 31, 1902 was: 8946, 22,222, 3072, 7784, 364, 443, 3831, 836, 1206, 27, 6460. Find the total.

8. For the week ending December 31, 1902, the number of bales of cotton received at all the United States ports was: 44,172, 71,879, 8082, 40,019, 3495, 5017, 15,143, 1787, 2955, 596, 6460, 3050. Find the total.

9. The great lakes of the world are:

	Area in Sq. Miles		Area in Sq. Miles
Caspian	169,000	Michigan	20,000
Superior	31,200	Tanganyika	12,650
Victoria Nyanza	30,000	Baikal	12,500
Aral	26,900	Erie	10,000
Huron	17,400	Chad	10,000

Find the total area.

10. The ten largest cities of the world are:

	Population
London (Census 1898)	4,504,766
New York (Census 1900)	3,437,202
Paris (Census 1896)	2,536,834
Canton	2,500,000
Chicago (Census 1900)	1,698,575
Berlin (Census 1895)	1,677,304
Vienna (Census 1890)	1,364,548
Tokio (Census 1896)	1,299,941
Philadelphia (Census 1900)	1,293,697
St. Petersburg (Census 1897)	1,267,023

Find the total population.

What number must be added to 5, 8, 6, 9, 7, 3, respectively, to make 21? 24? 32? 43? 54? 17? 37? 16? 36? 15? 45?

Find the remainder:

17 – 8
16 – 9
24 – 8
34 – 8
44 – 8
15 – 7
25 – 7
35 – 7

23 – 6
33 – 6
43 – 6
43 – 9
53 – 9
27 – 8
17 – 8
37 – 8

11 – 3
21 – 3
31 – 5
41 – 5
51 – 7
22 – 9
32 – 9
42 – 9

26 – 8
25 – 9
34 – 8
32 – 7
45 – 9
16 – 4
76 – 9
87 – 8

7893 – 2452 = ? To work this exercise, write the subtrahend under the minuend, placing units under units, tens under tens, hundreds under hundreds, and thousands under thousands. We then draw a horizontal line, and, utilizing our knowledge of addition, we find the number which, when added to 2 makes 3. We say 2 and 1, 3. Write the 1. Treating the next column in the same way, we have 5 and 4, 9. Write the 4. Continuing, 4 and 4, 8; 2 and 5, 7. The remainder is 5441.

$$\begin{array}{r}7893\\ \underline{2452}\\ 5441\end{array}$$

EXERCISE 10 (Written)

Find the remainder:

1. 3582 – 1232.
2. 7328 – 6114.
3. 9439 – 7214.
4. 8358 – 3221.
5. 9473 – 6241.
6. 9004 – 4002.
7. 8384 – 3282.
8. 4435 – 2123.
9. 6279 – 3221.
10. 15798 – 2164.
11. 29003 – 4001.
12. 93800 – 22400.
13. 70009 – 50000.
14. 69000 – 34000.
15. 34456 – 12345.

32. *Example* 1. In order to appreciate the essential nature of subtraction, consider a simple exercise in addition. Find the sum of 2739 and 3874. Writing the numbers so that units stand under units, tens under tens, etc., we have

$$\begin{array}{r} 2739 \\ \underline{3874} \\ 6613 \end{array}$$

Next adding, 4 and 9 are 13. Write 3 and carry 1. Adding the 1 to 7 we get 8, 8 and 3 are 11. Write 1 and carry 1. Adding the 1 to 8 we have 9, 9 and 7 are 16. Write 6 and carry 1, adding the 1 to 3 we get 4, 4 and 2 are 6. The sum is 6613. (The frequent use of *and*, *are* is merely for explanatory purposes.)

Next consider the inverse problem: Given the sum 6613 and one addend 3874, to find the other addend. This problem might be stated thus: (*a*) What number added to 3874 gives 6613? (*b*) What is left over when 3874 is taken away from 6613? (*c*) By how much does 6613 exceed 3874? (*d*) How much less is 3874 than 6613?

$$\begin{array}{r} 6613 \\ \underline{3874} \\ 2739 \end{array}$$

To solve our problem we write the subtrahend 3874 under the minuend 6613. Units should be under units, tens under tens, etc. Draw a horizontal line. Our knowledge of addition tells us that the only digit added to 4 which gives a 3 in the sum is 9. We therefore say 4 and 9 are 13. Write the 9 and carry 1 as we have done in the addition exercise. Adding the 1 to the 7 we see the only digit added to 8 which gives a 1 in the sum is 3. We therefore say 8 and 3 are 11. Write 3 and carry 1. Similarly for the next figure of the answer, 9 and 7 are 16. Write 6 and carry 1. 4 and 2 are 6. Write 2. A little consideration will enable one to see that this method of performing subtraction involves no principle beyond that of carrying in addition.

Example 2. Take 3427 from 4000, arranging the numbers in the usual way.

$$\begin{array}{r} 4000 \\ \underline{3427} \\ 573 \end{array}$$

Here 7 and 3 are 10. Write 3 and carry 1. 3 and 7 are 10. Write 7 and carry 1. 5 and 5 are 10. Write 5 and carry 1. 4 and 0 are 4.

The figure in the remainder is written at the same time that its name is uttered. Thus in the above exercise 7 and 3 are 10, the 3 is written as ten is uttered. 1 is carried in examples like this one whenever the sum is 10 or more than 10.

NOTE. This is the Austrian method of subtraction, known in some English books as the computers' method, and is considered the simplest and most expeditious of all methods.

EXERCISE II (Written)

1. 37 – 19.
2. 54 – 35.
3. 62 – 27.
4. 95 – 58.
5. 101 – 77.
6. 123 – 84.
7. 235 – 117.
8. 309 – 271.
9. 711 – 354.
10. 1010 – 643.
11. 2374 – 1237.
12. 4983 – 2495.
13. 3920 – 2946.
14. 7935 – 2947.
15. 6,838 – 2549.
16. 9,357 – 3449.
17. 4,454 – 3957.
18. 7,429 – 3938.
19. 8,254 – 4255.
20. 6,004 – 3205.
21. 5,001 – 3202.
22. 7,005 – 3006.
23. 9,004 – 6014.
24. 8,002 – 5004.
25. 3,000 – 2123.
26. 4,700 – 2711.
27. 10,000 – 3456.
28. 10,010 – 2734.

29. 11,000 – 7,301.
30. 12,000 – 9,011.
31. 73,288 – 24,299.
32. 46,234 – 32,237.
33. 94,301 – 92,204.
34. 60,100 – 40,127.
35. 84,000 – 79,104.
36. 100,000 – 27,841.
37. 203,406 – 104,409.
38. 507,602 – 208,705.
39. 808,305 – 798,406.
40. 500,207 – 201,209.
41. 600,010 – 399,803.
42. 800,007 – 299,919.
43. 100,000 – 7,314.
44. 400,000 – 4,002.
45. 900,101 – 6,192.
46. 705,208 – 5,703.

NOTE. By utilizing the identity

$$(a-b)+(b-c)+(c-d)+(d-e)=a-e,$$

a teacher can frame any number of examples for drill in subtraction and addition. As an illustration, the following is given:

$$\begin{array}{r} 83284 \\ 64597 \\ \hline 18687 \end{array} \qquad \begin{array}{r} 64597 \\ 43598 \\ \hline 20999 \end{array} \qquad \begin{array}{r} 43598 \\ 25873 \\ \hline 17725 \end{array} \qquad \begin{array}{r} 25873 \\ 24985 \\ \hline 888 \end{array}$$

Here we have four exercises in subtraction: if the work is correct, the sum of the four remainders equals the difference between the first minuend and the last subtrahend. Hence, this example gives four exercises in subtraction and one in addition. Furthermore, if we sum the four minuends, and also sum the four subtrahends, and then take the difference of the two sums, we again get a number equal to the difference of the first minuend and the last subtrahend. This furnishes two exercises in addition and one in subtraction.

EXERCISE 12 (Written)

1. A man owns two houses worth respectively 2390 dollars and 4575 dollars; he has deposited in bank 3280 dollars; he owes two notes, one for 783 dollars, and the other for 870 dollars. How much is he worth?

2. The area of the British Isles is 120,975 square miles; the area of Texas is 265,780 square miles. By how many square miles does the area of Texas exceed the area of the British Isles?

3. In the year 1899 the quantity of coal mined in the United States was 252,115,387 tons; in the same year the number of tons mined in Great Britain and in Germany was respectively 246,495,200 and 144,283,196 tons. How many more tons were mined in the United States than in Great Britain? In Great Britain than in Germany?

4. The population of the Chinese Empire is 402,680,000; of the British Empire, 383,900,000; of the Russian Empire, 129,000,000; of the United States, exclusive of colonial possessions, 76,000,000; of Germany, 52,279,000. How many more people are in the United States than in Germany? In the British Empire than in Russia, United States, and Germany combined? By how many does the population of China exceed the population of Russia, United States, and Germany together?

5. The production of cotton for the year 1898 was —

United States . . .	11,235,383 bales
India	2,275,200 bales
China	1,320,000 bales
Egypt	1,240,000 bales
Korea	400,000 bales

By how much does the total fall short of 17,000,000 bales? How much more cotton was produced in the United States than in the other four countries combined?

6. The total output of coffee for the year 1899 is given for the principal coffee-producing countries as follows:

Brazil	703,890 tons
Venezuela	52,800 tons
Java	45,012 tons
Guatemala	33,331 tons

How many more tons were produced in Brazil than in the other three countries combined?

7. The principal sugar-producing countries and the quantity each produces are given as follows:

Cuba	832,431 tons
Java	546,760 tons
United States	353,976 tons
Hawaii	264,579 tons
Philippine Islands	217,500 tons

Find the difference between the output in the United States and that in each of the other countries.

8. The area of Maine is 33,040 square miles; of New Hampshire, 9305 square miles; of Vermont, 9565 square miles; of Massachusetts, 8315 square miles; of Rhode Island, 1250 square miles; of Connecticut, 4990 square miles; of California, 158,360 square miles. By how many square miles does the area of California exceed the area of the six New England states?

9. What number taken from 10,000 will leave for remainder 2478?

10. By how much does a million exceed ten thousand?

11. A man has a salary of 1600 dollars a year. He spends for board 324 dollars; for room rent, 144 dollars; for clothes, 75 dollars; and for incidentals, 84 dollars. If he saves the rest, how much does he save?

12. In order to build a church 50,000 dollars are required. Five contributors give 12,000 dollars, 2 others give each 4450 dollars. The remainder is made up by general subscription. Find it.

13. A man buys the following articles: bookcase, 55 dollars; chairs, 27 dollars; wardrobe, 38 dollars; stationery, 3 dollars. He pays with twenty-dollar bills. How many does he need? What change should he receive?

14. A man pays a debt with 10 thousand-dollar bills, 3 twenty-dollar bills, 2 ten-dollar bills, and 4 dollars. What is the amount of the debt?

15. In going from Galveston to Chicago by rail, a distance of 1410 miles, a man travels the first day 345 miles; the next day, 201 miles; the third day, 290 miles. How far is he from Chicago at the end of the third day?

CHAPTER IV

MULTIPLICATION

33. *Example.* Consider the following problem in addition: $26 + 26 + 26 + 26 + 26 + 26$. Write the numbers in the usual way.

$$\begin{array}{r}26\\26\\26\\26\\26\\26\\\hline 156\end{array}$$

Here we have six 6's to add. Write 6 and carry 3; next we have six 2's and the 3 which was carried, making in all 15. This work may be shortened. In examples of this character we usually proceed as follows:

$$\begin{array}{r}26\\6\\\hline 156\end{array}$$

Write first the number to be repeatedly added, and underneath it write the number which indicates how many times it is to be taken as an addend. Then say 6 times 6 are 36. Write the 6 and carry 3; next say 6 times 2 are 12, and 12 and 3 are 15. Write the 15. We call this short method of adding **multiplication.** The result is called the **product.**

34. Multiplication of integers is a short method of performing addition when the addends are all equal to one another.

35. The number to be repeatedly added is called the **multiplicand.**

36. The number indicating how often the multiplicand is to be taken as an addend is called the **multiplier.**

37. The result of taking the multiplicand a number of times is called the **product.**

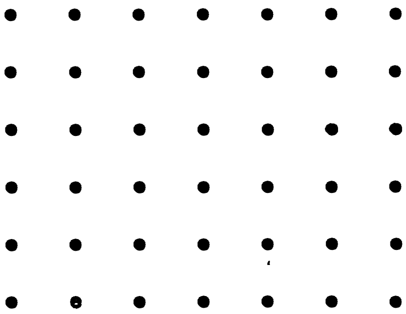
38. The **multiplicand** and **multiplier are** called **factors of the product.**

39. Since the multiplier denotes the number of times an addend is taken, it must always be an abstract number. To take a number 5 feet times is nonsense. How many inches are in 5 feet? In one foot there are 12 inches. In 5 feet there are 5 times as many inches as there are in one foot. Hence, we have 12 inches multiplied by the abstract number, 5.

The **multiplicand** may be either **concrete** or **abstract.** Since in the addition of concrete numbers, we can add only concrete numbers of the same kind, and since the result of such addition is a concrete number of the same kind as the addends, hence, whenever the multiplicand is an abstract number, the product is an abstract number.

To sum up: **The product is concrete or abstract, according as the multiplicand is concrete or abstract. The multiplier can never be a concrete number.**

40. Consider the following arrangement of dots:



Counting the dots in the horizontal rows, we find there are 7 dots in each row, and we find, furthermore, there are

6 horizontal rows. Hence, if we count all the dots, we have 7 + 7 + 7 + 7 + 7 + 7, or 6 times 7. Next count the dots in the vertical lines; we find 6 dots in each line, and there are 7 such lines. Hence, the total number of dots is 7 times 6. Therefore, 6 times 7 = 7 times 6.

In picturing the product of any other two abstract integers we shall always be led to the conclusion: **The product of two abstract integers remains the same, no matter which is taken as multiplier and which as multiplicand.**

41. The sign of multiplication is ×, and is read **multiplied by.** Thus, 12 × 7 means 12 multiplied by 7. This same indicated operation is sometimes read 12 times 7. This may be allowable, since 12 × 7 = 7 × 12.

4 × 5 × 6 means 4 multiplied by 5 and that product multiplied by 6.

NOTE. Before undertaking work in multiplication, pupils should be thoroughly drilled in the Multiplication Table.

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

EXERCISE 13 (Oral)

6 times 8 = ?
6 times 5 = ?
7 times 9 = ?
9 times 8 = ?
9 times 7 = ?
8 times 5 = ?
8 times 12 = ?
8 times 9 = ?
7 times 7 = ?
7 times 8 = ?

7 times 5 = ?
7 times 3 = ?
6 times 9 = ?
6 times 6 = ?
5 times 9 = ?
5 times 8 = ?
5 times 7 = ?
4 times 9 = ?
5 times 9 = ?
6 times 8 = ?

7 times 9 + 6 = ?
7 times 7 + 4 = ?
8 times 9 + 5 = ?
8 times 6 + 6 = ?
9 times 7 + 6 = ?
9 times 9 + 8 = ?
9 times 8 + 7 = ?
8 times 11 + 6 = ?
8 times 4 + 7 = ?
7 times 8 + 5 = ?

EXERCISE 14 (Written)

Multiply:

1. 273 by 2, 3, 5, 7, 8.
2. 378 by 4, 5, 6, 9, 7.
3. 493 by 3, 6, 9, 7, 8.
4. 449 by 5, 8, 11, 9, 7.
5. 2,012 by 3, 5, 8, 9, 7.
6. 4,298 by 4, 6, 9, 7, 10.
7. 5,039 by 3, 4, 6, 8, 7, 9.
8. 34,783 by 8, 4, 9, 7, 6, 5, 11.
9. 29,398 by 2, 4, 6, 8, 10, 12, 7.
10. 39,285 by 9, 11, 12, 8, 7, 6, 5.
11. 44,287 by 8, 10, 7, 11, 12, 9, 7.
12. 89,954 by 4, 6, 8, 10, 12.
13. 31,416 by 5, 3, 4, 2, 8, 6, 9, 12.

42. We have seen that the product of two factors is the same, either factor being taken as multiplier and the other as multiplicand.

Suppose that in the illustration, § 40, each dot represents a group of, say, 11 things. Each horizontal row would then represent 7 × 11 or 11 × 7. Hence the total is 7 × 11 × 6 or 11 × 7 × 6. Also since each vertical line has 6 dots and each one represents a group of 11 things, each vertical line has 6 × 11 or 11 × 6 things. Hence the total is 6 × 11 × 7 or 11 × 6 × 7.

Summing up, we have the same total represented by any one of the four combinations:

$$\begin{array}{r}
7 \times 11 \times 6 \\
11 \times 7 \times 6 \\
6 \times 11 \times 7 \\
11 \times 6 \times 7
\end{array}$$

Illustrations of this character will help one to see the truth of the general law: **The order in which factors are multiplied is immaterial; the product always remains the same.**

This is known as **The Commutative Law for Multiplication,** and is of great importance in subsequent work.

When the multiplier is a number consisting of two figures, and is itself the product of factors less than 12, the multiplication can be conveniently performed by multiplying in succession by the factors of the multiplier. Thus we can multiply a number by 54 by multiplying first by 9 and the resulting product by 6. The reason for the correctness of this process is that 54 equals 9 groups of 6 each. The multiplication by 9 gives the number of groups, and that by 6 gives the number of single things.

EXERCISE 15 (Written)

Multiply by using factors:

1. 672 by 42.
2. 1,728 by 48.
3. 3,136 by 49.
4. 2,025 by 81.
5. 5,103 by 63.
6. 15,625 by 25.
7. 5,832 by 72.
8. 3,136 by 64.
9. 4,032 by 28.
10. 1,225 by 25.
11. 4,536 by 56.
12. 3,087 by 63.
13. 4,928 by 77.
14. 10,164 by 84.
15. 7,776 by 96.
16. 5,376 by 84.
17. 5,324 by 44.
18. 1,498 by 49.
19. 96,428 by 66.
20. 81,126 by 56.
21. 64,325 by 35.

EXERCISE 16 (Written)

1. A barrel of sugar costs 6 dollars. How much will 4 barrels cost? How much will 8 barrels cost? 75 barrels? 93 barrels?

2. A chest of tea costs 15 dollars. How much will 6 chests of the same kind cost? 8 chests? 67 chests? 98 chests?

3. A bag of coffee costs 18 dollars. How much will 7 bags cost? 21 bags?

4. A barrel of pork costs 11 dollars. How much will 8 barrels cost? 25 barrels?

5. How many feet are in 5 yards? In 12 yards? In 28 yards?

6. How many inches are in 7 feet? In 9 feet? In 792 feet?

7. An office desk costs 25 dollars. How much will 3 such desks cost? 8 desks? 36 desks? 49 desks? 83 desks?

8. A book sells for 65 cents. How much will 9 such books cost? 32 books?

9. An acre of land rents for 2 dollars. Find the rent paid for 6 acres. For 94 acres.

10. A barrel of beef costs 13 dollars. Find the cost of 4 barrels. Of 24 barrels. Of 42 barrels.

11. Eggs sell for 26 cents a dozen. Find the cost of 8 dozen. Of 18 dozen.

12. In a mile there are 320 rods. How many rods are in 9 miles? How many are in 27 miles?

13. Oranges sell for 18 cents a dozen. What is the cost of 9 dozen? Of 21 dozen? Of 54 dozen?

14. There are 5280 feet in a mile. How many feet would a person travel in going from Newark to New York City, a distance of 10 miles?

15. Galveston is 1790 miles from New York City. A bicyclist starts from Galveston for New York, traveling at the rate of 49 miles a day. How far will he be from New York at the end of 36 days?

16. Wheat sells at 84 cents a bushel. Find the cost of 17 bushels of wheat.

17. Sugar sells at 6 cents a pound. How much change should a person receive who gives a dollar for 12 pounds?

43. Multiply 3079 by 10. Proceeding as in § 33, we have 10 nines are 90. Write 0 and carry 9. 10 sevens are 70; 70 and 9 are 79. Write 9 and carry 7. 10 naughts are 0; 0 and 7 are 7. Write 7. 10 threes are 30. Write 30.

$$\begin{array}{r} 3079 \\ \underline{10} \\ 30790 \end{array}$$

Notice the multiplicand and product are expressed by the same digits. Notice further that in multiplying by 10 each digit is moved one place to the left. What effect has moving a digit one place to the left upon the value of the digit?

To multiply an integer by 10, move each figure of the multiplicand one place to the left and write naught in the units' place.

44. Multiply 3079 by 100. This may be done by multiplying by 10 twice in succession.

$$\begin{array}{r} 3079 \\ \underline{10} \\ 30790 \\ \underline{10} \\ 307900 \end{array}$$

Notice each figure in the multiplicand is moved two places to the left, and that the units' and tens' places are occupied by naughts.

To multiply by 100, move each figure of the multiplicand two places to the left and write naughts in the tens' and units' places.

Similarly, to multiply by 1000, move each figure in the multiplicand three places to the left, and fill in the units', tens', and hundreds' places with naughts.

45. Multiply 2357 by 400. This may be done by multiplying first by 4 and then the product by 100. This work may be shortened in the following manner:

$$\begin{array}{r} 2347\\ 4\\ \hline 9388\\ 100\\ \hline 938800 \end{array} \qquad \begin{array}{r} 2347\\ 400\\ \hline 938800 \end{array}$$

4 sevens are 28. Write 8 directly under 4, carry 2. 4 fours are 16; 16 and 2 are 18. Write 8, carry 1. 4 threes are 12; 12 and 1 are 13. Write 3, carry 1. 4 twos are 8; 8 and 1 are 9.

The means of procedure in multiplying by any number of tens, by any number of hundreds, by any number of thousands, is exactly similar. Hence the general statement:

To multiply by any number of tens, by any number of hundreds, by any number of thousands, etc.:

First, write the multiplier under the multiplicand so that units stand in the same column with units, tens in the same column with tens, hundreds in the same column with hundreds, etc.

Second, write the first figure of the product directly under the figure of the multiplier which produced it, and then proceed as in § 33.

Third, fill in the vacant places with ciphers, one cipher for tens, two ciphers for hundreds, three ciphers for thousands, four ciphers for ten-thousands.

NOTE. In some text-books the statement is made that annexing a cipher to a number multiplies it by ten, annexing two ciphers multiplies it by one hundred, etc. This is nonsense, as will be seen later.

46. How many are 7 times a number and 3 times a number?

How many are 4 times a number and 9 times a number?

How many are 20 times a number and 7 times a number?

How many are 40 times a number and 6 times a number?

How many are 70 times a number and 8 times a number?

How many are 100 times a number, 10 times a number, and 7 times a number?

How many are 200 times a number, 50 times a number, and 7 times a number?

Multiply 3254 by 346. Here we break up the multiplier into three parts, indicated by the values of its digits, namely, 6, 40, 300. A knowledge of § 33 enables us to multiply by 4; a knowledge of § 45 enables us to multiply by 40, and also by 300.

3254
346
19524 = 6 times multiplicand.
13016 = 40 times multiplicand.
9762 = 300 times multiplicand.
1125884 = 346 times multiplicand.

Here we multiply first by 6; then by 4, as if the 4 were units; lastly by 3, as if the 3 were units. We take care, however, to move one place to the left for tens and two places to the left for hundreds. This we accomplish by writing the first figure of each partial product directly under the figure of the multiplier which produced it. Hence we have the following general rule for multiplication:

First, write multiplicand and multiplier so that units of the same name stand in the same column.

Second, multiply the multiplicand by each digit in the multiplier, placing the first figure of each partial product directly under the digit of the multiplier producing it.

Third, add the partial products.

NOTE. If there are ciphers in the multiplier, no notice should be taken of them, as any number of naughts makes naught.

We might have multiplied first by 300, then by 40, and finally by 6, and then added and obtained the same result.

EXERCISE 17 (Written)

Find the product:

1. 38 × 34.
2. 98 × 51.
3. 99 × 25.
4. 49 × 68.
5. 172 × 29.
6. 392 × 37.
7. 544 × 46.
8. 527 × 58.
9. 963 × 98.
10. 3,775 × 71.
11. 4,009 × 99.
12. 5,040 × 72.
13. 10,659 × 91.
14. 9,783 × 86.
15. 4,419 × 72.
16. 8,838 × 54.
17. 36,238 × 28.
18. 26,514 × 36.
19. 38,308 × 35.
20. 46,267 × 85.
21. 32,603 × 94.
22. 65,206 × 47.
23. 13,536 × 78.
24. 54,144 × 39.
25. 12,195 × 69.
26. 12,400 × 37.
27. 43,400 × 74.
28. 14,038 × 64.
29. 40,563 × 56.
30. 14,108 × 69.
31. 12,600 × 89.
32. 18,456 × 19.
33. 73,864 × 38.
34. 1,584 × 132.
35. 1,728 × 144.
36. 2,368 × 148.
37. 2,058 × 343.
38. 5,096 × 182.
39. 5,348 × 191.
40. 6,336 × 198.
41. 6,552 × 234.
42. 8,802 × 163.
43. 3,472 × 248.
44. 7,056 × 252.
45. 17,721 × 179.
46. 11,529 × 183.
47. 14,925 × 597.
48. 7,152 × 596.
49. 7,018 × 638.
50. 4,764 × 397.
51. 11,956 × 427.
52. 10,728 × 894.
53. 10,736 × 976.
54. 11,904 × 496.
55. 646 × 143 × 15.
56. 17 × 68 × 34.
57. 8,950 × 170.

47. A square whose side is 1 foot is called a square foot. A surface just as large would be said to contain an area of 1 square foot. What is a square yard? A square mile? A square inch?

Draw a square three inches on a side to represent a square yard. Divide each side into three equal parts. Draw straight lines connecting opposite points of division. How many small squares are thus made?

What is the size of each small square? What does each small square represent?

How many square feet are in 1 square yard?

48. A figure having the same shape as a page of this book, or the same shape as the floor of the ordinary room in a dwelling house, is called a rectangle.

Draw a rectangle 4 inches long and 3 inches wide. If the long side is divided into 4 equal parts, and the short side into 3 equal parts, and lines are drawn connecting the opposite points of division, how many squares does the figure contain? What is its area?

1. Draw a figure representing a rectangle 5 feet long and 3 feet wide. How many square feet does it contain?

2. Draw a figure representing a rectangle 4 yards long and 2 yards wide. Find how many square yards it contains.

3. Draw a square. If one of its sides is 6 yards, how many square yards does it contain?

4. A room is 6 yards long and 4 yards wide. How many square yards does the floor of the room contain?

5. Draw on the blackboard a square 1 foot on a side. Mark the inches. Draw lines so as to show the square inches. How many square inches are in the figure?

6. A rectangle is 7 yards long and 6 yards wide. How many square yards are in its area? Make a drawing of the rectangle, and divide it so as to show the square yards.

7. A hall is 12 yards long and 10 yards wide. How many square yards of surface does it contain?

8. A ranch is 5 miles long and 4 miles wide. Find its area in square miles. Make a drawing showing its division into square miles.

EXERCISE 18 (Written)

1. A city block is 100 yards long and 90 yards wide. Find its area.

2. Find the area of a square whose side is 84 yards.

3. Find the area of a square having 320 rods for a side.

4. Find the area of a rectangle, the length being 140 yards and the width 84 yards.

5. Find the area of a rectangle 238 yards long and 96 yards wide.

6. A farm, rectangular in shape, 440 yards long and 380 yards wide, contains how many square yards?

7. Find the area of a rectangle 75 rods long and 63 rods wide.

8. Find the area of a grass plot 240 feet by 84 feet.

9. A sheet of paper 18 inches long and 14 inches wide contains how many square inches?

10. A township is 6 miles long and 6 miles wide. How many square miles does it contain?

11. A county, having the shape of a rectangle, is 24 miles long and 18 miles wide. How many square miles are in its area?

12. A street is 1760 yards long and 23 yards wide. How many square yards does it contain?

13. A garden is 50 yards long and 44 yards wide. How many square yards are in its area?

14. A city lot is 43 feet wide and 124 feet deep. How many square feet are in its area?

15. A window is 60 inches by 48 inches. How many square inches are in its area?

16. A yard is 36 inches. How many square inches are in a square yard?

17. A mile is 1760 yards. How many square yards are in 1 square mile?

18. The playground of a school is 78 yards long and 54 yards wide. Find its area in square yards.

EXERCISE 19 (Written)

1. A degree on the earth's surface is 69 miles long. How many miles are in 17 degrees?

2. Springfield, Ill., is 10 degrees north of New Orleans. How many miles are between these cities?

3. Minneapolis, Minn., is 15 degrees north of Galveston. How far are these cities apart?

4. There are 7 days in a week. How many days are in 39 weeks?

5. There are 1440 minutes in a day. How many minutes are in the month of January?

6. 3240 freight cars are moved in a city in one day. At this rate how many are moved in the month of March?

7. A single line of barbed wire one mile long weighs about 315 pounds. How much will a single line 345 miles long weigh?

8. A railroad 285 miles long has a barbed wire fence on each side of it, each fence having four lines of wire. Find the weight of wire in the fences, supposing a single line of wire 1 mile long to weigh 318 pounds.

9. An architect receives 6 dollars a day. At this rate how much will he receive in 294 days?

10. A contractor pays in wages 78 dollars a day. How much will he pay in 78 days?

11. How many hours are in 1 day? In 89 days?

12. How many minutes are in 1 hour? In 24 hours? How many minutes are in 31 days?

13. There are 320 rods in a mile. How many rods are there from New York to Chicago, distance 920 miles?

14. How many rods are there from Honolulu to San Francisco, a distance of 2392 miles? How many from Paris to New York, a distance of 3624 miles?

15. From New Orleans to Rome is 5440 miles. Express the distance in rods.

16. In a section of land there are 640 acres. How many acres are in a ranch containing 98 sections of land? How many sheep can graze on this ranch, at 4 sheep to an acre?

17. Manila is 6940 miles from San Francisco. Express the distance in rods. (1 mile = 320 rods.)

18. In a long ton there are 2240 pounds. How many pounds are in 147 long tons?

19. A man buys 78 horses at 75 dollars each and sells them at 90 dollars each. How many dollars does he gain?

20. A train travels at the rate of 34 miles an hour. How far will it run in 50 hours if 3 hours are allowed for stops?

21. A train travels at an average rate of 28 miles an hour. How far will it travel in 48 hours at this rate?

22. There are 14 lots in a city block. If each is worth 2375 dollars, what is the value of the block?

23. Hammers are bought at 6 dollars a dozen. How much profit will a dealer make by selling 89 dozen at 8 dollars a dozen?

24. There are 5280 feet in a mile. How many feet are there in 140 miles?

25. There are 63,360 inches in a mile. How many inches are there from Philadelphia to New York, a distance of 90 miles?

26. A cubic foot of rock weighs 148 pounds. How many pounds do 3297 cubic feet weigh?

27. The area of New Hampshire is 9305 square miles. How many acres are in New Hampshire? (1 square mile = 640 acres.)

28. The area of Ohio is 41,060 square miles. How many acres are there in Ohio? There are in Ohio about 101 persons to the square mile. At this rate what is the population of Ohio?

29. The area of New York is 49,170 square miles. How many acres are there in New York?

30. An acre of land yields 18 bushels of wheat. At this rate what would be the yield of 640 acres?

31. A dealer sells 47 horses at 185 dollars each, 17 horses at 136 dollars each, and 28 horses at 96 dollars each. How much in all does he realize?

32. From the following table find the population of each of the states and countries mentioned:

	AREA IN SQUARE MILES	NUMBER OF PERSONS TO THE SQUARE MILE
New York	49,170	148
Massachusetts	8,315	337
Illinois	56,650	85
Louisiana	48,720	29
Japan	161,198	289
Germany	208,830	250
France	204,092	186
England	58,320	500

33. A carpenter receives 3 dollars a day. At this rate how much wages will he receive in 5 years if he works 299 days each year?

34. The rent of a dwelling is 27 dollars a month. Suppose a tenant occupies it for 3 years, how much rent will he pay?

35. In a ship's cargo are 1278 bushels of wheat. Express this quantity in pounds. (1 bushel of wheat weighs 60 pounds.)

36. A speculator buys 3785 sheep at 6 dollars a head. He sells them at 8 dollars a head. Find the profit if he spends in feed 295 dollars and for hire 180 dollars.

37. A clerk receives 125 dollars a month. If he spends 60 dollars a month, how much will he save annually?

38. A gallon of water contains 231 cubic inches. How many cubic inches are in 184 gallons?

39. Havre is about 3310 miles from New York City. A ship sails from New York for Havre on Monday morning at 9 o'clock, at the rate of 19 miles an hour. How far from Havre will it be at 9 o'clock the following Monday morning?

40. A man has 27 twenty-dollar bills, 15 ten-dollar bills, and 7 five-dollar bills. How many dollars has he?

41. A book has 350 pages, each page has 33 lines, and each line 9 words. How many words are in the book?

42. How many hours are there in 52 weeks?

43. An acre contains 160 square rods. How many square rods are there in 640 acres?

44. In a mile there are 1760 yards. How many yards are there in the distance from San Francisco to Yokohama, 5140 miles?

CHAPTER V

DIVISION

How many are 6 times 4 inches? Here 4 inches is the multiplicand, and 6 is the multiplier.

This problem leads at once to two other problems:

(*a*) How many times is 4 inches contained in 24 inches?

(*b*) What number of inches multiplied by 6 gives, as product, 24 inches? In other words, what is the sixth part of 24 inches? In (*a*) we are given the product and the multiplicand, and the multiplier is sought. In (*b*) we are given the product and the multiplier, and the multiplicand is sought. In either case we are given the product of two factors, and also one of those factors, and the other factor is sought.

The process by which this factor is obtained is called **division.**

To sum up, in multiplication two numbers are given and the product is sought. In division the product of two factors together with one of the factors is given, and the other factor is sought. Obviously, division is the inverse of multiplication.

49. Division is the process of finding either factor when the product and the other factor are given.

50. In a division problem, the product is called the **dividend.** The factor given is called the **divisor.** The result (the other factor) is called the **quotient.**

51. Three symbols are used for division, ÷, :, and the horizontal stroke, the dividend being written above the stroke and the divisor below it. Thus 18 ÷ 6, 18 : 6, $\frac{18}{6}$, all indicate that 18 is dividend and 6 divisor.

52. *Example* 1. How many times is 4 dollars contained in 28 dollars? In this case the divisor and dividend are both concrete numbers, while the quotient is an abstract number.

NOTE. The two phases of division must be clearly conceived. The first phase, in which we aim to find how many times one number or quantity is contained in another number or quantity, is often called **measuring**. The other phase in which we find a part of a number or quantity, is called **parting** or **dividing**.

Example 2. Divide the sum of 28 dollars among 7 boys, giving to each of them the same number of dollars. In this instance the divisor is an abstract number because it indicates the number of times a sum of money is to be taken in order to make the 28 dollars. By considering examples like the above, we are led to the following conclusions:

If the divisor is a concrete number, the dividend must be a concrete number of the same denomination, and the quotient is then an abstract number.

If the dividend is a concrete number, and the divisor an abstract number, the quotient is a concrete number of the same denomination as the dividend.

Since the divisor and quotient are factors of the dividend, and since one concrete number can never be multiplied by another concrete number, therefore **the divisor and quotient can never be both, at the same time, concrete numbers.**

To bring out the distinction between measuring and parting, and to impress firmly the idea of division, drill on the following examples :

EXERCISE 20 (Oral)

1. How many 3's in 12? 21? 24? 36? 18? 27?

2. How many 4's in 16? 24? 36? 40? 44? 48? 32?

3. How many 5's in 15? 25? 35? 40? 50? 60? 55?

4. How many 6's in 12? 30? 48? 60? 72? 42? 24?

5. How many 7's in 21? 35? 49? 56? 70? 84? 42?

6. How many 8's in 16? 32? 48? 64? 80? 72? 40?

7. How many 9's in 27? 54? 81? 72? 90? 108? 99?

8. How many 11's in 44? 55? 66? 88? 110? 132? 121?

9. How many 12's in 24? 36? 72? 84? 108? 120? 144?

10. What is the third part of 18? 24? 36? 21? 15? 12?

11. What is the fourth part of 24? 36? 44? 16? 20?

12. What is the fifth part of 30? 45? 60? 40? 35?

13. What is the sixth part of 18? 30? 42? 54? 60? 72?

14. What is the seventh part of 14? 28? 35? 49? 56? 63?

15. What is the eighth part of 24? 40? 56? 64? 80? 96?

16. What is the ninth part of 27? 45? 72? 81? 108? 36?

17. What is the tenth part of 90? 80? 70? 120? 110?

18. What is the eleventh part of 44? 66? 77? 110? 121? 132?

19. What is the twelfth part of 48? 60? 84? 96? 108? 132?

20. Divide 48 dollars equally among 4 boys. Among 8 boys. Among 12 boys. Among 6 boys.

21. If apples cost 12 cents a dozen, how many dozen can be bought for 60 cents?

22. Oranges cost 3 cents apiece. How many can be bought for 15 cents? For 30 cents? For 36 cents?

23. If sheep are bought at 6 dollars apiece, how many can be bought for 18 dollars? For 24 dollars? For 33 dollars? For 21 dollars? For 39 dollars?

24. If railroad picks cost 9 dollars a dozen, how many dozen can be purchased for 48 dollars? 72 dollars? 84 dollars? 120 dollars? 96 dollars?

25. Ice-cream freezers cost 4 dollars apiece. At this rate how many can be bought for 28 dollars? 16 dollars? 48 dollars? 24 dollars? 32 dollars?

26. Wrought wrenches sell for 9 dollars a dozen. How many dozen can be purchased with 27 dollars? 81 dollars? 36 dollars? 45 dollars? 63 dollars?

27. How many times is 4 yards contained in 28 yards? 36 yards? 48 yards?

28. Wood sells for 6 dollars a cord. How many cords can be bought for 48 dollars? For 72 dollars? For 54 dollars?

29. Sugar is worth 9 cents a pound. How many pounds can be bought for 81 cents? For 108 cents?

30. Divide 24 cents equally between 2 boys.

31. If hogs sell for 12 dollars apiece, how many can be bought for 96 dollars? For 120 dollars?

32. How many times is 11 square yards contained in 121 square yards?

53. *Example* 1. If lard is worth 12 cents a pound, how many pounds can I buy for 50 cents? Here the divisor is not contained an exact number of times in the dividend. I can buy 4 pounds. For these I pay 48 cents, and get back 2 cents in change.

Example 2. What is the sixth part of 51? Here the divisor is not contained an exact number of times in the dividend. 6 is contained in 51 eight times, with 3 left over.

Whenever the divisor is not contained an exact number of times in the dividend, what is left over is called the **remainder.**

The remainder being the undivided portion of the dividend must be a number of the same denomination as the dividend.

54. Division is called **short division** when the divisor is 12 or less than 12.

EXERCISE 21 (Oral)

Find quotients and remainders:

1. 11 ÷ 3.
2. 13 ÷ 4.
3. 20 ÷ 3.
4. 27 ÷ 4.
5. 31 ÷ 5.
6. 35 ÷ 6.
7. 39 ÷ 7.
8. 40 ÷ 6.
9. 41 ÷ 5.
10. 45 ÷ 8.
11. 49 ÷ 9.
12. 50 ÷ 11.
13. 55 ÷ 12.
14. 60 ÷ 9.
15. 70 ÷ 8.
16. 79 ÷ 10.
17. 81 ÷ 8.
18. 87 ÷ 6.
19. 88 ÷ 7.
20. 89 ÷ 9.
21. 91 ÷ 10.
22. 93 ÷ 12.
23. 99 ÷ 7.
24. 101 ÷ 11.
25. 109 ÷ 12.
26. 111 ÷ 11.
27. 114 ÷ 4.
28. 119 ÷ 9.
29. 121 ÷ 11.
30. 25 ÷ 6.
31. 30 ÷ 4.
32. 39 ÷ 7.
33. 44 ÷ 7.
34. 32 ÷ 8.
35. 29 ÷ 9.
36. 124 ÷ 11.
37. 118 ÷ 10.
38. 117 ÷ 12.
39. 115 ÷ 9.
40. 109 ÷ 10.

41. 173 ÷ 5.
42. 135 ÷ 4.
43. 200 ÷ 9.
44. 141 ÷ 7.
45. 217 ÷ 6.
46. 103 ÷ 4.
47. 234 ÷ 10.
48. 231 ÷ 11.
49. 150 ÷ 12.
50. 165 ÷ 8.
51. 182 ÷ 3.
52. 140 ÷ 12.

SHORT DIVISION

55. *Example* 1. Divide 3346 by 7.

Suppose the dividend consists of 3 thousand-dollar bills, 3 hundred-dollar bills, 4 ten-dollar bills, and 6 one-dollar bills, and we are asked to divide the 3346 dollars equally among 7 persons. First, divide the bills of largest denomination. There are not enough thousand-dollar bills to give each person one; we therefore change the 3 thousand-dollar bills to hundred-dollar bills, and get 30 hundred-dollar bills. Thirty and the 3 we already have make 33. Seven into 33 goes 4 times, and leaves 5 over. These 5 hundred-dollar bills we change to ten-dollar bills; doing so, we get 50 ten-dollar bills which, with the 4 we have, make 54 ten-dollar bills. Seven into 54 goes 7 times and leaves 5 over. Changing the 5 ten-dollar bills left over, and adding the 6 one-dollar bills, we get 56 dollars. Seven into 56, 8 times. Each person, therefore, gets 4 hundred-dollar bills, 7 ten-dollar bills, and 8 dollars, 478 dollars in all. Hence, 3346 ÷ 7 = 478.

$$\begin{array}{r} 7\underline{)3346} \\ 478 \end{array}$$

Looking over our work, we see at once that the actual work can be shortened. Notice the quotient is obtained by dividing 7 into 33, 54, and 56. Hence, we may proceed as follows: First, write the divisor to the left of the dividend and draw a horizontal line. Next, say 7 into 33 goes 4 times and 5 over. Write 4. Annex the remainder, 5, to the next figure of the dividend; this gives us 54. Seven into 54 goes 7 times and 5 over. Write the 7 and annex the remainder to the next figure of the dividend; this gives us 56. Seven into 56 goes 8 times. In writing the quotient, it is well to write each quotient figure directly under the right-hand figure of the number producing it. Thus, in our example, we write 4, 7, 8 directly under the units' figures of the 33, 55, 56, the numbers producing them.

Example **2.** Divide 220,234 by 5.

$$\begin{array}{r} 5\underline{)220234} \\ 44046\text{, remainder 4.} \end{array}$$

Here we say 5 into 24, 4 times and 4 over. Write the 4, annex the remainder, 2, to the next figure, 0. Five into 20 goes 4 times. Write 4. As there is no remainder, there is no figure to annex. Five into 2 goes 0 times. Write 0 and annex the remainder, 2, to the next figure, 5. Five into 23 goes 4 times and 3 over. Next, 5 into 34 goes 6 times and 4 over.

56. In Example 2, above, the divisor is not contained an exact number of times in the dividend. The quotient is written 44,046, remainder 4. The divisor would be contained an exact number of times, if the dividend was 220,230. In cases where the division is not exact, dividend = quotient × divisor + remainder. There is another way of expressing the quotient in Example 2. It is $44{,}046\frac{4}{5}$, which signifies that the divisor is contained in the dividend 44,046 times, and that 4 remains to be divided by 5. The quotient is $44{,}046\frac{4}{5}$. In this way of interpreting results, the dividend = divisor × quotient in all cases.

57. The best way to prove an exercise in division is to multiply the quotient by the divisor, and to this product add the remainder. The result should be equal to the dividend; if not, there is an error.

EXERCISE 22 (Written)

Divide and prove the correctness of your answers:

1. 478 ÷ 2.
2. 475 ÷ 3.
3. 464 ÷ 4.
4. 995 ÷ 5.
5. 696 ÷ 6.
6. 3,458 ÷ 7.
7. 4,988 ÷ 8.
8. 3,905 ÷ 7.
9. 7,854 ÷ 6.
10. 9,270 ÷ 8.
11. 54,239 ÷ 6.
12. 62,849 ÷ 7.
13. 74,753 ÷ 9.
14. 52,308 ÷ 4.
15. 98,157 ÷ 6.
16. 29,293 ÷ 5.
17. 14,156 ÷ 8.
18. 98,984 ÷ 3.
19. 92,735 ÷ 4.
20. 29,238 ÷ 7.
21. 94,001 ÷ 8.
22. 29,935 ÷ 9.
23. 73,245 ÷ 9.
24. 99,879 ÷ 2.
25. 799,680 ÷ 3.
26. 799,680 ÷ 7.
27. 38,576 ÷ 6.
28. 24,568 ÷ 4.
29. 64,739 ÷ 5.
30. 99,948 ÷ 6.
31. 100,000 ÷ 7.
32. 100,023 ÷ 8.
33. 275,987 ÷ 7.
34. 457,854 ÷ 4.
35. 903,904 ÷ 5.
36. 782,653 ÷ 6.
37. 867,508 ÷ 3.
38. 792,846 ÷ 10.
39. 673,584 ÷ 10.
40. 532,573 ÷ 11.
41. 493,594 ÷ 11.
42. 947,835 ÷ 12.
43. 897,631 ÷ 12.
44. 657,394 ÷ 12.
45. 393,948 ÷ 12.
46. 923,541 ÷ 12.
47. 100,000 ÷ 12.
48. 345,678 ÷ 12.

58. In case the divisor is a number of two figures which can be broken up into two factors, each less than 12, the division can be performed by dividing by the factors of the divisor in succession.

Example 1. Divide 78,253 by 24.

$$\begin{array}{r|l|l} 6 & 78253 & \\ \hline 4 & 13042,\ 1 & \\ \hline & \ \ 3260,\ 2 & 13 \end{array}$$

The first remainder consists of units, the second remainder consists of two groups of 6 each. Hence, the correct remainder is 1 + 2 sixes = 13. The remainder will always be obtained by adding the first remainder to the product of the second remainder and the first divisor.

EXERCISE 23 (Written)

Divide and prove your answers correct:

1. 77,354 ÷ 16.
2. 79,358 ÷ 18.
3. 97,854 ÷ 20.
4. 92,738 ÷ 21.
5. 100,000 ÷ 24.
6. 73,948 ÷ 25.
7. 69,593 ÷ 28.
8. 85,376 ÷ 32.
9. 99,392 ÷ 36.
10. 59,738 ÷ 35.
11. 49,399 ÷ 40.
12. 99,988 ÷ 42.
13. 68,698 ÷ 48.
14. 123,456 ÷ 49.
15. 876,543 ÷ 50.
16. 789,295 ÷ 54.
17. 828,374 ÷ 56.
18. 528,739 ÷ 64.
19. 629,394 ÷ 72.
20. 273,579 ÷ 81.
21. 179,246 ÷ 84.
22. 739,264 ÷ 90.
23. 543,293 ÷ 56.
24. 665,670 ÷ 81.

EXERCISE 24 (Written)

1. When sugar sells for 6 cents per pound, how many pounds can be bought for 84 cents?

2. If a boy walks at the rate of 3 miles per hour, how long will it take him to walk 87 miles?

3. In a peck there are 8 quarts. How many pecks are there in 3000 quarts?

4. If a bicyclist rides 9 miles an hour, how many hours will it take him to go from St. Louis to Indianapolis, a distance of 265 miles? After riding 19 hours, how far from Indianapolis will he be?

5. When coal costs 9 dollars a ton, how many tons can be bought for 3456 dollars?

6. A teacher receives a salary at the rate of 4 dollars a day for every day he teaches. His yearly salary is 716 dollars. How many days are in the school year?

7. A brick mason receives 4 dollars a day for every day he works. How many days must he work to earn 900 dollars?

8. How many feet are there in 25,000 inches?

9. How many weeks are there in 364 days?

10. If a dozen penknives cost 9 dollars, how many dozen penknives can be bought for 126 dollars?

11. Plows cost 12 dollars a piece. How many can be bought for 192 dollars?

12. A box of oranges is worth 3 dollars. How many such boxes can be bought for 111 dollars?

13. When a barrel of pork sells for 12 dollars, how many barrels must be sold to realize 5004 dollars?

14. A section foreman rides on a velocipede at the rate of 11 miles an hour. How long will it take him to go from Cincinnati to Cleveland, a distance of 264 miles?

15. Divide 1000 dollars among 8 persons, giving to each the same sum of money.

16. A flock of sheep sells for 966 dollars. How many sheep are in the flock, if each sheep sells for 6 dollars?

17. A man has 795 dollars in 5-dollar gold pieces. How many coins has he?

18. Hogs sell for 8 dollars apiece. At this price how many can be purchased for 360 dollars?

19. A box of soap is listed at 4 dollars. How many such boxes can be purchased for 980 dollars?

20. How many barrels of flour can be bought for 1002 dollars, when flour sells for 6 dollars a barrel?

21. Oyster crackers cost 5 cents a pound. How many pounds can be bought for 95 cents?

22. By buying horses at 75 dollars each and selling them at 84 dollars each, a jobber makes a profit of 324 dollars. How many does he sell?

23. A farmer rents land at 3 dollars an acre per annum. His yearly rent is 787 dollars. How many acres does he rent?

24. A mile equals 63,360 inches. How many feet are in a mile? In 7 miles?

25. A farmer exchanges 125 acres of land worth 50 dollars an acre for 5 city lots. Find the price of each lot.

26. When wood sells for 6 dollars a cord, how many cords must be sold to purchase 18 barrels of sugar at 16 dollars a barrel?

LONG DIVISION

59. *Example* 1. Divide 72,634 by 83. Here the divisor is too large for the work to be performed mentally as it is in short division. We do not know the multiplication table up to 83, and if we did, there would still be trouble in performing the subtractions. In this instance we proceed as follows:

```
     875
83)72634
   664
   ---
    623
    581
    ---
     424
     415
     ---
       9
```

83 is not contained in 7, nor in 72. Our first partial dividend is 726. By trial, we find 83 goes into 726, 8 times. We multiply the divisor by 8 and write the result, 664, under 726. Subtract 664 from 726, take down the next figure of the dividend. The first figure of the quotient, 8, is written directly over the right-hand figure of the first partial dividend. 83 into 623 goes 7 times. Write the 7 directly over 3. Multiply the divisor by 7. Write the result under 623 and subtract it from 623. Take down the next figure of the dividend. 83 into 424 goes 5 times. Write the 5 above 4 of the dividend. Multiply the divisor by 5 and subtract the result from 424. The quotient is 875 and the remainder is 9.

In this example the three partial dividends are 726 hundreds, 623 tens, and 424 units. 83 goes into 726 hundreds 8 hundreds times. By writing the 8, as above indicated, we see at once its value. Similarly for the other digits of the quotient, their positions indicate at once their values.

Example 2. Divide 400,123 by 39.

```
    10259
39)400123
   39
   ---
    101
     78
    ---
     232
     195
     ---
      373
      351
      ---
       22
```

39 goes into 40, 1 time. Write 1 and multiply 39 by 1. Subtract the result from 40. Take down the next figure, 0. 39 does not go into 10. Write 0 in the quotient and take down the next figure, 1. 39 into 101 goes 2 times. Write 2 in the quotient. Multiply 39 by 2 and subtract the product from 101. Take down the next figure, 2. Since 39 is near 40, we get the next figure most readily by trying mentally 40 into 232. It goes 5 times. Multiply 5 by 39. Subtract the result from 232. Write 5 in the quotient and take down the next figure. 39 into 373 goes 9 times. Write 9 in the quotient. Multiply 39 by 9 and subtract. The quotient is 10,259, and the remainder is 22. The remainder must, in every case, be less than the divisor. If the remainder happens, in any instance, to equal or exceed the divisor, then the quotient figure, which gave rise to it, is less than it should be.

EXERCISE 25 (Written)

Divide and prove in each case:

1. $252 \div 14$.
2. $288 \div 16$.
3. $357 \div 17$.
4. $396 \div 18$.
5. $441 \div 21$.
6. $575 \div 23$.

7. 2,448 ÷ 24.
8. 2,925 ÷ 25.
9. 4,096 ÷ 32.
10. 3,468 ÷ 34.
11. 5,476 ÷ 37.
12. 4,332 ÷ 38.
13. 5,547 ÷ 43.
14. 8,464 ÷ 46.
15. 9,608 ÷ 49.
16. 8,440 ÷ 53.
17. 19,494 ÷ 57.
18. 22,460 ÷ 67.
19. 33,330 ÷ 69.
20. 42,650 ÷ 73.
21. 38,350 ÷ 74.
22. 40,441 ÷ 76.
23. 62,057 ÷ 83.
24. 44,400 ÷ 86.
25. 60,600 ÷ 87.
26. 79,541 ÷ 94.
27. 45,140 ÷ 95.
28. 84,700 ÷ 97.
29. 34,242 ÷ 78.
30. 110,768 ÷ 86.
31. 254,736 ÷ 29.
32. 243,363 ÷ 69.
33. 190,213 ÷ 67.
34. 127,368 ÷ 58.
35. 352,002 ÷ 78.
36. 3,064,692 ÷ 47.
37. 971,560 ÷ 107.
38. 268,025 ÷ 355.
39. 969,969 ÷ 323.
40. 138,924 ÷ 908.
41. 804,075 ÷ 151.
42. 1,588,726 ÷ 234.
43. 3,621,600 ÷ 534.
44. 746,496 ÷ 432.
45. 529,984 ÷ 364.
46. 976,150 ÷ 247.
47. 874,240 ÷ 187.
48. 554,724 ÷ 342.
49. 277,425 ÷ 225.
50. 499,896 ÷ 524.
51. 184,041 ÷ 117.
52. 987,392 ÷ 896.
53. 298,112 ÷ 236.
54. 787,368 ÷ 636.
55. 691,440 ÷ 516.
56. 747,273 ÷ 669.
57. 336,960 ÷ 480.
58. 964,877 ÷ 817.
59. 970,045 ÷ 865.
60. 997,640 ÷ 980.

EXERCISE 26 (Written)

1. How many bags of Rio coffee can be bought for 882 dollars, if one bag costs 21 dollars?

2. Currants sell for 14 dollars a barrel; at this price how many barrels can be bought for 546 dollars?

3. Granulated sugar is worth 16 dollars a barrel. How many barrels must be sold to realize 1264 dollars?

4. There are 36 inches in one yard. How many yards are in 100,000 inches?

5. There are 32 quarts in one bushel. How many bushels are in 7712 quarts?

6. How many days are in 3000 hours?

7. A degree on a meridian of the earth's surface is 69 miles long. Two places on the same meridian are 2484 miles apart. How many degrees apart are they?

8. How many square yards are in 3276 square feet?

9. A gallon contains 231 cubic inches. How many gallons are in a barrel containing 8316 cubic inches?

10. Oolong tea costs 15 dollars a chest. How many chests can be purchased for 495 dollars?

11. A barrel of sugar weighs 325 pounds. How many barrels are in 105,625 pounds?

12. How long will it take a train, rate 30 miles an hour, to go from New York to San Francisco, a distance of 3270 miles, if 5 hours are allowed for stops?

13. There are 10 square chains in an acre. How many acres are in 10,000 square chains?

14. Rhode Island contains in round numbers 800,000 acres. Find its area in square miles. (640 acres = 1 square mile.)

15. The area of Ireland is 20,000,000 acres. Express its area in square miles.

16. The island of Guam has an area of 150 square miles. The area of Porto Rico is 3520 square miles. How many times larger is Porto Rico than Guam?

17. The area of Texas is 265,780 square miles. How many times larger is Texas than Porto Rico?

18. Sound travels at the rate of 1100 feet per second. How many seconds does it take sound to travel 6 miles?

19. Oregon has an area of 96,030 square miles. How many times larger is Oregon than Delaware whose area is 2050 square miles?

20. The area of Massachusetts is 8315 square miles, and the area of Alaska is 590,880 square miles. How many times larger is Alaska than Massachusetts?

21. The area of California is 158,360 square miles, and the area of Hawaii 6449 square miles. How many times larger is California than Hawaii?

22. The area of Connecticut is 4990 square miles; its population is 908,420. How many persons are there to the square mile in Connecticut?

23. From Paris to Moscow is 1843 miles. How long would it take a person to travel between those cities, traveling at the rate of 29 miles an hour?

24. From Paris to Constantinople is 1900 miles. A train leaving Paris reaches Constantinople in 64 hours. Find its average speed.

25. The distance from Berlin to Rome, 1048 miles, is made in 39 hours. Find the rate per hour.

26. The distance from Baltimore to Queenstown is 3118 knots. How long will it take a steamer to go from one port to the other, supposing its rate is 18 knots per hour?

27. The British Admiralty knot is 6080 feet. How many knots are there in 69 miles? (1 mile = 5280 feet.)

28. 193 francs are equivalent to 10 dollars. How many 10-dollar gold pieces are equivalent to 14,475 francs?

EXERCISE 27 (Review)

1. The acreage, production, and value of barley in five New England states in 1898 were:

	ACRES	BUSHELS	VALUE
Maine	12,109	326,943	183,088 dollars
New Hampshire. .	4,915	115,502	66,991 dollars
Vermont	17,739	532,170	250,120 dollars
Massachusetts . .	1,712	41,994	27,683 dollars
Rhode Island. . .	318	8,904	5,431 dollars

(*a*) Find the total number of acres.

(*b*) Find the total number of bushels.

(*c*) Find the total value.

(*d*) Find the average number of bushels produced by an acre of land in each of the five states.

(*e*) What was the crop worth in each state per acre?

2. In 1898 the number of tons of coal mined in Belgium was 21,492,446; this was 240,076 tons more than in 1896. How many tons were mined in Belgium in 1896?

3. The area of the Bermuda Islands is 20 square miles, and the population is 15,960; the whites number 6117. Find the number belonging to other races. Find the number of persons to a square mile.

4. During the three calendar years immediately preceding 1898, the coffee export from Brazil was as follows:

	1895	1896	1897
From Santos . . .	3,554,696 bags	4,156,567 bags	5,621,762 bags
From Rio de Janeiro	2,763,720 bags	2,784,958 bags	4,066,734 bags
From Victoria . .	307,438 bags	273,225 bags	372,221 bags
From Bahia . . .	264,775 bags	260,981 bags	292,480 bags
From Ceara . . .	20,202 bags	6,000 bags	6,568 bags

Add vertically and horizontally and verify your work by summing the vertical and horizontal totals.

5. In Example 4 find the increase in the total export for 1896 over that of 1895; also for the year 1897 over 1896.

6. In 1898 Brazil exported 11,360,173 bags of coffee. Find the increase in production for the year 1898 over each of the three previous years.

7. If coffee is worth 9 dollars a bag, find the value of the export for each of the 4 years above given.

8. The value of building stones produced in the United States in 1897 was:

Granite	8,905,075 dollars
Marble	3,870,584 dollars
Slate	3,524,614 dollars
Sandstone	4,065,445 dollars
Limestone	14,804,943 dollars
Bluestone	900,000 dollars

Find the total value.

9. In 1898 Delaware had 321 acres of buckwheat which yielded 5296 bushels valued at 2118 dollars. Find the number of bushels each acre produced.

10. The most valuable clay products of the United States for 1897 were:

Common bricks	26,353,904 dollars
Pressed bricks	3,931,336 dollars
Paving bricks	3,582,037 dollars
Fire bricks	4,094,704 dollars
Drain tile	2,623,305 dollars
Sewer pipe	4,069,534 dollars
Fire proofing	1,979,259 dollars
Pottery	9,450,859 dollars

Find the total value.

11. Connecticut had, in 1898, 1450 factories in operation, employing 136,200 persons. Find the average number of persons employed in a factory.

12. The following table gives the acreage, production, and value of corn for the year 1898 in the states named:

STATE	ACREAGE	PRODUCTION	VALUE
Kansas	8,302,628	132,842,048	$34,538,932
Nebraska	7,559,746	158,754,666	34,926,027
Iowa	7,285,710	254,999,850	58,649,966
Illinois	6,665,327	199,959,810	49,989,952
Missouri	5,951,211	154,731,486	41,777,501
Texas	4,213,468	105,336,700	35,814,478
Indiana	3,587,627	129,154,572	32,288,643
Georgia	2,954,072	26,586,648	12,761,591
Tennessee	2,941,067	76,467,742	22,175,645
Kentucky	2,747,653	85,177,243	22,997,856
Alabama	2,645,442	39,681,630	16,269,469
Ohio	2,779,147	102,828,439	27,763,679

Find the total number of acres, number of bushels, and their value.

13. How many more bushels did Iowa produce than Illinois?

14. How many more bushels did Texas produce than Ohio?

15. How many more acres in corn had Kansas than each of the other states given?

16. The total value of the exports of cotton fabrics, made in the United States, was, in 1897, 21,037,680 dollars; in 1898, it was 17,024,093 dollars. How much less was the value of exports in 1898 than in 1897?

17. The average daily attendance for all the public schools of Texas was, for the scholastic year 1895–96, 440,249, and the number of teachers for the same year was 13,217. Find the average number of pupils to one teacher.

18. There were, in 1897, 164 normal schools in the United States. The number of students attending these schools was, for the same year, 43,199. What was the average number of students in a normal school?

19. The value of the gold mined in 1898, in the principal gold-producing countries, is given as follows:

United States . . .	64,300,000 dollars
Canada	14,190,000 dollars
Mexico	7,668,866 dollars
Colombia	3,900,000 dollars
Austria-Hungary . .	2,178,556 dollars
Russia	25,136,994 dollars
China	6,641,190 dollars
British India . . .	7,753,150 dollars
South Africa . . .	79,329,443 dollars

Find the total value. The world's output of gold for the same year was 286,218,954 dollars. Find the value of the gold mined in all other countries.

20. The estimated area of Arizona is 54,369,023 acres. Of these 29,847,000 acres are grazing land and 15,000,000 acres are barren. The remainder is woodland. How many acres are in woodland?

21. The total population of the world is estimated at 1,429,682,189. The number of Christians is estimated at 477,080,158. How many are not Christians?

22. The value of silks imported into the United States in 1897 was 25,199,067 dollars. The value for the preceding year was 26,652,768 dollars. How much less was the expenditure for silks in 1897 than in 1896?

23. In the Spanish-American War the number of deaths in the American army from all causes, between May 1 and September 30, 1898, were: killed, 23 officers and 257 enlisted men; died of wounds, 4 officers and 61 enlisted men; died of disease, 80 officers and 2485 enlisted men. The total number of men enlisted was 274,717. Find the total number of deaths and the number of men that returned unharmed.

REVIEW QUESTIONS

1. What is arithmetic?

2. What is the first fundamental operation in arithmetic?

3. Give the derivation of calculate, digit, arithmetic, number.

4. Define number, unit, integer, quantity, numeration, notation.

5. Explain the terms concrete, abstract.

6. Name the digits.

7. Discriminate between digit and figure.

8. Is there any exception to the statement that the value of a figure depends on the position it occupies?

9. Upon what does the value of a digit depend?

10. How do we read numbers?

11. Define addition, addend, sum.

12. Give the rule for addition of integers.

13. Define subtraction, difference, remainder, subtrahend, minuend.

14. What relation exists between minuend, subtrahend, and remainder?

15. Explain the statement, "subtraction is the inverse of addition."

16. Define multiplication, multiplicand, multiplier, product.

17. Define factor.

18. When is a product concrete? When is a product abstract?

19. What is the commutative law for multiplication?

20. How do we multiply by 10? By 100? By 1000?

21. How do we multiply by 70? 700? 7000?

22. How do we multiply by using factors of the multiplier?

23. Give the rule for multiplication when the multiplier consists of two or more figures.

24. Define division, dividend, divisor, quotient.

25. What is the distinction between measuring and parting?

26. "Division is the inverse of multiplication;" explain.

27. In a division exercise when is the quotient abstract and when concrete?

28. What is meant by short division? long division?

29. Give the rule for long division.

30. Explain how division is performed by using factors of the divisor.

31. In this case give the rule for obtaining the correct remainder.

32. How do you prove an exercise in addition?

33. How do you prove an exercise in subtraction? How do you prove an exercise in multiplication? How do you prove an exercise in division?

34. In an exercise in multiplication which of the three terms, multiplicand, multiplier, and product, can never be a concrete number? Why?

CHAPTER VI

4

UNITED STATES MONEY

60. The coins used as money in the United States and the materials from which they are made are:

Bronze: 1-cent piece.
Nickel: 5-cent piece.
Silver: dime, quarter dollar, half dollar, dollar.
Gold: quarter eagle, half eagle, eagle, double eagle.

61. The **United States unit of money** is the gold dollar. The dollar symbol is $. It is always written before the number specifying how many dollars.

62.
10 mills (m.) = 1 cent (ct. ¢)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)
10 dollars = 1 eagle (E.)
20 dollars = 1 double eagle

There are no mills coined. They occur, however, in various questions of calculation. Thus, we have such questions as the following:

What is the amount of a person's tax when the rate is 5 mills on the dollar and the value of his property is 4000 dollars?

63. Notation of United States money.

To understand this, consider the notation of the decimal number system.

3333. If we write a single digit, 3, this represents 3 units of some kind. If we write a second 3 to the left of this one, this second 3 represents 3 tens, that is ten times as much as the first 3. If we write a third 3 to the left of the second one, this 3 represents ten times as much as the second 3. A fourth 3 written to the left of the third 3 would represent ten times as much as the third 3, and so on. A digit increases tenfold in value as we go to the left.

Begin now to the left of 3333 and go to the right. The second 3, *i.e.* the 3 immediately to the right of the one on the extreme left, is ten times less in value than the one to its left. The next 3 is ten times less in value than the one to its left. The next 3 is ten times less in value than the 3 to its left. Hence, as we go to the right, the value of a digit becomes 10 times less. Write now a 3 to the right of the units' place, separating it from the units' place by a period, and we have

3333.3

To preserve the unity of our decimal system of notation we agree to give this a value, so that it will be ten times less than the same digit to its left. Similarly a 3 to the right of this 3 represents a number ten times less than the 3 to its left. Another 3 to the right of this second 3, from the units' place, is ten times less than the 3 in the second place from the units, and so on indefinitely.

Take now the number 3333.333.

The right-hand figure represents 3 units of some kind; the next digit to the left represents a number ten times as great; the next one is ten times as great as the one to its right, and so on.

This is the law on which our number system is constructed. What has this to do with United States money?

The units of the United States money are made out on the same principle. Notice:

10 mills = 1 cent
10 cents = 1 dime
10 dimes = 1 dollar

A dime is ten times less in value than a dollar, a cent is ten times less in value than a dime, and a mill is ten times less in value than a cent. Hence dimes can be adequately represented by being written in the first place to the right of units' place, cents in the place next on the right, and mills in the next place to the right. Thus

$42.345

will represent 42 dollars, 3 dimes, 4 cents, and 5 mills. To repeat, dimes are written in the first place to the right of units' place, cents in the second place to the right of units' place, and mills in the third place to the right of units' place.

64. The dot to the right of the units' place is called the **decimal point.**

65. In reading sums of money, consisting of different denominations, we rarely hear any denominations spoken of except dollars and cents. Instead of a dime, we say 10 cents; instead of 2 dimes, we say 20 cents; instead of 3 dimes, we say 30 cents, and so on. Instead of five eagles, we say 50 dollars.

66. Read $40.25. This is read 40 dollars and 25 cents.

Read $20.034. This is read 20 dollars, 3 cents, and 4 mills.

EXERCISE 28 (Oral)

Read:

\$ 19.96
\$ 24.08
\$101.10
\$ 89.90
\$ 40.004
\$182.203
\$ 18.18
\$ 38.01
\$1280.34
\$3490.001
\$ 90.09
\$ 100.10

EXERCISE 29 (Written)

Write:

12 dollars and 20 cents
108 dollars and 18 cents
50 dollars and 50 cents
90 dollars and 4 cents
300 dollars and 10 cents
400 dollars, 4 cents, 4 mills
772 dollars, 3 cents, 7 mills
120 dollars and 5 cents
290 dollars, 8 cents, 9 mills
500 dollars, 12 cents, 2 mills
1000 dollars, 90 cents, 9 mills

67. **Reduction** is the process of changing concrete quantities from units of one denomination to units of another denomination.

68. *Example* 1. Reduce \$15.65 to cents.

Since 1 dime = 10 cents,
10 dimes = 100 cents,
but 10 dimes = \$1.
Hence, \$1 = 100 cents;
\$15 = 1500 cents;
adding the 65 cents, \$15.65 = 1565 cents.

Example 2. Reduce \$80.60 to cents.

\$1 = 100 cents;
\$80 = 8000 cents;
adding 60 cents, \$80.60 = 8060 cents.

To reduce dollars and cents to cents, we multiply the number of dollars by 100 and add the cents. This operation is at once accomplished by moving the decimal point two places to the right.

69. *Example.* Reduce $27.345 to mills.

First reduce this amount to cents, and we have by moving the decimal point two places to the right 2734.5¢, *i.e.* 2734 cents and 5 mills. Now 1 cent = 10 mills, therefore 2734 cents = 27340 mills; adding the 5 mills, 2734.5 cents = 27345 mills.

Notice, this reduction can be accomplished by moving the decimal point three places to the right.

70. To reduce dollars and cents to mills, move the decimal point three places to the right.

Example 1. Reduce $30.20 to mills.

Here we have 30 dollars, 20 cents, no mills, to reduce to mills, or $30.200 to reduce to mills. Moving the decimal point three places to the right, we have 30200 mills.

Example 2. Reduce 9 cents, 5 mills, to mills.

This amount, in our notation, is written $0.095. Moving the decimal point three places to the right we have 95 mills.

EXERCISE 30 (Written)

Reduce to cents:

$ 12.28
$ 15.05
$ 19.11
$ 84.29
$128.03
$ 27.275
$100.103
$ 75.39

$4720.14
$3280.15
$7290.13
$ 284.15
$ 390.90
$ 928.84
$ 23.252
$ 1.294

Reduce to mills:

$ .023
$ .328
$ .135
$ .732
$1.29
$3.84
$9.29

$ 7.34
$11.23
$14.15
$17.35
$ 8.80
$10.10
$ 5.53

71. *Example.* Reduce to dollars and cents 3280 ¢.

This number of cents can be read 32 hundred and 80. As there are 100 ¢ in a dollar, there will be as many dollars in 3280 ¢ as there are hundreds in 3280. Now 1 hundred is contained in 32 hundred thirty-two times, and hence it is contained in 3280 thirty-two times with a remainder of 80.

Hence, 3280 ¢ = $32.80.

To reduce cents to dollars, move the decimal point two places to the left. Never fail to write the dollar mark to the left of the result.

NOTE. If the decimal point is not written, it is always understood to be immediately to the right of the number.

72. *Example.* Reduce $12.325 to mills. The answer is obtained by moving the decimal point three places to the right. The result is 12325 mills.

Now suppose we are asked to reduce 12325 mills to dollars. The answer is obtained by retracing our steps, *i.e.* by moving the decimal point three places to the left.

To reduce mills to dollars, move the decimal point three places to the left.

EXERCISE 31 (Oral)

Reduce to dollars and cents :

3750¢ 999¢ 5200¢ 3450¢ 1210¢
2384¢ 5375¢ 3245¢ 7890¢ 1550¢
5003¢ 4428¢ 495¢ 1070¢ 5103¢
4010¢ 5173¢ 384¢ 325¢ 3821¢
7783¢ 821¢ 1111¢ 444¢ 6903¢

73. Since our system of currency is decimal, and since the decimal system of notation is constructed upon the principle that the value of a digit is ten times greater than the value of the same digit in the place to its immediate right, we can therefore add, subtract, multiply, and divide dollars and cents in the same manner that we can add, subtract, multiply, and divide integers.

ADDITION

74. In addition of United States money the only caution to be exercised is to see that units of the same denomination stand in the same vertical column and that the decimal points in the sum and in the addends stand in the same vertical line.

EXERCISE 32 (Written)

Add:

1. $19.84, $27.33, $84.04, $98.92, $100.10, $130.75, $200.05.
2. $47.47, $38.39, $59.92, $200.30, $440.45, $7.75, $83.92.
3. $34.35, $38.10, $50.90, $62.63, $10, $87.50, $119.19.
4. $47.10, $17.73, $18.18, $99.90, $503.94, $723.29, $32.35.
5. $75.75, $92.93, $16.17, $92.94, $35.20, $24.25, $49.50.

6. $90.90, $60.30, $58, $79, $.94, $1.38, $232.11.

7. $.77, $29.80, $95, $100.10, $.33, $199, $129.34.

8. $93, $73.90, $77.75, $98.92, $194.95, $382.18, $78.54.

9. $35, $90.35, $28.38, $.75, $200.20, $770.50, $329.54.

10. $17.95, $.76, $32.505, $2.305, $79.282, $39.998, $.125.

11. $234.99
378.98
678.54
888.88
999.92
347.98
732.94
692.51
778.76
239.84
78.79

12. $111.78
392.95
123.98
992.29
777.33
668.59
773.85
392.98
839.91
347.33
779.35

13. $200.90
357.91
369.96
928.95
777.39
666.74
554.38
667.77
732.89
984.49
792.98

14. $728.33
385.79
38.45
92.84
999.28
888.88
773.79
527.73
735.54
928.73
599.98

15. $392.92
775.54
877.79
399.99
666.66
392.74
799.77
345.67
973.98
825.54
900.00
.89
209.83

16. $299.77
987.68
578.93
929.94
666.84
775.95
939.54
392.98
938.79
209.09
777.77
667.93

17. $528.53
947.91
392.87
447.79
998.89
694.75
392.73
739.57
38.38
99.34
101.17
84.85

18. $283.28
792.75
398.76
924.57
84.83
399.98
400.10
799.89
888.88
99.99
528.51
39.46

SUBTRACTION

75. Subtract $18.24 from $93.

$93.00
18.24
$74.76

Write two ciphers to indicate there are no cents. Then say, 4 and 6, 10; 3 and 7, 10; 9 and 4, 13; 2 and 7, 9.

EXERCISE 33 (Written)

Find the difference:

1. $84.20 19.25	**2.** $100.00 17.70	**3.** $300.00 84.84	**4.** $238.10 138.94
5. $345.00 77.70	**6.** $78.79 18.25	**7.** $10000.00 50.50	**8.** $8000.00 100.75
9. $373.25 129.96	**10.** $792.53 94.95	**11.** $327.27 90.90	**12.** $373.50 129.94
13. $285.94 93.97	**14.** $5000.00 1000.98	**15.** $6000.00 771.19	**16.** $5003.00 1003.57
17. $200.10 100.25	**18.** $729.01 328.05	**19.** $929.17 529.69	**20.** $1000.00 99.92

MULTIPLICATION

76. Multiply $78.15 by 8.

$78.15
8
$625.20

Multiply as if multiplicand and multiplier were both integers, and then beginning at the right of the product, point off two places for cents. If there are mills in the multiplicand, three places should be pointed off.

EXERCISE 34 (Written)

Find the product:

1. $79.94 × 8.
2. $32.20 × 7.
3. $79.49 × 6.
4. $128.29 × 7.
5. $399.39 × 9.
6. $454.59 × 12.
7. $729.38 × 11.
8. $237.38 × 9.
9. $720.99 × 7.
10. $285.68 × 8.
11. $51.33 × 7.
12. $79.29 × 7.
13. $29.97 × 8.
14. $179.38 × 6.
15. $373.39 × 5.
16. $799.94 × 8.
17. $822.50 × 9.
18. $998.78 × 11.
19. $778.75 × 12.
20. $732.75 × 9.
21. $928.34 × 7.
22. $653.82 × 5.

23. When shoes sell for $3.90 a pair, how much will 24 pairs of shoes cost?

24. If overcoats sell for $7.98, find the price of 20 overcoats.

25. Mackintoshes sell for $6.95 apiece. How much will 27 mackintoshes cost?

26. When wheat is 84¢ per bushel, how much will 384 bushels bring?

27. Find the price of 349 bushels of corn at 56¢ a bushel.

28. Cheese costs 13¢ per pound. Find the price of 54 pounds.

29. Find the cost of 325 pounds of sugar at 6¢ per pound.

30. An acre of land is worth $60.75. Find the value of 100 acres.

31. It is estimated that the Galveston sea wall will require 86,700 tons of rock. The cost of transporting this rock by railway is 90¢ per ton. Find the total cost of transportation.

DIVISION

77. *Example* **1.** Divide $1.52 by 40.

$$\begin{array}{r} \$.038 \\ 40)\overline{\$1.520} \\ \underline{1\ 20} \\ 320 \\ \underline{320} \end{array}$$

Here the first partial dividend is 152. 40 goes into 152 three times. Write the 3 directly over the right-hand figure of the partial dividend. Multiply 40 by 3 and subtract. Annex a cipher. 40 into 320 goes eight times. Write the 8 in the quotient. The quotient is 3 cents 8 mills. Write a cipher in the quotient to show there are no dimes, and write decimal point in the quotient over decimal point in the dividend. The answer is $.038.

Example **2.** Divide $441.84 by 21.

$$\begin{array}{r} \$21.04 \\ 21)\overline{\$441.84} \\ \underline{42} \\ 21 \\ \underline{21} \\ 84 \\ \underline{84} \end{array}$$

The mechanical work is exactly the same as if both dividend and divisor were integers.

EXERCISE 35 (Written)

Divide:

1. $ 84.98 ÷ 7.
2. $891.28 ÷ 8.
3. $969.75 ÷ 9.
4. $400.07 ÷ 11.
5. $944.96 ÷ 4.
6. $598.32 ÷ 6.
7. $949.90 ÷ 10.
8. $432.96 ÷ 12.
9. $615.04 ÷ 31.
10. $595.36 ÷ 61.

11. $ 462.84 ÷ 38.
12. $ 660.55 ÷ 55.
13. $ 841.35 ÷ 71.
14. $ 995.35 ÷ 85.
15. $2080.75 ÷ 145.
16. $2221.44 ÷ 156.
17. $2771.79 ÷ 201.
18. $2507.79 ÷ 179.
19. $9027.85 ÷ 95.
20. $9414.82 ÷ 97.

78. *Example* **1.** If shoes are sold for $3 a pair, how many pairs can be bought for $27? Here $3 × some number = $27. Hence, we are given in this problem the product of two factors together with one of the factors, and we are required to find the other factor. The number of pairs of shoes will be the number of 3's in 27.

Example **2.** If wheat sells at 84¢ a bushel, how many bushels can be bought for $124.82? In this problem $.84 × some number = $124.82. Again, we have the product of two factors together with one of the factors, and we require the other factor. How shall we divide $.84 into $124.82? We first reduce the divisor and the dividend to units of the same denomination. Reducing both to cents, we have for divisor 84¢, and for dividend 12,482¢. Both numbers are now integers.

NOTE. The formal statement of relations such as are exemplified in the examples above is of vital importance in subsequent work.

EXERCISE 36 (Written)

1. If land is worth $68 an acre, how many acres can be bought for $4624?

2. A tract of land is sold for $4795.50 at the rate of $69.50 an acre. How many acres are in the tract?

3. When horses sell for $85.40 apiece, how many can be bought for $36,465.80?

4. If the price of wheat is 76¢ per bushel, how many bushels can be bought for $4043.20?

5. When asparagus cans sell for 35¢ each, how many can be bought for $85.75?

6. If a pair of patent leather shoes sells for $3.85, how many pairs must be sold to bring $1482.25?

7. A clothier invests in men's trousers $392.04. How many does he buy, supposing each pair to cost $1.98?

8. If a keg of pickles cost $1.70, how many kegs can be purchased for $28.90?

9. Chipped beef is bought at 17¢ a pound. At this rate how many pounds can be bought for $361.25?

10. A 12-pound sack of flour retails at 45¢. How many sacks can be bought for $322.75?

11. When a can of sardines retails for 27¢, how many cans will $218.70 buy?

12. A farmer gets for his apples $816.35 at the rate of $1.45 per barrel. How many barrels does he sell?

13. Corn is worth 45¢ per bushel. How many bushels can be bought for $7876.35?

14. Oats are worth 36¢ per bushel. How many bushels can be bought for $142.56?

15. A share of railway stock is quoted at $78.50. At this rate how many shares must be sold to realize $3061.50?

16. Gold is worth $20.67 an ounce. In the Malay Peninsula the gold mined in a recent year was valued at $516,750. Find the production in ounces.

17. The gold output for Germany for the same year was valued at $1,879,357. How many ounces of gold did Germany produce?

18. In a certain hospital, the cost of medicine for one day was $1240.25. The cost per patient was 41¢. Find the number of persons in the hospital.

19. How many bags of coffee at $8.75 a bag can be bought for $22,968.75?

20. The poll tax in Texas is $1.50. In a city the amount realized from this tax was $4869. How many persons paid poll tax?

EXERCISE 37 (Written)

REVIEW

1. Shoes are worth $3.75 a pair. A person buys 3 pairs at this price. Find his change, if he gives in payment $15.

2. A real estate agent buys 3 houses at $3875 each. He pays in $100 bills. How many does he need? Find his change.

3. Algebras cost 99¢ each. In the first-year classes in a high school there are 117 pupils studying algebra. Find the cost of their algebras.

4. The steamship fare from Galveston to New York is $45. How much will 98 passengers pay?

5. Eggs are 35¢ a dozen. At this price, how many dozen can be bought for $12.60?

6. If 1000 bricks cost $9.75, how much will 84,000 bricks cost?

7. Milk sells for 9¢ a quart. How many quarts must be sold to realize $2.34?

8. Find the price of 5 bushels of apples at 85¢ a bushel, and 25 bushels of potatoes at 95¢ a bushel.

9. Seven houses which cost $4925 each were sold for $40,000. Find the gain.

10. A man earns $1250 a year and spends $75.50 a month. How much does he save?

11. A tenant rents a house for $180 a year and pays his rent in monthly installments. How much will he pay in 5 months?

12. The sales in a store average $98.75 a day. How much will they amount to in a year, the store being open 312 days?

13. £1 sterling is worth $4.866. Find the value of £200 sterling in United States currency.

14. A Panama hat costs $12.50. What is the value of 234 Panama hats?

15. 9025 tons of fluorspar were mined in the United States in 1897. Its estimated value was $74,456. What was the value per ton?

16. 1690 tons of sulphur, worth $34,814, were mined in the United States in 1897. How much was sulphur worth per ton?

17. Assuming the population of the United States is 75,000,000 and the amount of gold per head is $12.42, find the total amount of gold in the United States.

18. Find the value of 5 barrels of sugar, each weighing 325 pounds, if sugar is worth 5¢ per pound.

19. Find the cost of 12 chests of tea, each weighing 48 pounds, if tea cost 54¢ per pound.

20. Two houses are bought for $10,000 and sold for $4275 each. Find the loss.

CHAPTER VII

MEASURES AND MULTIPLES

NOTE. Measures and Multiples in this book, unless the contrary is stated, have reference only to numbers which are both integral and abstract.

79. A number is **prime**, or is said to be a **prime number**, when it is exactly divisible by only itself and unity.

Thus, 1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc., are prime numbers, or prime factors.

80. A number which is exactly divisible by other numbers, as well as by itself and unity, is called a **composite number**.

Thus, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, etc., are composite numbers.

81. An **even number** is one which is exactly divisible by 2.

Thus, 4, 6, 8, 10, 12, etc., are even numbers.

82. An **odd number** is one which is not exactly divisible by 2.

Thus, 3, 5, 7, 9, 11, etc., are odd numbers.

83. One number is said to be a **measure** of another number when it is contained an exact number of times in that other number.

Instead of the word "measure," *factor*, *divisor*, and *sub-multiple* are often used. Examples: 5 is a measure, factor, divisor, or sub-multiple of 10, 15, 20, 25, etc.

84. By the **Greatest Common Measure of Two or More Numbers** is understood the greatest measure which these numbers have in common.

Thus, 6 is the greatest common measure of 12, 18, 30.

Greatest Common Measure is usually designated by the letters G. C. M. It is called also *Greatest Common Divisor* (G. C. D.) or *Highest Common Factor* (H. C. F.).

85. If a number measures each of two or more numbers, it is said to be a common measure of those numbers.

Thus, 2, 3, and 6 are common measures of 12, 18, 30.

86. Two or more numbers are **prime to each other** when they have no common measure but 1.

Thus, 8 and 9 are prime to each other; so also are 15 and 28 prime to each other. These numbers, while prime to each other, are not themselves prime numbers.

87. The result obtained by multiplying a number by an integer is called a **multiple** of the number.

Thus, the multiples of 8 are 8, 16, 24, 32, 40, 48, etc.

We say, for instance, 6 is a measure of 18. Whenever one number measures another number, that other number is a multiple of the number which measures it. 18 is a multiple of 6, and 6 is a measure, factor, or sub-multiple of 18.

88. The **Least Common Multiple of Two or More Numbers** is the least number which is a multiple of each of the numbers. In other words, the Least Common Multiple of two or more numbers is the least number which is exactly divisible by each of the two or more numbers.

By multiplying a number by each of the numbers in the natural number series, 1, 2, 3, 4, 5, 6, 7, etc., we get all its multiples.

Least Common Multiple is denoted by L. C. M.

89. *Example* **1.** What is the L. C. M. of 8 and 12?

Writing the multiples of 8 and 12, we have:

8	16	**24**	32	40	**48**	64	**72**	80
12	**24**	36	**48**	60	**72**	84		

Notice that **24** is a common multiple of 8 and 12. So also are **48** and **72** common multiples. We might get several other multiples. The L. C. M. is 24.

Example **2.** What is the L. C. M. of 12 and 18? Here the second multiple of 18 contains 12 as a factor. Hence 2 times 18 is the L. C. M.

Example **3.** A man buys two kinds of sugar done up in 4-pound bags and in 5-pound bags. What is the least number of pounds he can buy so as to have the same number of pounds of each kind?

Here the answer is obviously a multiple of 4 and 5. The L. C. M. of 4 and 5 is 20. Hence he buys 20 pounds of each kind.

EXERCISE 38 (Written)

1. A person has equal sums of money in dimes and in 25¢ pieces. Find the least amount he can have.

2. Fence posts in two fences are respectively 14 feet and 21 feet apart. What is the smallest distance corresponding to an exact number of feet in both fences?

3. A man earns $4 a day. How many days must he work so as to be paid in 10-dollar notes?

4. If a person earns $6 a day, how many days must he work to earn an exact number of $20 bills?

5. A housewife puts her flour into 10-pound and 6-pound sacks, and has the same quantity in the 10-pound sacks as she has in the 6-pound sacks. What is the least quantity of flour she can have?

6. A man buys two grades of sheep at $4 and $6 a head respectively. He spends the same amount in the purchase of the two grades of sheep. What is the smallest amount he can spend? How many sheep can he buy?

7. A boy buys oranges at 3¢, 4¢, 5¢ apiece. He spends the same amount on each kind of oranges. What is the least amount he can spend on each kind? How many oranges does he buy?

8. A person spends the same amount of money on eggs at 15¢ a dozen and at 20¢ a dozen. What is the smallest amount he can spend on each kind?

FACTORS AND MULTIPLES

90. Perform the following indicated operations:

$$2 \times 3 \times 4 \times 5 \times 7.$$

Multiplying 2 by 3, that product by 4, then the product of the first three factors by 5, and finally this product by 7, we get 840.

Suppose, now, we group the factors, first multiply 2 by 3 and get 6, next 4 by 5, and get 20, and then multiply 6 × 20 × 7, we get the same result; *i.e.* 840.

If we group the factors in any other way, for instance, first, multiply 3 × 4 × 5, and next multiply 7 × 2, and then multiply these products, we get again 840. No matter how we group the factors, the product remains the same. By taking any other factors, we find the same law holds.

The product of any number of factors is the same, no matter how the factors are grouped.

This is known as the **Associative Law.**

NOTE. A thorough understanding of this law is of great importance in future work.

91. How many times are 3 yards contained in 36 yards?

$$\begin{array}{r} 3\overline{)36} \\ 12 \end{array}$$

Reduce 3 yards to feet, and also 36 yards to feet. In 3 yards there are 9 feet. In 36 yards there are 108 feet. Now divide 108 by 9.

$$\begin{array}{r} 9\overline{)108} \\ 12 \end{array}$$

We again get the same result.

Reduce now divisor and dividend to inches, and we have 108 for divisor and 1296 for dividend.

Divide:

$$\begin{array}{r} 12 \\ 108\overline{)1296} \\ \underline{108} \\ 216 \\ \underline{216} \end{array}$$

Again we get the same result.

It appears, therefore, that $\frac{36}{3} = \frac{36 \times 3}{3 \times 3} = \frac{36 \times 3 \times 12}{3 \times 3 \times 12}$.

FACTORS AND MULTIPLES: CANCELLATION

If divisor and dividend are multiplied by the same number or numbers, the quotient remains the same. Conversely, we may divide dividend and divisor by the same number, and the quotient remains the same.

92. Cancellation is the shortening of the process of division by dividing dividend and divisor by the same factor or factors.

Find by cancellation the quotient:

$$\frac{18 \times 27 \times 16}{9 \times 8 \times 3} = \frac{2 \times 9 \times 2}{1 \times 1 \times 1}.$$

Dividing 9 into 18, 8 into 16, and 3 into 27, we find the quotient $\frac{2 \times 9 \times 2}{1 \times 1 \times 1} = 36.$

EXERCISE 39 (Written)

Find by cancellation the quotient:

1. $\frac{4 \times 18 \times 3 \times 24}{9 \times 4 \times 144}$.

2. $\frac{18 \times 90 \times 105}{14 \times 25 \times 3}$.

3. $\frac{27 \times 64 \times 8}{108 \times 32}$.

4. $\frac{343 \times 125}{35 \times 35}$.

5. $\frac{16 \times 16 \times 8 \times 81}{64 \times 32 \times 3}$.

6. $\frac{225 \times 216}{75 \times 9 \times 12}$.

7. $\frac{108 \times 27 \times 121}{22 \times 33 \times 18}$.

8. $\frac{22 \times 88 \times 15}{132 \times 4}$.

9. $\frac{1760 \times 99}{4 \times 88 \times 165}$.

10. $\frac{5280 \times 14}{176 \times 84}$.

11. $\frac{1728 \times 34}{27 \times 136 \times 8}$.

12. $\frac{640 \times 5200}{125 \times 512 \times 13}$.

13. $\frac{2380 \times 104}{119 \times 8 \times 13}$.

14. $\frac{111 \times 39 \times 12}{74 \times 27 \times 13}$.

15. A farmer exchanged 320 acres of land worth \$50 an acre for 25 city lots. How many lots did he get?

(price of a lot) × 25 = \$50 × 320.

Hence, price of lot $= \frac{\$50 \times 320}{25} = \$640$.

16. How many cows at \$40 a head cost as much as 15 horses at \$64 a head?

17. How many dozen eggs at 35¢ a dozen must be sold to pay for 7 barrels of apples at \$2.10 a barrel?

18. A laborer receives \$3.20 a day. How many days must he work to pay for 6 tons of coal at \$8 per ton?

19. A bicyclist rides at the rate of 9 miles an hour. How long will it take him to travel as far as a train goes in 6 hours at the rate of 33 miles an hour?

20. How many cattle at \$42 a head must be sold to pay for 11,200 bushels of wheat at 75¢ a bushel?

REVIEW QUESTIONS

1. What is the monetary unit of the United States?

2. What does the word *mill* mean? What does the word *cent* mean?

3. Name the United States coins.

4. Our monetary units conform to the decimal system of notation; explain.

5. How do you reduce dollars and cents to cents?

6. How do you reduce dollars and cents to mills?

7. How are mills expressed as dollars?

8. How are cents expressed as dollars?

9. How do you add dollars and cents?

10. How do you subtract dollars and cents?

11. How do you multiply dollars and cents by an integer?

12. How do you divide dollars and cents by an integer?

13. What is a composite number?

14. What is a prime number?

15. Explain the terms *measure* and *multiple.*

16. What other words are used for measure?

17. What is meant by greatest common measure?

18. What is meant by common measure?

19. Explain what is meant by least common multiple.

20. Name, in order, the first seven multiples of 11.

21. What is the Associative Law?

22. What is meant by cancellation?

CHAPTER VIII

FRACTIONS

93. In § 51 we have seen that division is indicated by writing the dividend above a horizontal line and the divisor below the same horizontal line.

Thus, $\frac{20}{4}$ implies that 20 is to be divided by 4, or that the fourth part of 20 is sought. Here the quotient is 5.

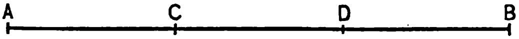
If we divide 23 by 4, the process is indicated $\frac{23}{4}$. The quotient, in this instance, is 5 with a remainder 3, or the quotient is $5\frac{3}{4}$; this means 4 is contained in 23 five times, and that 3 remains to be divided by 4.

From processes of this character, the term **fraction** (Latin *fractus*, broken in pieces) has arisen. $\frac{23}{4}$, $5\frac{3}{4}$, $\frac{3}{4}$, are all fractions.

A **fraction is an indicated division.**

94. There is another way of looking at a fraction; in other words, there is a second concept of a fraction.

Suppose the line *AB* to be divided into 3 equal parts.



One of these parts is called one-third of the whole line *AB*. Two of these equal parts are called two-thirds of *AB*, and three of them are called three-thirds of *AB*.

Take another line *AB* and divide it into 4 equal parts.

1 of these parts is called 1 fourth of the line.
2 of these parts are called 2 fourths of the line.
3 of these parts are called 3 fourths of the line.
4 of these parts are called 4 fourths of the line.

If a quantity is divided into 5 equal parts,

1 part is called what?
2 parts are called what?
3 parts are called what?
4 parts are called what?
5 parts are called what?

How many fifths are in any quantity?

If a quantity is divided into 6 equal parts,

What is 1 part called?
What are 2 parts called?
What are 3 parts called?
What are 4 parts called?
What are 5 parts called?
What are 6 parts called?

How many sixths are in a quantity?

If a quantity is divided into

7 equal parts,
8 equal parts,
9 equal parts,
10 equal parts,
11 equal parts,
12 equal parts,

what is each part called?

95. Suppose we have several oranges, and we divide each orange into 4 equal parts,

1 part is called 1 fourth of an orange.
2 parts are called 2 fourths of an orange.
3 parts are called 3 fourths of an orange.

4 parts are called 4 fourths of an orange.
5 parts are called 5 fourths of an orange.
6 parts are called 6 fourths of an orange.
7 parts are called 7 fourths of an orange.
8 parts are called 8 fourths of an orange.

What does the fraction 17 fourths mean? It means a number of things of the same kind are each divided into 4 equal parts, and 17 of the parts are taken.

EXERCISE 40 (Oral)

Add:

1. 1 third, 2 thirds, 4 thirds.
2. 1 half, 3 halves, 5 halves.
3. 3 fourths, 5 fourths, 7 fourths.
4. 1 fourth, 2 fourths, 3 fourths.
5. 2 fifths, 3 fifths, 7 fifths, 6 fifths.
6. 3 fifths, 4 fifths, 5 fifths, 6 fifths.
7. 1 sixth, 5 sixths, 2 sixths, 3 sixths.
8. 4 sevenths, 5 sevenths, 6 sevenths, 2 sevenths.
9. 1 eighth, 3 eighths, 5 eighths, 7 eighths.
10. 3 tenths, 4 tenths, 5 tenths, 7 tenths.

NOTATION OF FRACTIONS

96. 1 third is written $\frac{1}{3}$.
2 thirds is written $\frac{2}{3}$.
1 fourth is written $\frac{1}{4}$.
3 fourths is written $\frac{3}{4}$.
1 seventh is written $\frac{1}{7}$.
4 sevenths is written $\frac{4}{7}$.
7 eighths is written $\frac{7}{8}$.

How many thirds are in 1 thing?
How many fourths are in 1 thing?
How many sevenths are in 1 thing?
How many eighths are in 1 thing?

In the notation of fractions, what does the number below the line indicate?

What does the number above the line indicate?

97. If a unit quantity is divided into any number of equal parts, one of these parts is called a **fractional unit,** or **unit fraction.**

98. A **fraction is one fractional unit, or two or more fractional units of the same denomination.**

99. A fraction is expressed by two numbers, one number being written above a horizontal line and the other number being written below the same horizontal line.

The number above the horizontal line is called the **numerator,** because it *numbers* the parts taken, *i.e.* tells how many fractional units there are.

The number below the line is called the **denominator:** it *names* the fractional unit, and indicates how many fractional units there are in the unit quantity from which the fractional unit is derived.

Thus, $\frac{3}{4}$ signifies the unit quantity is broken into 4 equal parts and 3 of these parts are taken. Here the fractional unit is $\frac{1}{4}$ (one-fourth); 3 is the numerator, and 4 is the denominator.

100. The numerator and denominator are called the **terms of the fraction.**

NOTE. The definition of fraction in § 98 is correct so long as we are dealing with fractions whose terms are integral numbers. It must not be considered as a scientific definition of a fraction. It does not hold where the denominator is a fractional or a surd number. The real definition of a fraction is that given in § 93.

101. A **proper fraction** is one whose numerator is less than its denominator. Thus, $\frac{4}{7}$ is a proper fraction, because 4 is less than 7.

102. An **improper fraction** is one whose numerator is greater than or equal to its denominator. Examples: $\frac{14}{3}$, $\frac{7}{7}$.

103. A **mixed number** is a number, part integral and part fractional. Thus, $4\frac{2}{3}$ is a mixed number.

EXERCISE 41 (Oral)

Read and explain what each represents:

$\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{3}$, $\frac{3}{2}$, $\frac{7}{2}$, $\frac{9}{2}$, $\frac{7}{3}$, $\frac{9}{3}$, $\frac{7}{4}$, $\frac{11}{4}$, $\frac{13}{4}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{7}{5}$, $\frac{9}{5}$, $\frac{10}{5}$, $\frac{5}{6}$, $\frac{7}{6}$, $\frac{9}{6}$, $\frac{11}{6}$, $\frac{4}{7}$, $\frac{6}{7}$, $\frac{5}{7}$, $\frac{11}{7}$, $\frac{18}{7}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{9}{8}$, $\frac{11}{8}$, $\frac{15}{8}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{19}{10}$, $\frac{29}{10}$.

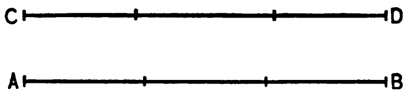
104. We have defined a fraction as an indicated division, and we have also regarded a fraction as the result of counting fractional units of the same name. Do these two points of view agree?

Take the fraction $\frac{2}{3}$. From the first point of view, this fraction is 2 divided by 3, *i.e.* the third part of 2. From the second point of view this fraction is read 2 thirds, *i.e.* 2 times one-third.

Now is

$$\text{one-third of } 2 = 2 \text{ times one-third?}$$

Let us see.



Take any line *AB*. Divide it into 3 equal parts. One of these parts is called 1 third of *AB*; 2 of these parts are called 2 thirds of *AB*.

Take another line *CD* equal to *AB*. Divide it into 3 equal parts. Counting the parts of the two lines, we find there are 6 parts in all. Now one-third of the two lines is equal to one-third of the 6 parts, *i.e.* 2 parts.

We have then one-third of the 2 lines equal to 2 parts. But any two of the parts equal 2 thirds of one line. Hence,

one-third of 2 = 2 thirds or 2 times one-third.

This same conclusion may be seen as follows:

1 = 3 thirds.

2 = 6 thirds.

Hence, one-third of 2 = 2 thirds.

For beginners a concrete illustration can be taken as follows:

Take any number, 12.

1 third of 12 = 4.

2 thirds of 12 = 8.

But one-third of 2 times 12 = 8. Hence, if the unit quantity is 12, it is seen at once that 2 thirds of it = one-third of twice it.

EXERCISE 42

Show by drawing lines, or illustrate in some way, the truth of the following:

1. 3 fourths = one-fourth part of 3.
2. 2 fifths = one-fifth part of 2.
3. 3 fifths = one-fifth part of 3.
4. 4 fifths = one-fifth part of 4.
5. 5 sixths = one-sixth part of 5.
6. 5 twelfths = one-twelfth part of 5.
7. 7 twelfths = one-twelfth part of 7.

105. Reduce $\frac{17}{3}$ to a mixed number. Performing the division, we have $5\frac{2}{3}$.

EXERCISE 43 (Oral)

Reduce to integers or mixed numbers:

1. $\frac{11}{8}$.
2. $\frac{13}{4}$.
3. $\frac{27}{4}$.
4. $\frac{24}{4}$.
5. $\frac{31}{5}$.
6. $\frac{32}{7}$.
7. $\frac{38}{9}$.
8. $\frac{40}{6}$.
9. $\frac{50}{7}$.
10. $\frac{60}{8}$.
11. $\frac{59}{9}$.
12. $\frac{19}{12}$.
13. $\frac{17}{11}$.
14. $\frac{44}{5}$.
15. $\frac{100}{9}$.
16. $\frac{100}{11}$.
17. $\frac{120}{12}$.
18. $\frac{15}{14}$.
19. $\frac{27}{7}$.
20. $\frac{39}{10}$.
21. $\frac{97}{13}$.
22. $\frac{78}{14}$.
23. $\frac{79}{12}$.
24. $\frac{72}{18}$.
25. $\frac{48}{16}$.
26. $\frac{94}{11}$.
27. $\frac{89}{9}$.
28. $\frac{88}{13}$.
29. $\frac{79}{14}$.
30. $\frac{80}{7}$.
31. $\frac{77}{14}$.
32. $\frac{95}{14}$.
33. $\frac{118}{18}$.
34. $\frac{127}{6}$.
35. $\frac{140}{8}$.
36. $\frac{120}{11}$.

106. Reduce to an improper fraction $4\frac{5}{6}$.

$$1 = 6 \text{ sixths.}$$
$$4 = 24 \text{ sixths.}$$
$$4\tfrac{5}{6} = 29 \text{ sixths} = \tfrac{29}{6}.$$

Another Method

What number divided by 6 gives 4 as a quotient and 5 as a remainder?

The answer is 6 times 4 + the remainder, 5. Therefore $4\frac{5}{6} = \frac{29}{6}$.

EXERCISE 44 (Written)

Reduce to improper fractions:

1. $3\frac{1}{2}$.
2. $3\frac{2}{3}$.
3. $5\frac{3}{7}$.
4. $9\frac{3}{8}$.
5. $9\frac{2}{7}$.
6. $10\frac{3}{11}$.
7. $8\frac{7}{8}$.
8. $5\frac{5}{6}$.
9. $9\frac{5}{12}$.
10. $10\frac{10}{11}$.
11. $11\frac{1}{10}$.
12. $10\frac{1}{4}$.
13. $19\frac{2}{7}$.
14. $18\frac{4}{9}$.
15. $27\frac{5}{11}$.
16. $38\frac{5}{12}$.
17. $39\frac{4}{5}$.
18. $19\frac{2}{3}$.
19. $20\frac{10}{11}$.
20. $16\frac{15}{16}$.
21. $14\frac{3}{8}$.
22. $15\frac{2}{7}$.
23. $19\frac{4}{9}$.
24. $29\frac{1}{9}$.
25. $27\frac{5}{12}$.
26. $13\frac{12}{13}$.
27. $14\frac{2}{7}$.
28. $37\frac{3}{8}$.
29. $44\frac{4}{11}$.
30. $30\frac{3}{10}$.

107. Add $\frac{2}{3}$, $\frac{5}{3}$, $\frac{7}{3}$. Here we have 2 thirds + 5 thirds + 7 thirds = 14 thirds, or $\frac{14}{3} = 4\frac{2}{3}$.

EXERCISE 45 (Written)

Add:

1. $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$.
2. $\frac{2}{3}$, $\frac{4}{3}$, $\frac{10}{3}$.
3. $\frac{3}{4}$, $\frac{5}{4}$, $\frac{9}{4}$.
4. $\frac{2}{4}$, $\frac{7}{4}$, $\frac{11}{4}$.
5. $\frac{3}{5}$, $\frac{4}{5}$, $\frac{5}{5}$, $\frac{6}{5}$.
6. $\frac{5}{6}$, $\frac{8}{6}$, $\frac{10}{6}$, $\frac{11}{6}$.
7. $\frac{4}{7}$, $\frac{3}{7}$, $\frac{9}{7}$, $\frac{11}{7}$.
8. $\frac{6}{7}$, $\frac{5}{7}$, $\frac{11}{7}$, $\frac{12}{7}$.
9. $\frac{8}{7}$, $\frac{9}{7}$, $\frac{10}{7}$, $\frac{11}{7}$.
10. $\frac{5}{8}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{12}{8}$.
11. $\frac{6}{9}$, $\frac{7}{9}$, $\frac{11}{9}$, $\frac{13}{9}$.
12. $\frac{5}{9}$, $\frac{7}{9}$, $\frac{14}{9}$, $\frac{19}{9}$.
13. $\frac{7}{10}$, $\frac{9}{10}$, $\frac{11}{10}$, $\frac{12}{10}$.
14. $\frac{5}{12}$, $\frac{7}{12}$, $\frac{9}{12}$, $\frac{11}{12}$.
15. $\frac{10}{12}$, $\frac{13}{12}$, $\frac{3}{12}$, $\frac{6}{12}$.

108. Subtract $\frac{2}{3}$ from $\frac{7}{3}$.

7 thirds − 2 thirds = 5 thirds = $\frac{5}{3} = 1\frac{2}{3}$.

EXERCISE 46 (Oral)

Find the remainder:

1. $\frac{4}{3} - \frac{2}{3}$.
2. $\frac{7}{3} - \frac{4}{3}$.
3. $\frac{7}{4} - \frac{1}{4}$.
4. $\frac{9}{4} - \frac{3}{4}$.
5. $\frac{11}{5} - \frac{3}{5}$.
6. $\frac{12}{7} - \frac{11}{7}$.
7. $\frac{12}{8} - \frac{5}{8}$.
8. $\frac{9}{8} - \frac{1}{8}$.
9. $\frac{11}{8} - \frac{2}{8}$.
10. $\frac{12}{9} - \frac{3}{9}$.
11. $\frac{17}{10} - \frac{5}{10}$.
12. $\frac{18}{11} - \frac{7}{11}$.
13. $\frac{13}{12} - \frac{2}{12}$.
14. $\frac{19}{12} - \frac{3}{12}$.
15. $\frac{20}{12} - \frac{1}{12}$.
16. $\frac{9}{15} - \frac{4}{15}$.
17. $\frac{14}{14} - \frac{1}{14}$.
18. $\frac{24}{13} - \frac{9}{13}$.
19. $\frac{20}{18} - \frac{9}{18}$.
20. $\frac{27}{14} - \frac{15}{14}$.
21. $\frac{29}{16} - \frac{13}{16}$.
22. $\frac{35}{16} - \frac{21}{16}$.
23. $\frac{29}{15} - \frac{14}{15}$.
24. $\frac{18}{9} - \frac{1}{9}$.

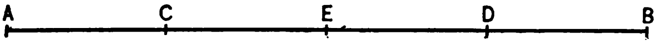
109. Multiply $\frac{3}{4}$ by 8.

3 fourths × 8 = 24 fourths = $\frac{24}{4} = 6$.

EXERCISE 47 (Written)

Find the value of:

1. $\frac{2}{3} \times 3$.
2. $\frac{3}{4} \times 4$.
3. $\frac{2}{5} \times 5$.
4. $\frac{5}{6} \times 12$.
5. $\frac{17}{2} \times 4$.
6. $\frac{19}{6} \times 7$.
7. $\frac{11}{4} \times 5$.
8. $\frac{13}{7} \times 14$.
9. $\frac{17}{8} \times 9$.
10. $\frac{3}{5} \times 14$.
11. $\frac{7}{3} \times 19$.
12. $\frac{11}{12} \times 12$.
13. $\frac{9}{10} \times 12$.
14. $\frac{7}{12} \times 15$.
15. $1\frac{1}{2} \times 8$.
16. $1\frac{2}{3} \times 9$.



Take a line *AB*. Suppose it is 12 inches long. Divide it into 2 equal parts. Let *AE* represent 6 inches and *EB* represent 6 inches. Divide *AE* into 2 equal parts; divide also *EB* into 2 equal parts. Let *C* and *D* be the points of division. Each of the lines *AC*, *CE*, *ED*, *DB* will represent 3 inches, since each will be one-fourth of *AB*.

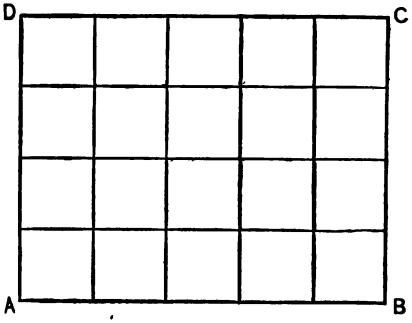
Divide each of the lines *AC*, *CE*, *ED*, *DB*, into 3 equal parts, making 12 parts in all.

$\frac{1}{4}$ of the whole line = 3 inches.
Hence, $\frac{2}{4}$ of the whole line = 6 inches.
$\frac{1}{2}$ of the whole line = 6 inches.

Therefore, $\frac{1}{2} = \frac{2}{4}$.

Also, 1 inch $= \frac{1}{12}$ of the whole line.
6 inches $= \frac{6}{12}$ of the whole line.

Therefore, $\frac{1}{2} = \frac{2}{4} = \frac{6}{12}$.



Take a rectangle *ABCD*. Divide it. How many parts are there in all?

What part of the whole rectangle is one of the smallest parts?

How many of the smallest parts are there in $\frac{1}{4}$ of the rectangle? $\frac{1}{2}$ of the rectangle?

From the figure *ABCD* can you infer that $\frac{1}{2} = \frac{2}{4} = \frac{5}{10}$?

What part of 10 inches is 1 inch?
What part of 10 inches is 2 inches?
What part of 10 inches is 3 inches?

What part of 10 inches is 4 inches?
What part of 10 inches is 5 inches?

What is $\frac{1}{2}$ of 10 inches? What is $\frac{1}{6}$ of 6 apples?
What is $\frac{1}{2}$ of 6 apples? What is $\frac{3}{6}$ of 6 apples?

From the above illustrations, it appears that $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$.

In other words, $\frac{1}{2}$ can be reduced to fourths, sixths, eighths, tenths, twelfths.

Can $\frac{1}{2}$ be reduced to fifths? We know $1 = 5$ fifths, hence $\frac{1}{2}$ is not equal to an integral number of fifths.

We cannot reduce $\frac{1}{2}$ to thirds, fifths, ninths, etc.

What is $\frac{1}{3}$ of 6 apples?
What is $\frac{1}{6}$ of 6 apples?
What is $\frac{2}{6}$ of 6 apples?
Is $\frac{1}{3} = \frac{2}{6}$?

Can $\frac{1}{3}$ be reduced to fifths, sevenths, tenths, etc.?

Show that $\frac{1}{3} = \frac{3}{9}$, $\frac{1}{3} = \frac{4}{12}$, $\frac{1}{3} = \frac{5}{15}$.

Show that $\frac{3}{4} = \frac{6}{8}$, $\frac{3}{4} = \frac{9}{12}$, $\frac{3}{4} = \frac{12}{16}$.

Can $\frac{1}{4}$ be reduced to fifths, sevenths, ninths, tenths, etc.?

110. The terms of a fraction may be multiplied by the same number and the value of the fraction remains the same.

This is merely a repetition of the principle stated in § 91; namely, that if the dividend and divisor be multiplied by the same number, the quotient remains the same.

111. Reduce $\frac{3}{7}$ to fourteenths.

$\frac{1}{7} = \frac{2}{14}$. Hence, multiplying by 3, $\frac{3}{7} = \frac{6}{14}$.

Or, proceed thus: $\frac{3}{7} = \frac{\text{some number}}{14}$. Here the divisor, 7, is doubled, therefore the dividend, 3, must be doubled. Hence, $\frac{3}{7} = \frac{6}{14}$.

EXERCISE 48 (Written)

Reduce:

1. $\frac{2}{3}$ to 9ths, to 15ths, to 24ths, to 30ths, to 36ths.
2. $\frac{3}{4}$ to 8ths, to 16ths, to 24ths, to 32ds, to 40ths.
3. $\frac{3}{5}$ to 10ths, to 20ths, to 25ths, to 35ths, to 40ths.
4. $\frac{4}{7}$ to 14ths, to 21sts, to 28ths, to 35ths, to 49ths.
5. $\frac{5}{8}$ to 16ths, to 24ths, to 40ths, to 48ths, to 64ths.
6. $\frac{3}{8}$ to 32ds, to 48ths, to 56ths, to 72ds, to 80ths.
7. $\frac{5}{9}$ to 18ths, to 27ths, to 45ths, to 63ds, to 72ds.
8. $\frac{9}{10}$ to 20ths, to 50ths, to 70ths, to 80ths, to 90ths.
9. $\frac{7}{12}$ to 24ths, to 48ths, to 72ds, to 84ths, to 96ths.
10. 3 to 7ths, to 10ths, to 12ths, to 20ths.
11. 5 to 4ths, to 9ths, to 14ths, to 25ths.
12. 7 to 3ds, to 8ths, to 10ths, to 12ths.

We can reduce $\frac{3}{4}$ to 28ths. $\frac{3}{4} = \frac{21}{28}$. This we do by multiplying the terms of the fraction $\frac{3}{4}$ by 7. Conversely, we can reduce $\frac{21}{28}$ to $\frac{3}{4}$ by dividing the terms of the fraction $\frac{21}{28}$ by 7.

112. A fraction is said to be **in its simplest form** when its terms are integers prime to each other. Thus, $\frac{3}{4}$ is in its simplest form because its terms 3 and 4 are prime to each other.

A fraction, in its simplest form, is also said to be **in its lowest terms.**

113. Reduce to lowest terms $\frac{96}{120}$.

$\frac{96}{120} = \frac{48}{60} = \frac{12}{15} = \frac{4}{5}$. Here we first divided the terms by 2, and got $\frac{48}{60}$; next we divided the terms of the fraction $\frac{48}{60}$ by 4 and got $\frac{12}{15}$. Then we divided by 3 and got $\frac{4}{5}$. In each case we find by inspection the factor which can be divided into the terms of the fraction.

EXERCISE 49 (Written)

Reduce to lowest terms:

1. $\frac{6}{12}$, $\frac{8}{18}$, $\frac{16}{24}$, $\frac{18}{27}$, $\frac{9}{24}$, $\frac{16}{28}$, $\frac{32}{40}$, $\frac{25}{30}$, $\frac{32}{48}$.
2. $\frac{18}{45}$, $\frac{24}{60}$, $\frac{30}{75}$, $\frac{44}{55}$, $\frac{35}{84}$, $\frac{30}{48}$, $\frac{40}{48}$, $\frac{84}{96}$.
3. $\frac{54}{81}$, $\frac{36}{54}$, $\frac{48}{64}$, $\frac{72}{96}$, $\frac{54}{72}$, $\frac{24}{64}$, $\frac{36}{96}$, $\frac{54}{144}$.
4. $\frac{36}{162}$, $\frac{48}{108}$, $\frac{60}{135}$, $\frac{49}{84}$, $\frac{63}{108}$, $\frac{84}{144}$, $\frac{126}{216}$, $\frac{140}{240}$.
5. $\frac{144}{216}$, $\frac{100}{150}$, $\frac{108}{162}$, $\frac{81}{108}$, $\frac{36}{64}$, $\frac{63}{81}$, $\frac{84}{108}$, $\frac{80}{192}$.

ADDITION

114. *Example* 1. What part of 1 foot is $\frac{1}{2}$ of a foot and $\frac{1}{3}$ of a foot?

$\frac{1}{2}$ of a foot = 6 inches; $\frac{1}{3}$ of a foot = 4 inches.

$\frac{1}{2}$ of a foot + $\frac{1}{3}$ of a foot = 10 inches.

1 inch = $\frac{1}{12}$ of a foot.

10 inches = $\frac{10}{12}$ of a foot = $\frac{5}{6}$ of a foot.

Hence, $\frac{1}{2}$ of a foot + $\frac{1}{3}$ of a foot = $\frac{5}{6}$ of a foot.

If we take any other unit quantity, we shall always find $\frac{1}{2}$ of it + $\frac{1}{3}$ of it = $\frac{5}{6}$ of it. Hence $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$.

We do not usually add fractions in this manner. In adding concrete quantities, we *always* reduce them to units of the same denomination. In adding fractions we first reduce them to fractions of the same denomination.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16} = \frac{9}{18}.$$

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{6}{18}.$$

We can reduce $\frac{1}{2}$ to 6ths, 12ths, 18ths.

We can reduce $\frac{1}{3}$ to 6ths, 12ths, 18ths.

Hence we can reduce $\frac{1}{2}$ and $\frac{1}{3}$ to fractions of the same denomination.

In adding fractions, we select, as a matter of convenience, the lowest denomination common to the fractions.

$$\tfrac{1}{2} = \tfrac{3}{6}.$$
$$\tfrac{1}{3} = \tfrac{2}{6}.$$

Hence, $\tfrac{1}{2} + \tfrac{1}{3} = \tfrac{3}{6} + \tfrac{2}{6} = \tfrac{5}{6}.$

Example 2. Add $\tfrac{3}{4}$, $\tfrac{5}{6}$.

4ths can be reduced to 8ths, 12ths, 16ths, 24ths.

6ths can be reduced to 12ths, 18ths, 24ths.

Here the lowest denomination common to both fractions is 12ths. Notice, 12 is L. C. M. of 4, 6.

$$\tfrac{3}{4} = \tfrac{9}{12}.$$
$$\tfrac{5}{6} = \tfrac{10}{12}.$$

Hence, $\tfrac{3}{4} + \tfrac{5}{6} = \tfrac{9}{12} + \tfrac{10}{12} = \tfrac{19}{12} = 1\tfrac{7}{12}.$

Example 3. Add $4\tfrac{1}{2}$, $2\tfrac{5}{6}$, $1\tfrac{5}{12}$.

$$\begin{array}{ll} 4\tfrac{1}{2} & = 4\tfrac{6}{12} \\ 2\tfrac{5}{6} & = 2\tfrac{10}{12} \\ \underline{4\tfrac{5}{12}} & = 1\tfrac{5}{12} \\ 8\tfrac{3}{4} & \end{array}$$

We first add the fractions $\tfrac{1}{2}$, $\tfrac{5}{6}$, $\tfrac{5}{12}$. To do this, we first find the L. C. M. of the denominators. The L. C. M. of 2, 6, 12, is 12. $\tfrac{1}{2} = \tfrac{6}{12}$, $\tfrac{5}{6} = \tfrac{10}{12}$, $\tfrac{5}{12} = \tfrac{5}{12}$. Adding $\tfrac{6}{12}$, $\tfrac{10}{12}$, $\tfrac{5}{12}$, we get $\tfrac{21}{12} = \tfrac{7}{4} = 1\tfrac{3}{4}$.

Write $\tfrac{3}{4}$ in the sum and carry 1. Add next the integers 4, 2, 1, and the 1 carried. The sum is $8\tfrac{3}{4}$.

The work may be contracted as follows:

$$\begin{array}{ll} & \underline{12} \\ 4\tfrac{1}{2} & 6 \\ 2\tfrac{5}{6} & 10 \\ \underline{1\tfrac{5}{12}} & \underline{5} \\ 8\tfrac{3}{4} & \tfrac{21}{12} = 1\tfrac{9}{12} = 1\tfrac{3}{4} \end{array}$$

Find first the L. C. M. of the denominators and write it above a horizontal line. Underneath this line write the number of 12ths in each of the fractions. Add these numbers, and underneath the sum write the common denominator. Reduce this fraction to its simplest form. Add its integral part to the integral part of the addends.

115. To add fractions, first reduce them to equivalent fractions having the same denominator. Then add the numerators, and underneath the sum write the common denominator.

If the resulting fraction is not in its simplest form, reduce it to its simplest form.

To add mixed numbers, first add the fractions, and to this sum add the sum of the integers.

EXERCISE 50 (Written)

Find the sum of:

1. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$.
2. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{3}$.
3. $\frac{1}{2}$, $\frac{1}{6}$, $\frac{3}{4}$.
4. $\frac{1}{2}$, $\frac{5}{8}$, $\frac{3}{4}$.
5. $\frac{1}{3}$, $\frac{5}{6}$, $\frac{7}{12}$.
6. $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{12}$.
7. $\frac{1}{6}$, $\frac{5}{12}$, $\frac{1}{4}$.
8. $\frac{7}{15}$, $\frac{2}{5}$, $\frac{1}{5}$.
9. $\frac{3}{8}$, $\frac{3}{4}$, $\frac{1}{2}$.
10. $\frac{2}{5}$, $\frac{1}{2}$, $\frac{9}{10}$.
11. $\frac{7}{10}$, $\frac{1}{2}$, $\frac{3}{5}$.
12. $\frac{3}{10}$, $\frac{2}{5}$, $\frac{1}{2}$.
13. $\frac{2}{3}$, $\frac{5}{9}$.
14. $\frac{7}{24}$, $\frac{7}{12}$, $\frac{5}{6}$.
15. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{11}{16}$.
16. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$.
17. $\frac{7}{12}$, $\frac{1}{3}$, $\frac{1}{6}$.
18. $\frac{11}{18}$, $\frac{2}{3}$, $\frac{7}{9}$.
19. $\frac{3}{4}$, $\frac{7}{12}$, $\frac{17}{24}$.
20. $\frac{5}{6}$, $\frac{5}{12}$, $\frac{13}{24}$.
21. $\frac{3}{4}$, $\frac{7}{10}$, $\frac{17}{20}$.
22. $\frac{1}{4}$, $\frac{4}{5}$, $\frac{9}{10}$.
23. $\frac{1}{4}$, $\frac{2}{5}$, $\frac{7}{10}$, $\frac{11}{20}$.
24. $\frac{3}{5}$, $\frac{3}{10}$, $\frac{19}{20}$.
25. $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{5}{24}$.
26. $\frac{7}{8}$, $\frac{2}{3}$, $\frac{3}{4}$.
27. $\frac{5}{8}$, $\frac{1}{3}$, $\frac{7}{12}$.
28. $\frac{3}{8}$, $\frac{2}{3}$, $\frac{5}{12}$.
29. $\frac{3}{4}$, $\frac{7}{12}$, $\frac{5}{6}$.
30. $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{12}$, $\frac{13}{24}$.
31. $1\frac{1}{2}$, $2\frac{1}{3}$, $3\frac{3}{4}$.
32. $2\frac{1}{6}$, $5\frac{1}{2}$, $7\frac{7}{12}$.
33. $3\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{7}{8}$.
34. $5\frac{2}{3}$, $7\frac{5}{6}$, $11\frac{11}{12}$.
35. $9\frac{1}{6}$, $7\frac{2}{3}$, $10\frac{1}{12}$.
36. $10\frac{5}{12}$, $9\frac{3}{4}$, $2\frac{2}{3}$.
37. $9\frac{1}{2}$, $1\frac{2}{5}$, $6\frac{7}{10}$.
38. $2\frac{5}{12}$, $3\frac{1}{6}$, $7\frac{5}{24}$.
39. $9\frac{1}{3}$, $7\frac{3}{8}$, $2\frac{7}{12}$.
40. $4\frac{1}{4}$, $5\frac{1}{6}$, $7\frac{11}{12}$.
41. $8\frac{2}{5}$, $3\frac{3}{4}$, $12\frac{9}{10}$.
42. $6\frac{7}{8}$, $4\frac{13}{32}$, $3\frac{5}{16}$.

43. A boy has \$$\frac{1}{2}$ and \$$\frac{1}{5}$. What part of \$1 has he?

44. How much is $\frac{1}{2}$ of an hour, $\frac{1}{3}$ of an hour, $\frac{1}{4}$ of an hour?

45. Which is the largest and which the smallest of the three fractions, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$?

46. How much is $\frac{3}{4}$ of a foot and $\frac{1}{3}$ of a foot?

47. How much is $\frac{3}{4}$ of a dollar and $\frac{2}{5}$ of a dollar?

48. How much is $\frac{1}{2}$ of a quart, $\frac{1}{3}$ of a quart, and $\frac{5}{6}$ of a quart?

49. Find the price of 4 books, if each book cost \$$\frac{3}{5}$.

50. A boy studies $1\frac{1}{2}$ hours in the morning, $1\frac{2}{3}$ hours in the afternoon, and $\frac{3}{4}$ of an hour at night. How many hours does he study in all?

51. A grammar cost \$$\frac{1}{2}$, and a geography cost \$$\frac{3}{5}$. What was the cost of the two books?

52. If sugar costs $5\frac{1}{4}$ cents a pound, how much will 4 pounds of sugar cost?

SUBTRACTION

116. *Example* **1.** From $\frac{4}{5}$ take $\frac{3}{4}$.

$$\begin{array}{r} \frac{4}{5} = \frac{16}{20} \\ \underline{\frac{3}{4} = \frac{15}{20}} \\ \frac{1}{20} \end{array}$$

Reduce, first, the fractions to the same denomination. L. C. M. of 4 and 5 is 20. We have then $\frac{16}{20} - \frac{15}{20} = \frac{1}{20}$.

Example **2.** Find the difference between 20 and $7\frac{5}{8}$.

$$\begin{array}{ll} 20 & \text{(sum of two numbers)} \\ \underline{7\frac{5}{8}} & \text{(one of the numbers)} \\ 12\frac{3}{8} & \end{array}$$

$\frac{5}{8}$ and $\frac{3}{8} = 1$. Write $\frac{3}{8}$ and carry 1, as we do in addition. 8 and 2, 10. Write 2. 1 and 1, 2. Write 1.

Example **3.** Find the value of $\$17\frac{3}{10} - \$12\frac{3}{4}$.

If a person had $\$17\frac{3}{10}$ and had to pay a bill amounting to $\$12\frac{3}{4}$, the way his change would be made out is somewhat as follows:

$$\begin{array}{ll} \$17\frac{3}{10} & \text{(sum of two numbers)} \\ \underline{\$12\frac{3}{4}} & \text{(one of the numbers)} \\ \$\ \ 4\frac{11}{20} & \end{array}$$

First take $12\frac{3}{4}$ from 17. $\frac{3}{4}$ and $\frac{1}{4} = \frac{4}{4} = 1$. 3 and 4, 7. To this sum $\frac{3}{10}$ is then added. $\frac{3}{10} + 4\frac{1}{4} = \frac{6}{23} + 4\frac{5}{20} = 4\frac{11}{20}$.

This method works in all cases but one. An example where we could not proceed in this way is, $\$4\frac{3}{4} - \$4\frac{2}{5}$. In cases where the integral parts of the minuend and subtrahend are the same we subtract the fractions at once. This is what is done in actual business.

Another Method

$$\begin{array}{rr}
 & \underline{20} \\
\$17\frac{3}{10} & 6 \\
\underline{12\frac{3}{4}} & \underline{15} \\
\$\ 4\frac{11}{20} & \frac{11}{20}
\end{array}$$

Reduce to the same denomination. Write the L. C. M. of the denominators above a horizontal line; underneath write the number of fractional units in each fraction. Add the number to 15, which makes the sum $20 + 6$. Write the result for the numerator of the fraction in the remainder.

EXERCISE 51 (Written)

Find the value of:

1. $\frac{3}{4} - \frac{1}{2}$.
2. $\frac{2}{3} - \frac{1}{2}$.
3. $\frac{2}{3} - \frac{1}{6}$.
4. $\frac{5}{6} - \frac{2}{3}$.
5. $\frac{9}{10} - \frac{1}{2}$.
6. $\frac{7}{8} - \frac{3}{4}$.
7. $\frac{9}{10} - \frac{2}{5}$.
8. $\frac{11}{12} - \frac{3}{4}$.
9. $\frac{7}{12} - \frac{1}{3}$.
10. $\frac{11}{12} - \frac{5}{6}$.
11. $\frac{19}{24} - \frac{2}{3}$.
12. $\frac{4}{5} - \frac{3}{10}$.
13. $3 - 1\frac{1}{2}$.
14. $4 - 1\frac{3}{4}$.
15. $5 - 3\frac{3}{8}$.
16. $6 - 4\frac{7}{10}$.
17. $11 - \frac{4}{5}$.
18. $13 - \frac{7}{8}$.
19. $14 - 5\frac{1}{6}$.
20. $15 - 3\frac{7}{12}$.
21. $28 - 21\frac{9}{10}$.
22. $17\frac{3}{4} - 10\frac{1}{2}$.
23. $18\frac{2}{3} - 11\frac{1}{4}$.
24. $5\frac{4}{5} - 2\frac{3}{10}$.
25. $29\frac{7}{8} - 24\frac{3}{4}$.
26. $33\frac{2}{3} - 17\frac{1}{6}$.
27. $9\frac{5}{12} - 3\frac{1}{4}$.
28. $32\frac{17}{24} - 30\frac{5}{12}$.
29. $31\frac{13}{24} - 20\frac{1}{12}$.
30. $40\frac{2}{3} - 30\frac{11}{24}$.
31. $4\frac{1}{2} - 1\frac{2}{3}$.
32. $7\frac{2}{3} - 3\frac{3}{4}$.
33. $9\frac{1}{2} - 5\frac{7}{12}$.
34. $10\frac{1}{3} - 9\frac{3}{4}$.
35. $16\frac{1}{4} - 9\frac{2}{3}$.
36. $4\frac{2}{5} - 1\frac{7}{10}$.
37. $9\frac{3}{5} - 2\frac{9}{10}$.
38. $8\frac{1}{2} - 3\frac{9}{10}$.
39. $9\frac{3}{4} - 4\frac{11}{12}$.
40. $18\frac{1}{3} - 9\frac{3}{5}$.
41. $28\frac{1}{3} - 8\frac{4}{9}$.
42. $17\frac{1}{4} - 9\frac{7}{20}$.

43. What number must be added to $1\frac{1}{2}$ to make $7\frac{1}{8}$?

44. A man buys a suit of clothes for \$$12\frac{3}{4}$ and gives 3 five-dollar bills in payment. How much change should he receive?

45. From a piece of cloth containing $17\frac{3}{8}$ yards $14\frac{3}{4}$ yards are sold. How many yards are left?

46. A boy buys two books costing \$$\frac{3}{4}$ and \$$\frac{3}{5}$. How much change should he get out of a \$$2\frac{1}{2}$ gold piece?

MULTIPLICATION OF A FRACTION BY AN INTEGER

117. *Example* **1.** Multiply $\frac{3}{4}$ by 18.

$\frac{3}{4} \times 18 = 3$ fourths $\times\, 18 = 54$ fourths $= \frac{54}{4} = 13\frac{2}{4} = 13\frac{1}{2}$.

The mechanical work may be contracted as follows:

$$\frac{3}{4} \times 18 = \frac{3 \times \overset{9}{\cancel{18}}}{\underset{2}{\cancel{4}}} = \frac{27}{2} = 13\frac{1}{2}.$$

Here we cancel 2 in the numerator and denominator. **To multiply a fraction by an integer,** multiply the integer by the numerator of the fraction, and divide by the denominator. Cancel factors in the numerator and denominator whenever possible.

MULTIPLICATION OF A MIXED NUMBER BY AN INTEGER

118. *Example* **2.** Multiply $3\frac{3}{10}$ by 14.

Since multiplication in which the multiplier is an integer has been defined as a short method of addition wherein the addends are each equal to the multiplicand, we have

$$3\tfrac{3}{10} \times 14 = (3 \times 14) + (\tfrac{3}{10} \times 14) = 42 + 4\tfrac{1}{5} = 46\tfrac{1}{5}.$$

Another Method

$$3\tfrac{3}{10}=\tfrac{33}{10}.\quad 3\tfrac{3}{10}\times 14=\tfrac{33}{10}\times 14=\frac{33\times \overset{7}{\cancel{14}}}{\underset{5}{\cancel{10}}}=\tfrac{231}{5}=46\tfrac{1}{5}.$$

We first reduce the mixed number to an improper fraction, and then multiply by 14. The first method is to be preferred.

To multiply a mixed number by an integer, first multiply the fractional part of the mixed number by the multiplier, next multiply the integral part of the mixed number by the multiplier; add the two results for the final product.

EXERCISE 52 (Written)

Find the value of:

1. $\frac{2}{3}\times 18$.
2. $\frac{3}{4}\times 14$.
3. $\frac{4}{5}\times 7$.
4. $\frac{9}{10}\times 18$.
5. $\frac{7}{10}\times 16$.
6. $\frac{3}{8}\times 12$.
7. $\frac{5}{8}\times 19$.
8. $\frac{7}{8}\times 28$.
9. $\frac{3}{14}\times 24$.
10. $\frac{5}{16}\times 40$.
11. $\frac{9}{16}\times 34$.
12. $\frac{11}{12}\times 42$.
13. $\frac{13}{16}\times 44$.
14. $\frac{19}{20}\times 50$.
15. $\frac{17}{20}\times 45$.
16. $1\frac{3}{4}\times 9$.
17. $1\frac{3}{8}\times 10$.
18. $1\frac{7}{8}\times 12$.
19. $3\frac{3}{8}\times 10$.
20. $5\frac{7}{12}\times 54$.
21. $6\frac{7}{8}\times 36$.
22. $9\frac{1}{2}\times 16$.
23. $7\frac{5}{8}\times 44$.
24. $10\frac{5}{16}\times 24$.
25. $7\frac{4}{11}\times 33$.
26. $8\frac{5}{11}\times 19$.
27. $9\frac{5}{12}\times 80$.
28. $6\frac{11}{12}\times 102$.
29. $7\frac{7}{16}\times 60$.
30. $11\frac{1}{10}\times 15$.
31. $12\frac{3}{10}\times 18$.
32. $10\frac{4}{9}\times 13$.

33. Find the cost of a dozen cans of baking powder at $37\frac{1}{2}$ cents a can.

34. A pail of mackerel cost $2\frac{1}{8}$ dollars. Find the cost of 20 pails.

35. Find the cost of a barrel of sugar weighing 325 pounds, if the cost per pound is $4\frac{11}{16}$¢.

36. When starch sells for $3\frac{1}{2}$¢ a pound, find the price of 15 pounds of starch.

37. When wheat is $79\frac{1}{8}$ cents a bushel, how much will 164 bushels bring?

38. Find the cost of a 75-pound chest of Hyson tea at $42\frac{1}{2}$¢ per pound.

39. If a dozen cakes of yeast cost $42\frac{3}{4}$¢, find the cost of 9 dozen cakes of yeast.

40. Pepper sells for $14\frac{3}{4}$¢ a pound. Find the cost of 2 bags, each containing 120 pounds.

41. A pound package of chocolate costs $31\frac{1}{4}$¢. Find the cost of 25 such packages.

42. A square rod equals $30\frac{1}{4}$ square yards. Reduce 160 square rods to square yards.

43. A link of a surveyor's chain is $7\frac{23}{25}$ inches. If the chain contains 100 links, how many inches long is the chain?

44. When silk sells at $\$\frac{17}{20}$ a yard, what is the cost of 14 yards of silk?

45. A degree on a meridian of the earth's surface is about $69\frac{1}{10}$ miles. How many miles are in 15 degrees? In 40 degrees?

46. A person fails for $9800. His creditors receive $\$\frac{3}{4}$ on every dollar that he owes. How much in all do they receive?

47. A mass of copper and lead weighs 2240 pounds. $\frac{4}{5}$ of the mass is copper. How much copper and how much lead is in the mass?

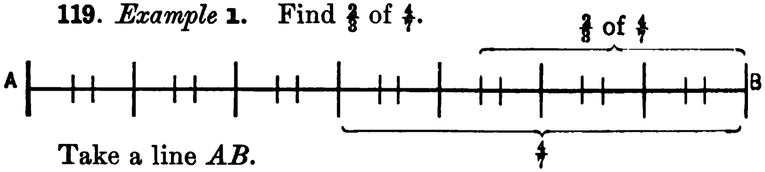
48. A man invests $2300. At the end of a year his gain is $\frac{7}{20}$ of his investment. Find his gain and the value of the investment at the end of the year.

49. A clerk gets a salary of $900 a year. He spends $\frac{7}{8}$ of his salary. How much does he spend?

50. A stenographer gets a salary of $50 a month, and spends each month 38\frac{3}{4}$. How much will he save in a year?

51. A mass of coal contains $\frac{9}{20}$ carbon. How much carbon is in 2240 pounds of coal?

119. *Example* **1.** Find $\frac{2}{3}$ of $\frac{4}{7}$.



Take a line AB.

Divide it into 7 equal parts. Subdivide each of these parts into 3 equal parts. We shall then have 21 subdivisions in all. One of these subdivisions will be equal to $\frac{1}{21}$ of AB.

Hence, $\frac{1}{3}$ of $\frac{1}{7} = \frac{1}{21}$,

therefore $\frac{1}{3}$ of $\frac{4}{7} = \frac{4}{21}$,

therefore $\frac{2}{3}$ of $\frac{4}{7} = \frac{8}{21}$.

Or, $\frac{2}{3}$ of $\frac{4}{7} = \frac{2}{3}$ of $\frac{12}{21}$,

$\frac{1}{3}$ of $\frac{12}{21} = \frac{4}{21}$,

therefore $\frac{2}{3}$ of $\frac{12}{21} = \frac{8}{21}$,

therefore $\frac{2}{3}$ of $\frac{4}{7} = \frac{8}{21}$.

In a similar manner we should find $\frac{4}{7}$ of $\frac{2}{3} = \frac{8}{21}$.

Example **2.** What is the product of $\frac{2}{3}$ multiplied by $\frac{4}{7}$?

We know from § 117, $\frac{2}{3} \times 4 = \frac{8}{3}$. If our multiplier, instead of being 4, were $\frac{1}{7}$ of 4 or $\frac{4}{7}$, we should naturally expect the product to be $\frac{1}{7}$ of the previous result.

Hence, $\frac{2}{3} \times \frac{4}{7} = \frac{1}{7}$ of $\frac{8}{3} = \frac{8}{21}$.

But
$$\frac{8}{21} = \frac{2 \times 4}{3 \times 7}.$$

The **result of multiplying a fraction by a fraction** is another fraction whose numerator is the product of the numerators of the two fractions, and whose denominator is the product of the denominators of the two fractions.

Notice also $\frac{2}{3}$ of $\frac{4}{7}$ is equivalent to $\frac{2}{3} \times \frac{4}{7}$.

120. *Example* 3. Find the product of $\frac{2}{5}, \frac{2}{7}, \frac{3}{11}$. Find first the product of $\frac{2}{5}, \frac{2}{7}$. $\frac{2}{5} \times \frac{2}{7} = \frac{2 \times 2}{5 \times 7}$. Then multiply this product by $\frac{3}{11}$.

$$\frac{2 \times 2}{5 \times 7} \times \frac{3}{11} = \frac{2 \times 2 \times 3}{5 \times 7 \times 11} = \frac{12}{385}.$$

To find the **continued product of several fractions,** find the product of the numerators, find the product of the denominators; write the first result as numerator of the final product, and the second result as its denominator.

Another Point of View

Assuming that the **Associative Law,** § 90, is true in all cases, the above results can be more elegantly deduced. Suppose we wish to find the product of $\frac{5}{8}, \frac{3}{4}$.

$$\frac{5}{8} \times \frac{3}{4} = 5 \times \frac{1}{8} \times 3 \times \frac{1}{4} = 15 \times \frac{1}{32} = \frac{15}{32}.$$

121. *Example* 4. Multiply $3\frac{3}{4}$ by $2\frac{7}{8}$. Reduce the mixed numbers to improper fractions and then proceed as in the previous exercises.

NOTE. Multiplication has been already defined as the process of taking a number a given number of times. To take $\frac{5}{8}$, $\frac{3}{4}$ of a time as addend is absurd. The term *multiplication* now takes on an additional meaning. To multiply by $\frac{3}{4}$ means to take the fourth part of the multiplicand and afterwards to multiply this result by 3. The authors are of the opinion that a formal definition of multiplication is as yet too difficult for the beginner to understand.

EXERCISE 53 (Written)

Multiply:

1. $\frac{1}{2}, \frac{2}{3}$.
2. $\frac{1}{2}, \frac{3}{4}$.
3. $\frac{4}{5}, \frac{10}{11}$.
4. $\frac{7}{8}, \frac{9}{10}$.
5. $\frac{3}{8}, \frac{17}{25}$.
6. $\frac{7}{12}, \frac{19}{28}$.
7. $\frac{16}{19}, \frac{57}{74}$.
8. $\frac{17}{31}, \frac{93}{119}$.
9. $\frac{14}{25}, \frac{75}{112}$.
10. $\frac{12}{27}, \frac{45}{64}$.
11. $\frac{15}{29}, \frac{87}{105}$.
12. $\frac{16}{35}, \frac{105}{112}$.
13. $\frac{3}{7}, \frac{11}{16}, \frac{8}{33}$.
14. $\frac{4}{9}, \frac{15}{18}, \frac{27}{40}$.
15. $\frac{4}{13}, \frac{26}{45}, \frac{9}{16}$.
16. $\frac{9}{20}, \frac{70}{81}, \frac{27}{28}$.
17. $\frac{11}{20}, \frac{90}{121}, \frac{33}{45}$.
18. $\frac{16}{55}, \frac{11}{48}, \frac{15}{17}$.
19. $\frac{1}{4}, \frac{7}{9}, \frac{72}{101}$.
20. $\frac{3}{5}, \frac{3}{9}, \frac{135}{243}$.
21. $1\frac{2}{3}, 1\frac{2}{3}$.
22. $1\frac{1}{2}, 1\frac{1}{2}, 1\frac{1}{2}$.
23. $2\frac{1}{7}, \frac{7}{15}, \frac{3}{4}$.
24. $3\frac{3}{4}, \frac{4}{15}, \frac{4}{5}$.
25. $2\frac{2}{3}, 2\frac{2}{3}, \frac{5}{64}$.
26. $7\frac{1}{4}, 1\frac{3}{4}, \frac{16}{29}$.
27. $2\frac{5}{8}, 3\frac{3}{5}, \frac{20}{21}$.
28. $4\frac{1}{6}, 2\frac{3}{5}, \frac{64}{65}$.
29. $7\frac{1}{2}, 7\frac{1}{2}, \frac{4}{15}$.
30. $3\frac{1}{8}, 3\frac{1}{8}, \frac{7}{25}$.
31. $5\frac{1}{2}, 5\frac{1}{2}, \frac{8}{11}$.
32. $7\frac{3}{4}, 3\frac{1}{2}, \frac{61}{62}$.
33. $1\frac{4}{5}, \frac{5}{18}, \frac{2}{3}$.
34. $7\frac{2}{3}, 1\frac{7}{8}, \frac{45}{46}$.
35. $4\frac{1}{4}, 4\frac{1}{4}, 3\frac{5}{9}$.
36. $1\frac{1}{5}, 1\frac{1}{6}, \frac{5}{7}$.
37. $20\frac{1}{2}, 20\frac{1}{2}$.
38. $4\frac{1}{2}, 5\frac{1}{2}, 1\frac{3}{11}$.
39. $6\frac{3}{4}, 1\frac{7}{9}, \frac{7}{12}$.
40. $2\frac{2}{5}, 1\frac{5}{12}, \frac{10}{17}$.
41. $2\frac{5}{7}, 8\frac{3}{5}, 11\frac{9}{13}$.
42. $6\frac{7}{8}, 4\frac{2}{9}, 13\frac{1}{15}$.

43. Find the value of $63\frac{1}{2}$ acres of land at \$$55\frac{1}{2}$ an acre.

44. Find the cost of $12\frac{1}{2}$ pounds of meat at $15\frac{1}{2}$ cents per pound.

45. Find the price of $4\frac{1}{2}$ bushels of wheat at $81\frac{1}{4}$ cents per bushel.

46. A speculator buys 10,000 bushels of wheat at $79\frac{5}{8}$ cents per bushel and sells it when wheat is $81\frac{1}{4}$ cents per bushel. Find his profit.

47. Coal cost \$$9\frac{1}{2}$ a ton. Find the price of $5\frac{1}{2}$ tons.

48. A man travels $3\frac{1}{2}$ miles an hour. At this rate how far would he travel in $3\frac{1}{7}$ hours?

49. A train runs at the rate of $37\frac{1}{2}$ miles an hour. How far will it go in $3\frac{3}{5}$ hours?

50. A house is valued at $3500. It is insured for $\frac{3}{4}$ of its value. How much is it insured for?

51. Find the cost of 2500 barrels of Beaumont oil at $25\frac{1}{4}$ cents a barrel.

52. A ton of hay cost $6.75. Find the price of $6\frac{3}{4}$ tons.

DIVISION

There are three symbols to indicate division. Thus,

$$(a)\ 17 \div 4,\quad (b)\ \tfrac{17}{4},\quad (c)\ 17:4,$$

all indicate that 17 is to be divided by 4. The indicated division in (*b*) is called a fraction. That in (*c*) is called a ratio.

122. By the **ratio of one number to another** is meant the quotient of the first number divided by the second number.

Ratio is generally defined as the mutual relation of two magnitudes of the same kind with respect to quantity. This definition is borrowed from Euclid, a celebrated Greek writer on geometry, who lived about 2200 years ago. To a beginner it conveys no definite meaning. Nowadays we represent all magnitudes by numbers. A ratio is simply a quotient.

123. *Example* 1. Divide 23 by 7. This problem might be stated in another way. What is $\frac{1}{7}$ of 23? But $\frac{1}{7}$ of $23 = \frac{1}{7} \times 23$. Hence we can divide by 7 by multiplying by $\frac{1}{7}$.

Example 2. Divide $\frac{3}{5}$ by $\frac{4}{7}$.

If we multiply dividend and divisor by the same number, the quotient remains the same. Multiplying by the L. C. M. of the denominators, we have:

$$\tfrac{3}{5} \times 35 = 3 \times 7.$$
$$\tfrac{4}{7} \times 35 = 5 \times 4.$$

Hence the quotient is $\frac{3 \times 7}{5 \times 4} = \frac{21}{20} = 1\frac{1}{20}$.

Since $\frac{3 \times 7}{5 \times 4} = \frac{3}{5} \times \frac{7}{4}$, the quotient can be readily obtained by inverting the terms of the divisor and multiplying. By inverting we mean writing the numerator in place of the denominator, and the denominator in place of the numerator.

124. To divide by a fraction, invert the terms of the divisor and then proceed as in multiplication.

To divide by an integer, multiply the dividend by the fraction having 1 for numerator and the integer for denominator.

Another Method

$$\tfrac{3}{5} \div \tfrac{4}{7} = \tfrac{3}{5} \div \tfrac{1}{7} \text{ of } 4.$$
$$\tfrac{3}{5} \div 4 = \tfrac{3}{20}.$$

If the divisor is $\frac{1}{7}$ of 4, the quotient will be 7 times as great as when the divisor is 4.

Hence, $\frac{3}{5} \div \frac{1}{7}$ of $4 = 7 \times \frac{3}{20} = \frac{21}{20} = 1\frac{1}{20}$.

EXERCISE 54 (Written)

Divide:

1. $\frac{3}{4}$ by 4.
2. $\frac{3}{7}$ by 5.
3. $\frac{5}{8}$ by 6.
4. $\frac{5}{9}$ by 7.
5. $\frac{3}{11}$ by 10.
6. $\frac{5}{12}$ by 6.
7. $\frac{9}{14}$ by 6.
8. $\frac{15}{19}$ by 20.
9. $\frac{8}{11}$ by 12.

10. $\frac{16}{21}$ by 24.
11. $\frac{18}{25}$ by 27.
12. $\frac{20}{27}$ by 16.
13. $\frac{9}{14}$ by 12.
14. $\frac{12}{13}$ by 16.
15. $\frac{10}{13}$ by 15.
16. $\frac{10}{11}$ by 25.
17. $\frac{14}{15}$ by 21.
18. $\frac{18}{21}$ by 36.
19. 12 by $\frac{2}{3}$.
20. 14 by $\frac{2}{7}$.
21. 16 by $\frac{4}{7}$.
22. 18 by $\frac{6}{7}$.
23. 21 by $\frac{3}{4}$.
24. 25 by $\frac{5}{6}$.
25. 26 by $\frac{4}{5}$.
26. 26 by $\frac{13}{14}$.
27. 33 by $\frac{6}{11}$.
28. 40 by $\frac{5}{12}$.
29. 60 by $\frac{8}{9}$.
30. 11 by $\frac{2}{9}$.
31. 1 by $\frac{1}{2}$.
32. 1 by $\frac{3}{4}$.
33. 1 by $\frac{2}{11}$.
34. 1 by $\frac{5}{13}$.
35. 1 by $\frac{6}{17}$.
36. 1 by $\frac{9}{82}$.
37. $3\frac{3}{4}$ by 10.
38. $2\frac{1}{8}$ by 3.
39. $3\frac{5}{9}$ by 8.
40. $4\frac{5}{11}$ by 14.
41. $5\frac{5}{13}$ by 21.
42. $9\frac{7}{12}$ by 46.
43. $9\frac{9}{10}$ by 77.
44. $9\frac{3}{13}$ by 45.
45. $16\frac{4}{5}$ by 63.
46. $18\frac{1}{5}$ by 26.
47. $17\frac{1}{7}$ by 75.
48. $10\frac{2}{7}$ by 48.
49. $9\frac{2}{7}$ by 52.
50. $9\frac{1}{11}$ by 75.
51. $8\frac{1}{3}$ by 15.
52. $7\frac{9}{13}$ by 80.
53. $5\frac{15}{17}$ by 50.
54. $8\frac{5}{8}$ by 46.
55. $\frac{5}{14}$ by $\frac{3}{7}$.
56. $\frac{1}{3}$ by $\frac{5}{12}$.
57. $\frac{9}{10}$ by $\frac{7}{12}$.
58. $\frac{7}{8}$ by $\frac{8}{9}$.
59. $\frac{3}{4}$ by $\frac{4}{3}$.
60. $\frac{3}{7}$ by $\frac{3}{11}$.
61. $\frac{2}{5}$ by $\frac{4}{13}$.
62. $\frac{3}{10}$ by $\frac{6}{11}$.
63. $\frac{7}{12}$ by $\frac{14}{15}$.
64. $\frac{8}{9}$ by $\frac{4}{5}$.
65. $\frac{9}{14}$ by $\frac{7}{12}$.
66. $\frac{8}{15}$ by $\frac{4}{25}$.
67. $\frac{9}{28}$ by $\frac{18}{35}$.
68. $22\frac{10}{13}$ by 8.
69. $4\frac{4}{5}$ by 9.
70. $4\frac{2}{3}$ by $1\frac{3}{4}$.
71. $3\frac{3}{4}$ by $2\frac{5}{7}$.
72. $4\frac{3}{8}$ by $9\frac{2}{7}$.
73. $7\frac{1}{5}$ by $3\frac{1}{2}$.
74. $4\frac{1}{17}$ by $17\frac{1}{4}$.
75. $3\frac{1}{3}$ by $\frac{7}{9}$.
76. 1 by $4\frac{3}{4}$.
77. 1 by $7\frac{1}{7}$.
78. 1 by $9\frac{2}{3}$.
79. $10\frac{2}{3}$ by $2\frac{2}{3}$.
80. $14\frac{2}{3}$ by $1\frac{11}{12}$.
81. $\frac{19}{36}$ by $1\frac{7}{12}$.
82. $\frac{14}{15}$ by $1\frac{3}{11}$.
83. $9\frac{1}{11}$ by $11\frac{1}{9}$.
84. $7\frac{1}{7}$ by $4\frac{1}{6}$.

EXERCISE 55 (Written)

1. What is the ratio of 6 inches to 12 inches?

2. What is the ratio of 1 foot to 1 yard?

3. What is the ratio of 1 square foot to 1 square yard?

4. There are $30\frac{1}{4}$ square yards in 1 square rod. What is the ratio of 1 square yard to 1 square rod?

5. What is the ratio of 3 weeks to 10 days?

6. What is the ratio of 1 hour to 1 minute?

7. What is the ratio of 4 days to 15 hours?

8. What is the ratio of 325 to 100?

Find the values of the following ratios:

9.	$2 : \frac{1}{2}$.	15.	$6 : \frac{2}{3}$.	21.	$3\frac{1}{7} : 2\frac{4}{9}$.	27.	$10\frac{2}{5} : 7\frac{1}{9}$.
10.	$3 : \frac{1}{3}$.	16.	$2\frac{1}{2} : 3\frac{3}{4}$.	22.	$9\frac{1}{7} : 31\frac{1}{2}$.	28.	$7\frac{9}{13} : 6\frac{1}{4}$.
11.	$4 : \frac{1}{4}$.	17.	$7\frac{1}{8} : 21\frac{3}{8}$.	23.	$5\frac{1}{6} : 7\frac{3}{4}$.	29.	$4\frac{1}{6} : 7\frac{1}{7}$.
12.	$4 : \frac{3}{4}$.	18.	$5\frac{5}{6} : 4\frac{3}{8}$.	24.	$4\frac{7}{9} : 2\frac{1}{6}$.	30.	$7\frac{8}{9} : 7\frac{1}{10}$.
13.	$5 : \frac{1}{5}$.	19.	$9\frac{2}{3} : 7\frac{1}{4}$.	25.	$5\frac{1}{4} : 1\frac{1}{6}$.	31.	$8\frac{1}{4} : 1\frac{3}{8}$.
14.	$7 : \frac{6}{7}$.	20.	$3\frac{3}{8} : 5\frac{2}{5}$.	26.	$6\frac{1}{8} : 3\frac{1}{9}$.	32.	$6\frac{5}{11} : 4\frac{1}{6}$.

125. *Example* 1. Find the price of 2000 pounds of wheat at 84 ¢ a bushel.

To solve this question there are two steps to take.

Step 1. Find the number of bushels by dividing the number of pounds by the number of pounds in one bushel.

Step 2. Multiply the price of one bushel by the number of bushels.

SOLUTION. Number of bushels $= \frac{2000}{60}$.

$$\text{Price of the wheat} = 84¢ \times \frac{2000}{60} = \frac{\overset{14}{\cancel{84}} \times 200\cancel{0}}{\underset{\cancel{6}}{\cancel{6}\cancel{0}}}$$

$$= 2800¢ = \$28.$$

126. *Example* **2.** Three-fifths of a man's money is $2437. How much money has he?

$$\tfrac{3}{5} \text{ of his money} = \$2437.$$

$$\tfrac{1}{5} \text{ of his money} = \frac{\$2437}{3}.$$

$$\tfrac{5}{5} \text{ of his money} = \frac{\$2437}{3} \times 5 = \frac{\$2437 \times 5}{3}$$

$$= \frac{\$12185}{3} = \$4061.66\tfrac{2}{3}.$$

Hence, his money = $4061.67 (to nearest cent).

This method of solving a problem is known as the **analytical method.** It is called also the **unit method** because the value of the unit of the quantity under consideration is first sought and from this the value of any number of units is then obtained.

Another Method

$\frac{3}{5} \times$ amount of money = $2437. Here we have the product of two factors together with one of the factors.

Hence, amount of money = $2437 ÷ $\frac{3}{5}$.

EXERCISE 56 (Written)

1. Find the price of 78 acres of land if 25 acres are worth $1375.

2. When 18 pounds of sugar sell for $1, find the cost of 45 pounds.

3. When 7 bushels of wheat sell for $5.95, how much can a person get for 255 bushels?

4. If 5 bushels of barley sell for $2, how much will 343 bushels sell for?

5. If 6 barrels of flour are sold for $45, at this rate how much will 84 barrels sell for?

6. Seven barrels of pork sell for $80.50. Find the cost of 50 barrels of pork.

7. Nine barrels of salt cost $11.70. Find the cost of 19 barrels of salt.

8. Eleven bushels of oats are sold for $4.51. Find the value of 168 bushels.

9. Six barrels of lard bring $115. How much will 46 barrels bring?

10. When 7 yards of sheeting cost 50¢, find how much must be paid for 98 yards.

11. Six yards of cambric sell for 75¢. How much must be given for 34 yards of cambric?

12. Four yards of flannel cost $1.16. How much will 29 yards of flannel cost?

13. Eight yards of gingham cost 60¢. How much will 103 yards cost?

14. Nine yards of cotton fabric cost 75¢. How much will 69 yards cost?

15. Six yards of cotton cheviot cost $1. How much will 81 yards cost?

16. Five-eighths of a man's money is $75. How much money has he?

17. Three-fourths of the length of a pole is 81 feet. Find the length of the pole.

18. The eighth and the twelfth of a number are 15. What is the number?

19. A dealer sold $\frac{1}{3}$ of his coal and had 170 tons left. How many tons had he at first?

20. The fourth part and the sixth part of a number are 25. What is the number?

21. A man spends $\frac{1}{4}$ of his money and has left $342. How many dollars had he at first?

22. A's money is $1\frac{2}{3}$ times B's money. If A has $120, how much money has B?

23. Three-eighths of an estate was left to A, $\frac{1}{3}$ of it to B, and the remainder, amounting to $1400, to C. What was the value of the estate? Find A's share and B's share.

24. A man bequeaths $\frac{1}{5}$ of his property to his son, $\frac{1}{6}$ to his daughter, and the remainder, amounting to $2400, to his widow. Find the value of the property.

25. After gaining one-fourth of his outlay, a person is worth $5475. Find his outlay.

26. If $\frac{5}{7}$ of a farm contains 235 acres, how many acres are there in $\frac{1}{2}$ of the farm?

27. Three-eighths of an investment is $1260. Find $\frac{1}{2}$ of the investment.

28. A sum of money is divided between A and B. A gets $\frac{3}{5}$, B $\frac{2}{5}$ of the money. If A gets $900, find B's share.

29. After $\frac{7}{12}$ of his marbles are lost, a boy has 15 left. How many had he at first?

30. If $\frac{7}{12}$ of a yard of satin is worth $1, find the price of 8 yards.

EXERCISE 57 (Written)

REVIEW

1. By how much does $\frac{1}{4}$ exceed $\frac{1}{6}$?

2. Reduce to an improper fraction $94\frac{5}{8}$.

3. Which is the greatest and which is the least, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$?

4. If a man spends $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{12}$ of his money, what part of his money has he left?

5. A clerk gets a salary of \$65 a month. If he spends $\frac{7}{12}$ of his salary, how much does he save in a year?

6. Find the value of $\frac{3}{4}$ of $\frac{8}{9}$ of 285.

7. From the sum of $1\frac{1}{2}$ and $1\frac{2}{3}$ take their difference.

8. What number divided by $1\frac{2}{3}$ gives as a quotient $\frac{6}{7}$?

9. How often is $\frac{3}{4}$ of a yard contained in 12 yards?

10. What must be added to $3\frac{3}{5}$ to give $6\frac{3}{4}$?

11. What is the ratio of $1\frac{7}{8}$ to $\frac{3}{4}$?

12. What is the ratio of $1\frac{2}{3}$ yards to 7 feet?

13. A number is multiplied by $1\frac{7}{8}$, and the product is $4\frac{2}{7}$. Find the number.

14. What number divided by $1\frac{9}{14}$ gives $16\frac{2}{3}$ for a quotient?

15. Reduce $\frac{3}{4}$, $\frac{2}{3}$, $\frac{7}{24}$ to fractions of the same denomination.

16. Add $9\frac{1}{5}$ to $\frac{3}{4}$ of $13\frac{2}{3}$.

17. A man spends $\frac{3}{7}$ of his money on board, and $\frac{1}{3}$ of it on lodging. He saves \$375. Find his income.

18. It takes $2\frac{1}{4}$ yards to make a coat. How many coats can be made out of a piece of cloth containing $60\frac{3}{4}$ yards?

19. A, B, and C own a cotton press. At the end of a year A got $\frac{1}{2}$ the profits, B $\frac{1}{3}$, and C the remainder, which amounted to \$675. Find A's and B's respective shares of the profits.

20. A and B enter into business as partners. A gets $\frac{7}{16}$ of the profits, B gets the remainder, amounting to \$810. Find A's share of the profits.

21. A man spends $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$ of his money, and has left \$440. How much had he at first?

22. Two-thirds of a city lot is valued at $840. Find the value of the lot. Find also the value of $\frac{3}{4}$ of the lot.

23. If $1\frac{1}{2}$ acres of pasture will graze 5 sheep, how many sheep will $28\frac{1}{2}$ acres of this land graze?

24. When $\frac{3}{4}$ of a ton of coal costs $\$6\frac{3}{4}$, what will 11 tons of this kind of coal cost?

25. If $1\frac{1}{2}$ dozen eggs sell for 40¢, find the selling price of 24 dozen.

26. I can buy $1\frac{2}{3}$ acres of timber land for $80. If I buy 74 acres of this land, how much will it cost?

27. How many pounds of tea, at $66\frac{2}{3}$¢ per pound, can be bought for 1 ten-dollar bill?

28. A person buys $6\frac{3}{4}$ yards of silk at $1.20 per yard, and $8\frac{3}{4}$ yards of cloth at 96¢ per yard. How much does his bill amount to?

29. Find the difference between the greatest and the least of the fractions $\frac{5}{8}$, $\frac{13}{16}$, $\frac{3}{4}$, $\frac{27}{32}$. Find also the difference between the other two.

30. How long will it take a person to walk $12\frac{1}{4}$ miles, if he walks at the rate of $3\frac{1}{2}$ miles an hour?

31. A train travels at the rate of $31\frac{1}{2}$ miles an hour. How long will it take it to go $393\frac{3}{4}$ miles?

32. In $\frac{2}{3}$ of a pound avoirdupois there are $4666\frac{2}{3}$ grains. How many grains are in $5\frac{1}{2}$ pounds avoirdupois?

33. In one ounce avoirdupois weight there are $437\frac{1}{2}$ grains. How many grains are in $\frac{2}{5}$ of a pound avoirdupois? (16 ounces = 1 pound.)

34. How many minutes would it take to count 100,000,000, if one can count 500 in 4 minutes?

35. How long will it take a person to walk 5280 feet, if he takes 120 steps of $2\frac{1}{3}$ feet each minute?

36. The polar diameter of the earth is 41,709,790 feet. Reduce this to miles. (1760 yards = 1 mile.)

37. Subtract \$$\frac{3}{8}$ from \$$\frac{4}{5}$. Give your answer in cents.

38. Subtract \$$\frac{3}{4}$ from \$$\frac{4}{5}$. Give your answer in cents.

39. Find the difference in feet between $1\frac{7}{8}$ yards and $2\frac{2}{3}$ yards.

40. What is the smallest sum of money which exactly contains 10¢, 15¢, 24¢?

41. What is the smallest sum of money which exactly contains 12¢, 18¢, 8¢?

42. Find the sum of $\frac{1}{8}$ of \$10 and $\frac{1}{6}$ of \$17.28. Give your answer in dollars and cents.

43. Telegraph poles are $58\frac{2}{3}$ yards apart. A train passes 20 of them in 1 minute. How long will it take the train to go 1 mile? (1 mile = 1760 yards.)

44. A bicycle wheel in one revolution goes $7\frac{1}{3}$ feet. How many revolutions will it make in going 1 mile?

45. If the price of a duck is $\frac{5}{16}$ of the price of a turkey, and ducks sell for \$1.25 a pair, find the price of a turkey.

46. One-fortieth part of the weight of sea water is salt. Find the weight of sea water which will yield $1\frac{3}{4}$ tons of salt.

47. In a pound Troy there are 5760 grains, and in a pound avoirdupois there are 7000 grains. How many pounds avoirdupois are there in $17\frac{1}{2}$ pounds Troy?

48. How much will 40 men earn in 30 days, if 3 men earn \$8 in one day?

49. Coal is bought for \$$5\frac{1}{2}$ a ton, and sold at \$$8\frac{1}{4}$ a ton. Find the profit made in selling 240 tons.

50. If coal is bought at \$$6\frac{3}{4}$ a ton, and sold at \$$8\frac{1}{2}$ a ton, how many tons must be sold to make a profit of \$245?

51. How many yards in 1024 meters? (32 meters = 35 yards.)

52. How many minutes in $7\frac{4}{15}$ hours?

53. If $\frac{3}{4}$ of a pound of meat cost 12¢, find the price of $3\frac{3}{4}$ pounds of meat.

54. If $\frac{2}{3}$ of a yard of carpet cost 50¢, find the price of $5\frac{1}{5}$ yards of the carpet.

55. A bankrupt's debts amount to $3750, and his assets amount to $2150. How many cents can he pay on every dollar of debt?

56. Find the price of 6720 pounds of wheat at 83¢ a bushel. (60 pounds = 1 bushel of wheat.)

57. Find the value of 40 sacks of wheat, 118 pounds to the sack, at 78¢ a bushel.

58. A man's property is assessed at $3260. If he pays taxes on this sum at the rate of $1\frac{3}{4}$¢ on every dollar, how much taxes does he pay?

59. Find the product of the sum and difference of the fractions $\frac{3}{4}$, $\frac{1}{2}$.

60. Three bells toll at intervals of 3, 4, 10 seconds. If they begin to toll together, after how many seconds will they again toll together?

REVIEW QUESTIONS

1. Give the derivation of *fraction.*

2. Give two definitions of a fraction.

3. Explain the terms *numerator*, *denominator*, and *fractional unit.*

4. What is the distinction between proper and improper fractions?

5. What is a mixed number?

6. What are the terms of a fraction?

7. How would you show that $\frac{1}{6}$ of $4 = 4$ times $\frac{1}{6}$ of 1?

8. How are mixed numbers reduced to improper fractions?

9. How are improper fractions reduced to mixed numbers?

10. How is a fraction, proper or improper, multiplied by an integer?

11. What is the most important property of fractions?

12. How are 8ths reduced to 40ths?

13. When is a fraction in its simplest form?

14. How is a fraction reduced to its simplest form?

15. Upon what does the value of a fractional unit depend?

16. Give the rule for the addition of fractions.

17. How are mixed numbers added?

18. Give the rule for the subtraction of fractions.

19. How is a mixed number multiplied by an integer?

20. How do you multiply two fractions?

21. What do you mean by multiplication of fractions?

22. Give a rule for division of fractions.

23. Define ratio.

24. What is Euclid's definition of ratio?

CHAPTER IX

DECIMALS

Note. In previous chapters we have dealt with the decimal system of numeration and notation. At the risk of repetition, we shall now enter into a fuller explanation of the decimal system of notation, and undertake to show that the so-called decimal fractions are really a part of our ordinary number system.

NUMERATION AND NOTATION

127. To read any integer expressed in figures, we divide the figures into periods of three figures, commencing at the right. The period to the extreme right is called the units' period, the next period to the left is called the thousands' period, the next period to the left of the thousands' period is called the millions' period, the next to the left is called the billions' period, and so on. Each period has its units', its tens', and its hundreds' place. Thus the number expressed by the figures 1234567890 is divided into periods 1,234,567,890 and is read one billion, two hundred thirty-four million, five hundred sixty-seven thousand, eight hundred ninety.

The period in which 1 represents one is called the units' period.

The period in which 1 represents a thousand is called the thousands' period.

The period in which 1 represents a million is called the millions' period.

The place in which 1 represents ten ones is called the tens' place of the units' period.

The place in which 1 represents ten-thousands is called the tens' place of the thousands' period.

The place in which 1 represents ten-millions is called the tens' place of the millions' period.

The place in which 1 represents one hundred ones is called the hundreds' place of the units' period.

The place in which 1 represents one hundred-thousands is called the hundreds' place of the thousands' period.

The place in which 1 represents one hundred-millions is called the hundreds' place of the millions' period.

The figure 0 in any place has no value.

128. Consider next the integer 7777. The right-hand digit represents 7 ones. The 7 in the second place has ten times as great a value as the 7 in the units' place. The 7 in the third place has ten times as great a value as the 7 in the second place, and it has 100 times as great a value as the 7 in the units' place. The 7 in the fourth place has 10 times as great a value as the 7 in the third place, 100 times as great a value as the 7 in the second place, and 1000 times as great a value as the 7 in the units' place.

If we begin at the left, and consider the value of each digit, we shall find that the value of any 7 is one-tenth of the value of the 7 to its immediate left. **The value of any digit in any place is always one-tenth of the value of the same digit written in the next place to the left.** This is the **law** which pervades the decimal system of notation.

129. How shall we represent the tenth part of a unit? We have already represented it by the fraction $\frac{1}{10}$. Is there another way to represent $\frac{1}{10}$?

Consider the number represented by the digits 11. This is eleven. If we write a period, called the decimal point, after eleven and another 1, we have 11.1. To be consistent, this right-hand 1 must stand for $\frac{1}{10}$ of the 1 to its immediate left. A second 1 must stand for $\frac{1}{10}$ of the 1 to its immediate left, *i.e.* $\frac{1}{10}$ of $\frac{1}{10} = \frac{1}{100}$. The number 11.11 is then eleven, one-tenth, and one-hundredth.

Similarly 11.111 is eleven, one-tenth, one-hundredth, and one-thousandth. These numbers are not generally read in this way. We know $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} = \frac{100}{1000} + \frac{10}{1000} + \frac{1}{1000} = \frac{111}{1000}$, and this is read one hundred eleven thousandths. We can get the same result in another way. Take .111. The digit in the third place is 1 thousandth. The next is 10 times as great in value, hence 10 thousandths, the next to the left, is 10 times as great as the second, and hence, 100 thousandths. Hence .111 is read 111 thousandths.

130. Consider next 6.6666666. The first 6 following the decimal point is 6 tenths.

The 6 in the second place to the right of the decimal point is 6 hundredths.

The 6 in the third place to the right of the decimal point is 6 thousandths.

The 6 in the fourth place to the right of the decimal point is 6 ten-thousandths.

The 6 in the fifth place to the right of the decimal point is 6 hundred-thousandths.

The 6 in the sixth place to the right of the decimal point is 6 millionths.

The 6 in the seventh place to the right of the decimal point is 6 ten-millionths.

Beginning now at the right and writing the value of each figure in succession, we have

6 ten-millionths.
60 ten-millionths.
600 ten-millionths.
6000 ten-millionths.
60000 ten-millionths.
600000 ten-millionths.
6000000 ten-millionths.

Adding, we get 6,666,666 ten-millionths. Hence we read a decimal in the same way we read an integer, taking care, however, to give the decimal the name of the right-hand figure in it.

131. Read 22.234. Twenty-two and two hundred thirty-four thousandths. It might also be read: Twenty-two thousand two hundred thirty-four thousandths, and in various other ways.

In reading numbers and in writing them in words *and* should never appear except to connect the integral and decimal parts of a number.

132. Before commencing to read and write decimals, it is well to **fix firmly in mind the following facts**:

Tenths occupy the first place to the right of the decimal point; hundredths, the second place; thousandths, the third place; ten-thousandths, the fourth place; hundred-thousandths, the fifth place; millionths, the sixth place.

EXERCISE 58 (Oral)

Read:

1. 7.3.
2. 3.24.
3. 4.201.
4. 6.321.
5. .012.
6. .007.
7. 9.2354.
8. 10.2398.
9. .0295.

10. .0294.
11. 12.31456.
12. 19.26378.
13. 18.73025.
14. 100.01234.
15. 27.09001.
16. 28.00123.
17. 29.00201.
18. 25.00045.
19. 84.01010.
20. 97.02053.
21. 5.01709.
22. .001251.
23. .002325.
24. .010267.
25. .002304.
26. .000305.
27. .100102.
28. .3000107.
29. .2001003.
30. .3010102.
31. .2320101.
32. .200011.
33. .270235.
34. .0101201.
35. 1.111001.
36. 2.235121.
37. 1.473598.
38. 3.000101.
39. 5.001029.

133. *Example* **1.** Write twenty-four tenths.

We write 24 as if it were an integer. Tenths occupy the first place to the right of the decimal point. Placing 4 in the first place, we have 2.4. Verify by writing 24 tenths as an improper fraction, and then reducing to a mixed number.

Example **2.** Write two thousand three hundred four hundredths.

Write 2304 as if it were an integer. Point off two places for hundredths, beginning at the right. We then have 23.04.

If we reduce $\frac{2304}{100}$ to a mixed number, we get $23\frac{4}{100} =$ 23.04, a result which agrees with the first.

Example **3.** Write eleven hundred-thousandths. We write 11 as if it were an integer and then point off five places, beginning at the right. The places having no digits are filled in with ciphers: .00011.

Example **4.** Write five hundred and five millionths.

First write 500, and then write 5 in the sixth place for 5 millionths. Doing this, we have 500.000005. (See § 131.)

Example 5. Write five hundred five millionths. Here the number of units is 505, the denomination is millionths. Writing 505 as if it were an integer, and remembering millionths occupy the sixth place, we have .000505.

EXERCISE 59 (Written)

Write:

1. 33 hundredths.
2. 2005 thousandths.
3. 329 thousandths.
4. 101 thousandths.
5. Two hundred three thousandths.
6. Seven hundred and four thousandths.
7. Nine hundred three thousandths.
8. Nine hundred and three thousandths.
9. Six thousand seven hundred ten-thousandths.
10. Six thousand seven hundred and one ten-thousandth.
11. Five hundred ninety ten-thousandths.
12. Five hundred and ninety ten-thousandths.
13. Six thousand one ten-thousandths.
14. Seven hundred ten-thousandths.
15. Seven hundred ten thousandths.
16. Five hundred thousandths.
17. Five hundred-thousandths.
18. Two hundred seven hundred-thousandths.
19. Two hundred and seven hundred-thousandths.
20. Five thousand two hundred two hundred-thousandths.
21. Six thousand four hundred-thousandths.
22. Three thousand ten thousandths.
23. Three thousand one ten-thousandths.
24. Five hundred seventeen ten-thousandths.
25. One hundred eleven hundred-thousandths.
26. Seventy-eight ten-thousandths.

27. Ninety-one hundred-thousandths.

28. One hundred twenty-seven hundredths.

29. Twenty-four millionths.

30. One thousand one millionths.

134. Consider the two numbers,

(*a*) 320.12,
(*b*) 3201.2.

Both are expressed by the same figures written in the same order. The number (*b*) can be obtained from the number (*a*) by moving each figure one place to the left. But moving a digit one place to the left makes its value ten times as great, and, hence, moving several digits each one place to the left makes the number they represent ten times as great.

The number (*b*) can also be obtained from (*a*) by moving the decimal point in (*a*) one place to the right. Also (*a*) can be obtained from (*b*) by moving the decimal point in (*b*) one place to the left.

To multiply a number by 10, move the decimal point in the number one place to the right.

To divide a number by 10, move the decimal point in the number one place to the left.

135. Consider the numbers,

(*a*) 320.12
(*b*) 32,012.

The number (*b*) is obtained from (*a*) by moving each digit in (*a*) two places to the left. This multiplies each digit by 100.

(*b*) may also be obtained from (*a*) by moving the decimal point in (*a*) two places to the right. Also (*a*) from (*b*) by moving the decimal point two places to the left.

To multiply a number by 100, move the decimal point in the number two places to the right.

To divide by 100, move the decimal point in the dividend two places to the left.

136. Consider the numbers,

(*a*) 320.12,
(*b*) 320,120.

(*b*) is here obtained from (*a*) by moving each digit in (*a*) three places to the left. It can also be obtained from (*a*) by moving the decimal point in (*a*) three places to the right.

To multiply a number by 1000, move the decimal point in the number three places to the right.

To divide a number by 1000, move the decimal point in the number three places to the left.

The rules for multiplying by 10,000, 100,000 are left for the reader to determine.

NOTE. In the last three articles "number" does not include so-called common fractions.

137. *Example* 1. Multiply 86.4 by 10,000.

Moving the decimal point four places, we have 864,000.

Example 2. Divide 12.3 by 100,000. Moving the decimal point five places to the left, we have .000123.

EXERCISE 60

Multiply by 10:

1. 120, 14.2, .1431, .00012, 1.7320, .01234.

Multiply by 100:

2. 173, 172.8, 19.23, .001237, 8654, 17.1.

Multiply by 1000:

3. 1156, 32.5, 7.123, .93891, .01275, .00011.

Multiply by 10,000:

4. 345, 34.25, 5.1739, 6.001, .01793, .12.

5. Divide each of the following numbers by 10, by 100, by 1000, by 10,000, by 100,000:

32734
3745.3
928.49
72173.5
12.792
99999.9
3728.3
12.7564
1.201
.345
9285.773
325.298
1.27
.325
.017
18.326
7.294
.067
.1

Find the values of the following ratios:

6. 22.3 : .223.
7. 3.74 : .374.
8. 173.2 : 1.732.
9. 7.3 : .073.
10. 1.25 : .0125.
11. 9.28 : .00928.
12. 11.34 : .01134.
13. 7.04 : .0704.
14. 100 : .01.
15. 1000 : .001.
16. .012 : .12.
17. 1.24 : 124.
18. 9.53 : 9530.
19. 7.1 : 7100.
20. 6.5 : 65,000.
21. 11.79 : 11,790.
22. .001 : 10.
23. .005 : 100.
24. 9.265 : 926.5.
25. 12.325 : 1232.5.
26. 1.534 : 153.4.
27. 1001 : .1001.
28. 54 : .054.
29. 792 : .0792.
30. 113 : .0113.
31. 79.28 : .7928.
32. 6.45 : 6450.
33. 99.29 : 99,290.
34. 7.35 : 73,500.
35. 9.24 : 92,400.
36. 8.123 : .008123.
37. .04567 : 45.670.

ADDITION

138. Find the sum of 3.4, 2.38, 5.005, 6.2374, 11.1.

```
  3.4
  2.38
  5.005
  6.2374
 11.1
 ------
 28.1224
```

Write the numbers so that units of the same denomination stand in the same vertical column. Then add in the same way that we add integers. Write the decimal point in the sum in the same vertical line with the decimal points in the addends.

EXERCISE 61 (Written)

Add:

1. 2.2, .025, 37.3, 5.284, 6.294, 538.1, 77.77.
2. 3.5, 7.12, .339, 47.35, 39.28, .123, 54.275.
3. 9.28, 11.18, .999, 39.28, 7.451, 94.354, 98.76.
4. 12.49, 1.492, 38.75, 53.41, 98.69, 845.5, 892.9.
5. .009, 5.976, 40.99, 6.385, 9.278, 8.239, 64.271.
6. .098, 9.853, 19.47, 17.392, 28.394, 8.01, 77.47.
7. .285, 11.95, 29.99, 94.931, 1.732, 64.6, 78.75.
8. 11.4, 17.5, 99.37, 15.273, 9.394, 71.3, 92.95.
9. 1.21, 12.1, .121, 8.295, 7.777, 68.7, 78.28.
10. 15.9, 9.158, 91.58, 9.158, 2.293, 84.5, .139.
11. 98.5, 11.667, 66.66, 8.394, 9.928, 76.8, 9.359.
12. 77.8, 88.88, 99.99, 6.325, 7.384, 94.9, 1.798.

SUBTRACTION

139. Find the difference between 4001 and 1.7003.

```
4001.0000
   1.7003
---------
3999.2997
```

Arrange the numbers so that units of the same denomination stand in the same vertical column. Ciphers may be inserted after the decimal point in the minuend. Proceed next as in the subtraction of integers.

EXERCISE 62 (Written)

Find the remainder and verify your answer in each case:

1. 7.73 – 6.78.
2. 9.29 – 3.47.
3. 6.34 – 1.95.
4. 9.82 – 7.78.
5. 7.45 – 3.59.
6. 10.71 – 7.79.
7. 8.94 – 3.95.
8. 5.012 – 2.9.
9. 10.943 – 7.97.
10. 8.325 – 4.378.
11. 8.924 – 5.938.
12. 7.312 – 2.7.
13. 9.419 – 5.57.
14. 10.1 – 7.325.
15. 9.24 – 5.3481.
16. 8.73 – 4.4444.
17. 12.32 – 5.6741.
18. 19.33 – 6.2734.
19. 9.271 – 4.3847.
20. 3.213 – .9875.
21. 4.321 – .73201.
22. 5.204 – 1.3256.
23. 8.731 – 5.4557.
24. 9.21 – 7.2349.
25. 7.29 – 3.4551.
26. 6.001 – 5.112.

27. From seven hundred four thousandths take two hundred five ten-thousandths.

28. From five hundred ten thousandths take five hundred ten-thousandths.

29. From two thousand take two thousandths.

30. How much does one thousandth exceed one hundred-thousandth?

31. Find the difference between a hundred and a hundredth.

32. From 39 tenths take 39 thousandths.

33. From 100 hundredths take 100 ten-thousandths.

34. How much must be added to one and five-tenths to make ten?

35. By how much does 175 hundredths exceed 175 hundred-thousandths? What is the ratio of the first number to the second?

36. By how much does $1 exceed 1 mill?

37. By how much does $2 exceed 15 mills?

MULTIPLICATION

140. *Example* 1. Multiply 3.23 by 25.

3.23 = 323 hundredths (see § 131).
323 hundredths × 25 = 8075 hundredths = 80.75.

Example 2. Multiply 3.23 by .25.

Since the multiplier is $\frac{1}{100}$ of 25, we should naturally expect 3.23 × .25 = $\frac{1}{100}$ of 3.23 × 25. $\frac{1}{100}$ of 80.75 = .8075.

The mechanical work of multiplying may be performed as follows:

$$\begin{array}{r} 3.23 \\ .25 \\ \hline 1615 \\ 646 \\ \hline .8075 \end{array}$$

Multiply as if both numbers were integers, and point off in the product, commencing at the right, as many places as there are decimal places in both multiplicand and multiplier.

141. *Example* 1. Multiply .32 by .018.

$$\begin{array}{r} .018 \\ .32 \\ \hline 36 \\ 54 \\ \hline .00576 \end{array}$$

Here we point off five places.

Another Explanation

$\frac{32}{100} \times \frac{18}{1000} = \frac{576}{100000} = .00576.$

142. To square a number means to multiply the number by itself or to take the number twice as a factor.

To cube a number means to take the number three times as a factor.

NOTATION

143. 2^2 is a short way of writing 2 × 2.
2^3 is a short way of writing 2 × 2 × 2.
2^4 is a short way of writing 2 × 2 × 2 × 2.
2^5 is a short way of writing 2 × 2 × 2 × 2 × 2.

144. The result of taking a number any number of times as factor is called a **power of the number.** Thus $7^4 = 7 \times 7 \times 7 \times 7 = 2401$.

2401 is the 4th power of 7.

The 4 written to the right of 7 and slightly above it is called the **index** or **exponent** of the power.

EXERCISE 63 (Written)

1. Find .04 of $108; .05 of $274; .06 of $720; .07 of $144.

2. Find .09 of $34.50; .3 of $75.30; .08 of $75.80; .07 of $84.70.

3. Find .4 of $29.75; .5 of $69.48; .6 of $68.32; .1 of $328.50.

4. Find .125 of $80.80; .75 of $54; .6 of $300.50; .25 of $98.84.

5. Find .625 of $688; .875 of $792.80; .375 of $900.80.

6. Find .375 of 84 acres; .0625 of 64 acres; .3125 of 96 acres.

7. Find .1 of .1; .3 of 4; .3 of .3; .01 of .2; .01 of 1.2.

8. Multiply 27.9 by 18.

9. Multiply 1327 by 1.6.

10. Multiply 3927 by .46.

11. Multiply 120.01 by 3.6.

12. Multiply 25 by .017.

13. Multiply 37.5 by .07.

14. Multiply 11.9 by 2.4.

15. Multiply 182.54 by 1.49.

16. Multiply .286 by 1.96.

17. Multiply 92.24 by 2.7.

18. .148 × 1.15 = ?

19. .82 × .51 = ?

20. 1.875 × .32 = ?

21. 1.78 × 1.89 = ?

22. 18.24 × .95 = ?

23. 1.18 × .1695 = ?

24. .97 × .97 = ?

25. .68 × .68 = ?

26. .373 × .373 = ?

27. .901 × .901 = ?

28. .808 × .808 = ?

29. .693 × .693 = ?

30. .1 × .1 × .1 = ?

31. .3 × .3 × .3 = ?

32. .4 × .4 × .4 = ?

33. .7 × .7 × .7 = ?

34. 1.04 × 1.04 × 1.04 = ?

35. 1.06 × 1.06 × 1.06 = ?

36. 1.08 × 1.08 × 1.08 = ?

37. .25 × .25 × .25 = ?

38. The composition of coal mined in the Eagle Pass Coal Field, Texas, is given as follows:

Moisture = .094 of the mass.
Combustible matter = .3308 of the mass.
Fixed carbon = .4009 of the mass.
Ash = .1743 of the mass.

Find the amount of moisture, combustible matter, etc., in 112 tons of this coal. Check your answer.

39. A piece of coal taken from the mine at Coos Bay, Oregon, had the following composition:

Moisture = .1[illegible]42 of the mass.
Combustible matter = .4221 of the mass.
Fixed carbon = .[illegible] of the mass.
Ash = .[illegible] of the mass.

Find the amount of each in [illegible] tons of this coal; in [illegible] tons. Check your answers.

40. .7645 of the asphalt found in West Virginia is composed of carbon, .0783 is hydrogen, .1346 is oxygen, and the remainder is ash. How much of each constituent is in 254 tons of asphalt? Check your answers.

41. .7217 of the asphalt found in Oregon is composed of carbon, .079 of hydrogen, .1461 of oxygen, and the remainder of ash. Find the amount of each in 385 tons of asphalt. Check your answer.

42. Multiply the square of 14 by .7854.

43. The area of the surface of a sphere is obtained by multiplying the square of the diameter by 3.1416. Find the area of the surface of the earth, taking the diameter to be 7920 miles. Compare your answer with the area given in your geography.

44. The moon is nearly 2200 miles in diameter. Find the area of its surface in square miles.

45. The velocity of the earth in its orbit is 18.5 miles per second. How far does it go in 1 minute? In 1 hour?

46. A hurricane moves at the rate of 146.6 feet per second. How far does it travel in 1 minute? In 1 hour?

47. One meter = 39.3704 inches. Find in inches the difference between 64 meters and 70 yards.

DIVISION

145. Before undertaking Division, it may be well to lay stress on the fact that numbers in the decimal system of notation may be read in as many ways as we please. Thus, 32.25 may be read, (*a*) 32 and 25 hundredths; (*b*) 3225 hundredths; (*c*) 32250 thousandths; (*d*) 322500 ten-thousandths; (*e*) 322.5 tenths; (*f*) 3.225 tens.

146. *Example* 1. Divide 1.293 by 8.

$$\begin{array}{r} 8\overline{)1.293000} \\ .161625 \end{array}$$

8 into 12 goes 1 time with remainder 4. Write 1 in the quotient in the same column with 2, because 8 into 12 tenths gives 1 tenth. 8 into 49 goes 6 times with remainder 1; 8 into 13 goes 1 time, remainder 5; 8 into 50 goes 6 times, remainder 2; and so on.

Example 2. Divide .01234 by 4.

$$\begin{array}{r} 4\overline{).012340} \\ .003085 \end{array}$$

The work calls for no explanation.

EXERCISE 64 (Written)

Divide:

1. 73.21 by 8.
2. 3.45 by 4.
3. 19.362 by 6.
4. 1.791 by 9.
5. 4.564 by 5.
6. 3.927 by 8.
7. .015 by 5.
8. 8.846 by 6.
9. 8.218 by 7.
10. 3.942 by 6.
11. 6.475 by 7.
12. 9.143 by 8.
13. .1234 by 5.
14. .73206 by 6.
15. 1.1466 by 7.
16. 6.2751 by 8.
17. 5.472 by 6.
18. 8.2548 by 9.
19. .34794 by 9.
20. .67356 by 9.
21. .999999 by 7.
22. 7.3745 by 7.
23. 6.2676 by 6.
24. 1.7346 by 7.

147. Find the difference between .07858 and .078; also find the difference between .07858 and .079.

$$\begin{array}{r} .07858 \\ \underline{.078} \\ .00058 \end{array} \qquad \begin{array}{r} .07900 \\ \underline{.07858} \\ .00042 \end{array}$$

Hence .07858 is nearer to .079 than it is to .078. If, therefore, we are asked to give the value of .07858 correct to three figures, we write for answer .079.

Express .73948 correct to three figures. *Ans.* .739.

Express .25764 correct to three figures. *Ans.* .258.

Whenever we are asked to give a decimal correct to any number of figures, discard the remaining figures if the first one of them is less than 5; if it is 5 or more than 5, increase the last figure by 1.

148. *Example* 1. Divide .0732 by .8.

Make the divisor an integer by moving the decimal point one place to the right. Make a corresponding change in the dividend. This change is equivalent to multiplying divisor and dividend by 10. Making this change, we have:

$$\begin{array}{r} 8\underline{).7320} \\ .0915 \end{array}$$

Example 2. Divide 12 by .125.

Moving the decimal point in the divisor and in the dividend three places to the right, *i.e.* multiplying each by 1000, we have

$$\begin{array}{r} 96 \\ 125)12000 \\ \underline{1125} \\ 750 \\ \underline{750} \end{array}$$

Example 3. Divide 3.274 by 6.25.

Moving the decimal points two places to the right, we have

$$\begin{array}{r} .523 \\ 625)327.400 \\ \underline{3125} \\ 1490 \\ \underline{1250} \\ 2400 \\ \underline{1875} \\ 525 \end{array}$$

The quotient correct to three figures is .524, because the next figure is obviously more than 5.

Whenever the divisor is a decimal, make it an integer by moving the decimal point to the right. Make a corresponding change in the dividend. After doing this, proceed in exactly

the same manner as we have done in long division of integers. Write the decimal point in the quotient in the same vertical line with the decimal point in the dividend transformed.

EXERCISE 65 (Written)

Divide :

1. 2.34 by .8.
2. .012 by .5.
3. 3.475 by .4.
4. 1.2348 by .6.
5. .1798 by .5.
6. 3.144 by 1.2.
7. 5.96 by 1.6.
8. 3.2903 by 1.3.
9. .27 by .2.
10. 5.376 by 1.6.
11. 9.4851 by 1.5.
12. 3.2 by 6.4.
13. 20 by .5.
14. 10 by .16.
15. 40 by .32.
16. 56 by 1.12.
17. 84 by 5.6.
18. 56 by 1.12.
19. 100 by .625.
20. 100 by .008.
21. 400 by .05.
22. 144 by .288.
23. 15.4 by .616.
24. .096 by .192.
25. 1 by .001.
26. 5 by .004.
27. .1 by .0001.
28. .04 by .0008.
29. .32 by .00128.
30. .45 by .0018.
31. .078 by .00312.
32. .067 by .0268.
33. .01 by .8.
34. .002 by 1.6.
35. .018 by 45.
36. .54 by 81.
37. .243 by 1.944.
38. .216 by 1.44.
39. 5.12 by .16.
40. 7.29 by 270.
41. 34.7231 by .713.
42. 31.8791 by 3.97.
43. .267584 by 2.96.
44. .348336 by .492.
45. .190256 by .188.
46. 59.4204 by 5860.
47. 55.9911 by 108.3.
48. .575484 by 54.6.
49. .461071 by 122.3.
50. 4.50775 by 123.5.

EXERCISE 66 (Written)

Find the following quotients correct to four decimal figures:

1. 1.2801 ÷ 1.131.
2. 1.6796 ÷ 1.5216.
3. 3.1139 ÷ 2.6851.
4. 2.4325 ÷ 20.464.
5. 18.73 ÷ 153.95.
6. 21.07 ÷ 1732.
7. 33.731 ÷ 219.1.
8. 27.725 ÷ 18.01.
9. 7.5744 ÷ 487.7.
10. 2.786 ÷ 17.1.
11. 3.7335 ÷ 17.96.
12. 14.636 ÷ 94.34.
13. 120.41 ÷ 739.2.
14. 5.516 ÷ 27.86.
15. 3.0791 ÷ 3.1416.
16. 2.2167 ÷ 3.1416.
17. .054821 ÷ .07958.
18. .1416 ÷ 15.915.
19. 3.0788 ÷ 62.832.
20. 8.2886 ÷ 28.79.
21. 2.99384 ÷ 7.738.
22. 2.72912 ÷ 7.388.
23. 136.5874 ÷ 1652.8.
24. 137.614 ÷ 165.9.
25. .1533 ÷ .3502.
26. 3.4258 ÷ 17.347.
27. .9273 ÷ 27.35.
28. .6994 ÷ 94.58.
29. .8375 ÷ 9.241.
30. .01998 ÷ .9387.
31. .0473 ÷ .8479.
32. 1 ÷ .07958.
33. 10 ÷ 1.732.
34. 85 ÷ 1.4142.
35. .834 ÷ 2.245.
36. 7.28 ÷ 9.652.

FRACTIONS AND DECIMALS

149. A fraction such as $\frac{3}{4}$ or $\frac{11}{19}$ is called, in American text-books, a **common fraction**, and in English text-books on arithmetic, a **vulgar fraction**. A number such as .327 is called a **decimal number**, or **decimal fraction**. If .327 is written $\frac{327}{1000}$, it then becomes a common, or vulgar fraction.

Common fractions have been in existence as far back as historic records go. Decimals, on the other hand, are comparatively recent innovations. They were not generally taught in schools until the last century.

150. *Example* 1. Reduce $\frac{7}{8}$ to a decimal.

Dividing 7 by 8, we get $\frac{7}{8} = .875$.

Example 2. Reduce $\frac{7}{11}$ to a decimal.

$\frac{7}{11} = .636363 + = .6364$ correct to four figures.

Example 3. Reduce $\frac{37}{5000}$ to a decimal.

$$\frac{37}{5000} = \frac{7.4}{1000} = .0074.$$

We divide numerator and denominator by 5; then divide the numerator by 1000 by moving the decimal point three places to the left.

EXERCISE 67 (Written)

Reduce to decimals :

1. $\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{5}{16}, \frac{6}{25}, \frac{7}{32}, \frac{11}{64}, \frac{79}{128}$.
2. $\frac{17}{20}, \frac{19}{50}, \frac{41}{400}, \frac{83}{200}, \frac{67}{125}, \frac{87}{250}$.
3. $\frac{3}{10}, \frac{17}{100}, \frac{19}{1000}, \frac{11}{10000}, \frac{101}{100000}, \frac{1279}{1000000}$
4. $\frac{23}{80}, \frac{17}{40}, \frac{9}{20}, \frac{37}{50}, \frac{103}{500}, \frac{109}{4000}$.
5. $\frac{1}{4}, \frac{1}{40}, \frac{1}{400}, \frac{13}{2000}, \frac{31}{4000}, \frac{3}{160}$.
6. $\frac{7}{320}, \frac{59}{160}, \frac{79}{640}, \frac{83}{8000}, \frac{109}{1250}$.

Reduce to decimals correct to four figures :

7. $\frac{1}{3}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{12}, \frac{1}{13}, \frac{1}{14}, \frac{1}{15}, \frac{1}{17}$.
8. $\frac{5}{27}, \frac{11}{23}, \frac{9}{28}, \frac{4}{31}, \frac{16}{35}, \frac{7}{78}$.
9. $\frac{8}{33}, \frac{7}{88}, \frac{4}{13}, \frac{3}{7}, \frac{9}{19}, \frac{10}{21}$.
10. $\frac{5}{13}, \frac{6}{17}, \frac{8}{99}, \frac{181}{999}, \frac{23}{990}, \frac{47}{900}$.
11. $\frac{5}{9}, \frac{13}{99}, \frac{14}{990}, \frac{19}{900}, \frac{32}{999}, \frac{81}{9990}$.

151. *Example* 1. Reduce .0625 to a common fraction. .0625 is read 625 ten-thousandths; $\frac{625}{10000}$ is read in the same way.

$$.0625 = \frac{625}{10000} = \frac{125}{2000} = \frac{25}{400} = \frac{5}{80} = \frac{1}{16}.$$

EXERCISE 68 (Written)

Reduce to common fractions:

1. .3, .8, .25, .125, .1875.
2. .07, .0125, .00875, .0625, .0075.
3. .009, .0225, .1125, .0275.
4. .072, .0104, .035, .0119, .0375.
5. .144, .0504, .0768, .162, .0112.
6. .288, .0176, .0325, .0175, .425.
7. .2875, .3375, .5125, .7375.

EXERCISE 69 (Written)

1. A man walks 3 miles an hour. At this rate how long will it take him to walk 12 miles?

2. A train goes 25 miles an hour. How long will it take it to go 300 miles at this rate?

3. A bicyclist travels at the rate of 9 miles an hour. How long will it take him to go 60 miles?

4. How would you find the time to go any given distance if you knew the distance gone in a unit of time?

5. A man walks 3.5 miles an hour. At this rate how long would it take him to go 49 miles?

6. The distance from London to Glasgow is 401.5 miles. An express train goes this distance in 8 hours. Find its rate per hour.

7. From London to Edinburgh is 393.5 miles. The daily mail train takes 7.75 hours to go this distance. Find its rate per hour.

8. The Empire State Express goes from New York City to Buffalo, a distance of 440 miles, in 8.25 hours. Find its rate per hour.

9. The mail train from Paris to Bayonne goes 486.25 miles in 8.983 hours. Find its rate per hour.

10. The distance from New York City to Cleveland is 568 miles. A train goes this distance in 19.5 hours. Find its average speed.

11. A steamer goes from New York City to Bremen, a distance of 4235 miles, in 7.75 days. Find its rate per day. Also its rate per hour.

12. The earth moves in its orbit at the rate of 1110 miles a minute. How many times faster does the earth move than a train which goes 54 miles an hour?

13. A city lot is worth $1800. If this sum is .75 of the value of the house on it, what is the value of the house?

14. If .7 of a sum of money is $196, what is the sum of money?

15. Cast iron is 7.2 times as heavy as water. How many cubic feet of cast iron weigh as much as 6120 cubic feet of water?

16. Coal is 1.3 times as heavy as water. How many cubic feet of coal weigh as much as 546 cubic feet of water?

17. There are 231 cubic inches in a gallon. How many gallons are in 1 cubic foot? (1 cu. ft. = 1728 cu. in.)

18. If 2000 pounds of coal cost $8.75, find the price of 8750 pounds of this kind of coal.

19. If 3.5 yards of cloth cost $12.25, find the price of 7.5 yards of this cloth.

20. If 1.6 yards of velvet cost $2.88, find the price of 9.75 yards of velvet.

21. If 7.75 acres of land are sold for $193.75, find the value of 16.9 acres of this land.

22. If .65 of a ton of coal cost $5.20, find the cost of 1 ton of this coal.

23. If .75 of a man's property is worth $3240, what is the value of his property?

24. When 16 pounds of sugar are bought for $1, find the cost of 28 pounds of sugar.

25. If wheat sells for $.6825 per bushel, how many bushels must be sold to bring $465.80?

26. (*a*) The mean distance of the earth from the sun is 92,790,000 miles. Light comes from the sun to the earth in 8.3 minutes. Find how many miles light travels in 1 second.

(*b*) The sun is 3,112,000 miles nearer to us in midwinter than it is in midsummer. Find how many seconds longer it takes light to travel to us from the sun in midsummer than in midwinter.

27. A kilometer equals .62137 of a mile. How many kilometers in 9 miles?

EXERCISE 70 (Oral)

(Inductive-deductive Lesson)

1. What fraction of a yard is 1 foot? What fraction of a yard is 2 feet?

2. What fraction of 1 foot is 1 inch? 3 inches? 4 inches? 5 inches? 7 inches? 8 inches? 9 inches? 10 inches?

3. What fraction of 1 yard is 1 inch? What fraction of a yard is 2 inches? 3 inches? 4 inches? 5 inches? 6 inches? 9 inches? 12 inches? 16 inches? 17 inches? 19 inches? 24 inches? 27 inches?

4. There are 8 quarts in 1 peck. What fraction of a peck is 1 quart? What fraction of a peck is 2 quarts? 3 quarts? 4 quarts? 5 quarts? 6 quarts?

5. What fraction of a square yard is 2 square feet? 3 square feet? 4 square feet? 5 square feet? 6 square feet? 7 square feet?

6. What fraction of 10 is 2? What fraction of 10 is 7?

7. What fraction of 11 is 4? What fraction of 13 is 9?

8. What fraction of 100 is 80?

9. Which of the four fundamental rules enables us to solve a problem of this character: What fraction of a number is some other number?

10. If 4 men can do a piece of work in 7 days, how long will it take 1 man to do the same work?

11. If a team of horses can plow a 40 acre lot in 16 days, how long will it take 4 teams working together to plow the same lot?

12. If a man can do a piece of work in 9 days, what fraction of the work can he do in 1 day? in 2 days? in 3 days? in 4 days? in 6 days?

13. A laborer can dig a trench in 15 days. In how many days can he dig $\frac{1}{5}$ of the trench? $\frac{2}{5}$ of the trench? $\frac{3}{5}$ of the trench? $\frac{2}{3}$ of the trench?

14. Reduce to decimals: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{2}{9}$, $\frac{5}{9}$, $\frac{7}{9}$, $\frac{6}{11}$, $\frac{5}{12}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

EXERCISE 71 (Written)

1. A surveyor's chain is 66 feet long, and it contains 100 links. What fraction of a chain is 44 feet? 21 feet? 54 feet? 15 yards?

2. What fraction of a chain is 40 links? $12\frac{1}{2}$ links? $37\frac{1}{2}$ links? $62\frac{1}{2}$ links? $87\frac{1}{2}$ links?

3. What fraction of a chain is a length of $14\frac{2}{7}$ links? $56\frac{1}{7}$ links? $85\frac{5}{7}$ links? $44\frac{4}{9}$ links? $6\frac{2}{3}$ links? $46\frac{2}{3}$ links? $73\frac{1}{3}$ links?

4. What fraction of a chain is $6\frac{1}{4}$ links? $31\frac{1}{4}$ links? $43\frac{3}{4}$ links? $56\frac{1}{4}$ links? $68\frac{3}{4}$ links? $81\frac{1}{4}$ links? $93\frac{3}{4}$ links?

5. What fraction of a chain is $4\frac{1}{2}$ links? $20\frac{5}{6}$ links? $45\frac{5}{6}$ links? $70\frac{5}{6}$ links? $95\frac{5}{6}$ links?

6. What fraction of an hour is $5\frac{5}{11}$ minutes? $4\frac{8}{13}$ minutes? $51\frac{3}{7}$ minutes? $56\frac{2}{3}$ minutes? $57\frac{1}{2}$ minutes? $48\frac{1}{3}$ minutes?

7. A gallon contains 231 cubic inches. What fraction of a gallon is 49 cubic inches? 154 cubic inches? 99 cubic inches? 132 cubic inches?

8. What fraction of a square yard is $10\frac{2}{5}$ square inches? $28\frac{4}{5}$ square inches? $20\frac{4}{7}$ square inches? $82\frac{2}{7}$ square inches?

9. What fraction of a dollar is 35 cents? $37\frac{1}{2}$ cents? $62\frac{1}{2}$ cents?

10. One mile equals 1760 yards. What fraction of a mile is 88 yards? 704 yards? 1408 yards? 336 yards? 924 yards? 588 yards?

11. A knot or nautical mile is, in round numbers, 6080 feet. What fraction of a nautical mile is an ordinary or statute mile?

12. One square mile equals 640 acres. What fraction of a square mile is 560 acres? 448 acres? 576 acres? $9\frac{1}{7}$ acres? 12.8 acres? 51.2 acres? 19.2 acres?

13. If 7 laborers do a piece of work in 12 days, how long will it take 6 laborers to do the same work?

14. If 5 men do a piece of work in 14 days, how long will it take 7 men to do the same amount of work?

15. If 2 teams plow 25 acres in 6 days, how long will it take 3 teams to plow the same number of acres?

16. If 3 pounds of sausage cost 24¢, find the cost of $10\frac{1}{2}$ pounds of the sausage.

17. If 4 cans of sardines cost 25¢, find the cost of 15 cans.

18. If a 12-pound sack of flour sells for 45¢, at this rate what should a 100-pound sack sell for?

19. If I can buy 3.5 pounds of butter for $1.05, at this rate how many pounds of butter can I buy for $2.70?

20. If 1.75 pounds of ham cost 25¢, how many pounds of this ham can be bought for $2.75?

21. When 6 cans of salmon can be bought for $1, how many cans can be bought for $3.50?

22. If 5 pounds of mince-meat cost 72¢, how many pounds can be bought for $1.62?

23. What is the ratio of 50¢ to 75¢?

24. What is the ratio of $87\frac{1}{2}$¢ to $1?

25. What is the ratio of $213\frac{1}{3}$ acres to 640 acres?

26. What is the ratio of 18 minutes to 2 hours?

27. What is the ratio of 1.2 to $3\frac{1}{2}$?

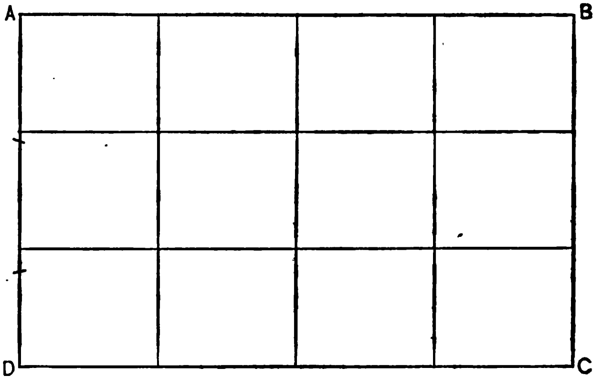
28. If .75 of a ton of coal cost $6, what will 2 tons of coal cost at the same price per ton?

29. A business man invests $5000, and makes a profit of $1750. What is the ratio of his gain to his investment?

30. A and B engage in business. A puts in .58 of the entire capital, and B the remainder. Find each person's gain if they make a profit of $3500.

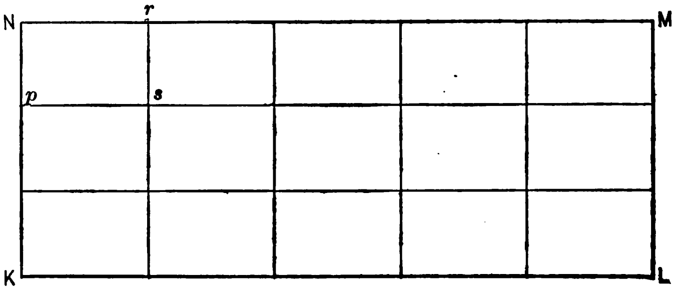
AREAS OF RECTANGULAR FIGURES

152. Take the rectangle *ABCD*. Let *AB* represent 4 feet, *BC* represent 3 feet.



Area of *ABCD* = 4 square feet × 3, or 3 square feet × 4. Hence the number of square feet is obtained by multiplying the number of feet in the length by the number of feet in the breadth.

Take another rectangle *KLMN*. Let *KL* = 9 yards. Let *LM* = 4 yards. Area *KLMN* = 36 square yards. Divide *KL* into any number of equal parts, say 5; and *LM* into any number of equal parts, say 3. Draw lines through the points of division.



Then

Area of *psrN* $= \frac{36}{15}$ square yards.
Length of *psrN* $= \frac{1}{5}$ of 9 yards.
Breadth of *psrN* $= \frac{1}{3}$ of 4 yards.

Take the product of the number of yards in the length and in the breadth of *psrN*, and we have $\frac{1}{3}$ of $4 \times \frac{1}{5}$ of $9 = \frac{36}{15}$.

Hence the area of *psrN* $= \frac{36}{15}$ square yards.

From these illustrations it appears that the **area of a rectangle** is obtained by multiplying the number of units in the length by the number of units in the breadth; the resulting product times the square unit will be the area.

153. **Number** is the ratio of one quantity to another quantity of the same kind taken as a standard.

For example, take 154 square feet. Here the number is 154. The unit is a square foot. The ratio of 154 square feet to 1 square foot is 154.

Take another example, 17 bushels. The unit or standard is 1 bushel. The ratio of 17 bushels to 1 bushel is 17. The expression "17 bushels" is designated a **concrete number** or **quantity**.

NOTE. The definition of number in § 153 is due to Leibnitz, a German philosopher and mathematician. In advanced work in mathematics, the idea of number is divorced from measurement, and is made to spring from counting.

EXERCISE 72 (Written)

1. The dimensions of a rectangle are 125 feet by 100 feet. Find its area in square feet. Find also the area in square yards.

2. A rod is 5½ yards long. How many square yards are in 1 square rod? Find the number of square yards in 1 acre (1 acre = 160 square rods).

3. A chain is 66 feet long. How many square feet are in a square chain? How many square yards are in 1 square chain?

4. Find how many square inches are in 1 square yard.

5. In 1 mile there are 1760 yards. How many square yards are in 1 square mile?

6. A farm in the form of a rectangle is 53⅓ rods long and 48 rods wide. How many acres does it contain?

7. A wheat field, rectangular in shape, is 64 rods long and 40 rods wide. How many bushels of wheat will it produce at 18 bushels to the acre? Find the value of the crop at $.84⅓ a bushel.

8. A cornfield is 213⅓ rods long and 96 rods wide. How many bushels of corn will it produce at 32 bushels to an acre? Find the value of the crop at $.48¾ per bushel.

9. A city block is 110 yards long by 90 yards wide. How many acres are in a park which extends 7 blocks one way and 5 blocks the other way?

10. A street is 1760 yards long and 20 yards wide. How many thousand bricks, 8 inches by 4 inches, will be needed to pave it?

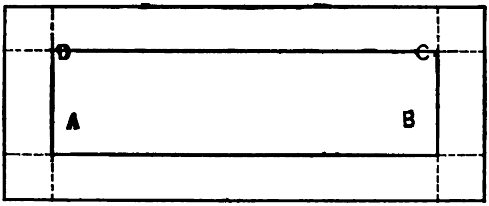
11. How many square tiles, 4 inches on a side, will be required to tile a hall 60 feet by 16 feet?

12. The dimensions of a room are 16 feet by 12 feet, and 10 feet high. How many square yards are in the four walls of the room? How many square yards are in the walls and ceiling?

13. The dimensions of a room are 40 feet by 30 feet, and 18 feet high. How many square yards are in its walls and ceiling?

14. Find the area, in square yards, of the walls and ceiling of a room 24 feet by 16 feet, and 12 feet high.

15. *ABCD* is a rectangular plot of ground 400 feet by 160 feet. Surrounding it is a road 15 feet wide. Find the area of the road.



16. A rectangular park, 600 feet long by 560 feet wide, has a road surrounding it. Find the area of the road if its width is 24 feet. Suppose the road is fenced in, how many feet of wire will it take to go once round?

17. A rectangular grass plot 252 feet by 180 feet has a walk around it. The width of the walk is 9 feet. How many flags 9 inches square will be required to flag the walk?

18. Find the area of each of the following rectangles, in square feet, correct to two decimal figures:

(*a*) 136 feet 8 inches by 115 feet 4 inches.

(*b*) 225 feet by 93 feet 10 inches.

(*c*) 78 feet 5 inches by 56 feet 6 inches.

(*d*) 25 feet 9 inches by 50 feet 2 inches.

(*e*) 104 feet 2 inches by 153 feet 11 inches.

(*f*) 203 feet by 53 feet 9 inches.

(*g*) 223 feet 10 inches by 78 feet.

(*h*) 618 feet 1 inch by 130 feet 7 inches.

HINT. Reduce the inches in each example to the fraction of 1 foot.

19. Find the area of the following rectangles, giving the results in square yards, correct to two decimal figures:

(*a*) 84.5 feet by 76.75 feet.

(*b*) 90.67 by 84.33 feet.

(*c*) 96.34 by 85.28 feet.

(*d*) 177.33 feet by 82.54 feet.

(*e*) 129.55 feet by 79.63 feet.

BILLS

154. A **bill** is an itemized statement in writing of articles sold, or of services rendered, specifying the price of each article, and of all the articles of each kind, and also the total amount.

155. The party who owes is called a **debtor**, and the party to whom anything is owed is called a **creditor**. Dr. and Cr. stand respectively for *debtor* and *creditor*. The symbol @ used in bills stands for *at*. Thus, 7 bushels of wheat @ 81 ¢ means 7 bushels of wheat at 81 cents per bushel.

156. The words *Received Payment* are printed on some bills. When a bill is paid, the party to whom payment is made acknowledges payment by writing his name and the date of payment underneath the words *Received Payment*. Or he may simply stamp on the face of the bill *Paid*, the date of payment, and his name.

SPECIMEN BILL

GALVESTON, TEXAS, Jan. 31, 1903.

MR. A. B. C.

In account with K. M. & CO.,

DEALERS IN

FURNITURE, CARPETS, RUGS, &C.

Jan.	*2*	*3 Chairs @ $2.25*	*$6*	*75*		
	10	*1 Library Table*	*25*	*00*		
	15	*3 Rugs @ $6.75*	*20*	*75*		
	20	*40 yd. Matting @ 45¢*	*18*	*00*		
	24	*2 Wardrobes @ $17.50*	*35*	*00*		
					$105	*00*
		Paid				
		Feb. 1, 1903.				
		K. M. & CO.				
		Per M.				

DALLAS, TEXAS, Feb. 5, 1903.

MR. P. Q. R.

Bought of M. R. S.,

RETAIL GROCERS.

Jan.	*5*	*3 lb. Tea @ 50¢*	*$1*	*50*		
	"	*28 lb. Sugar @ 5¼¢*	*1*	*47*		
	"	*3 pks. Potatoes @ 40¢*	*1*	*20*		
	"	*7 lb. Bacon @ 15¢*	*1*	*05*		
	10	*8 lb. Butter @ 35¢*	*2*	*80*		
	12	*3 cans Salmon @ 17¢*		*51*		
	13	*6 lb. Sausage @ 12¢*		*72*	*$9*	*25*
		Paid				
		Feb. 8.				
		M. R. S.				
		Per X.				

EXERCISE 73 (Written)

Make out the following bills and receipt them:

1. Mr. John Rye bought of William Merchant,

12 yd. Calico	@	9¢
15 yd. Sheeting	@	7¢
11 yd. Flannel	@	35¢
2 Hats	@	$3.75
18 yd. Carpet	@	75¢
3 Smyrna Rugs	@	$10.50

2. Mr. J. Hill bought of F. Warner & Co.,

5 Stoves	@	$6.50
3 doz. Knives	@	$4.80
2 Saws	@	$1.50
5 Iron Beds	@	$15.75
6 Wrenches	@	$1.25

3. H. Van Oppen bought of Hegel & Co.,

2 bu. Potatoes	@	$1.50
5 lb. Tea	@	75¢
2 boxes Herring	@	$1.95
25 lb. Ham	@	15¢
45 lb. Sugar	@	$4\frac{1}{3}$¢

4. Mr. James Kay bought of Simpson, Perdue & Co.,

50 lb. Sugar	@	$4\frac{1}{6}$¢
15 cans Tomatoes	@	13¢
27 cans Corn	@	16¢
10 packages Breakfast Food	@	$12\frac{1}{2}$¢
8 cans Salmon	@	18¢
5 gal. Maple Sirup	@	$1.30
25 lb. Butter	@	$37\frac{1}{2}$¢
6 lb. Y. H. Tea	@	75¢

5. Mr. H. Scharp bought of Matt, Schorn & Co.,

350 bu. Oats	@	45½¢
278 bu. Barley	@	69¢
340 bu. Rye	@	94¢
10 bbl. Flour	@	$6.50
847 bu. Wheat.	@	81¼¢
450 bu. Corn	@	54¢

REVIEW QUESTIONS

1. What is the derivation of the word *decimal?*

2. What is the law pervading the decimal system of notation?

3. What is meant by the decimal point?

4. What is the origin of the decimal system of notation?

5. Give the different names for the symbol 0.

6. Of what use is 0 in the decimal system of notation?

7. Name, in order, the first eight places to the left of the decimal point.

8. Name, in order, the first six places to the right of the decimal point.

9. How do you write any number of 10ths? any number of 100ths? any number of 1000ths?

10. Give the rule for writing, in figures, a decimal number.

11. How do you multiply a number by 10?

12. In the notation of what numbers is the decimal point omitted?

13. How do you multiply a number by 1000? by 10,000?

14. What effect upon the value of a combination of figures has moving the decimal point two places to the left in the number expressed by those figures? What effect has moving the decimal point three places to the right?

15. What is the value of the ratio 7.01: .00701?

16. Give the rule for the addition of decimals.

17. Give the rule for the multiplication of decimals.

18. What is meant by a power of a number?

19. What is meant by the index of a power?

20. Give three ways in which 2.7 may be written.

21. Express the decimal .079577 correct to four figures; correct to three figures; correct to two figures.

22. What is meant by a vulgar fraction?

23. How do you reduce a fraction to a decimal?

24. How do you reduce a decimal to a common fraction?

25. Give the ratio definition of number.

26. How do you find the area of a rectangle?

27. What is a bill?

28. Explain the terms *Dr.* and *Cr.*

CHAPTER X

COMPOUND QUANTITIES

157. Concrete quantities of the same kind, but consisting of units of different denominations, are called **compound quantities.**

17 days, 10 hours, and 30 minutes is a compound quantity. Here, we have three units of measurement; namely, a day, an hour, and a minute. These units are of different denominations, but each is of the same kind, inasmuch as it stands for a definite portion of time.

Compound quantities are also called **compound denominate quantities.** Quantities composed of units of one denomination are generally called **simple quantities.**

158. AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweight, or 2000 pounds = 1 ton (T.)
In Great Britain, 112 pounds = 1 hundredweight (cwt.)
20 hundredweight, or 2240 pounds = 1 ton (T.)

Avoirdupois weight is used in weighing all commercial quantities excepting the precious metals, jewelry, and drugs sold by retail druggists.

The **unit in Avoirdupois weight** is the **pound** of 7000 grains.

The English ton, known in the United States as the long ton, is used in the United States Custom Houses, and in weighing coal and mineral products at the mines.

EXERCISE 74 (Oral)

1. How many ounces are in 3 lb.? in 5 lb.? in 8 lb.?

2. How many ounces are in 2 lb. 3 oz.? in 3 lb. 2 oz.?

3. How many hundredweight are in 3 T.? in 7 T.? in 9 T.?

4. How many pounds are in 5 cwt.? in 9 cwt.? in 11 cwt.?

5. How many pounds are in 3 cwt. 20 lb.? in 7 cwt. 30 lb.?

6. How many pounds and ounces are in 57 oz.? in 90 oz.? in 84 oz.? in 73 oz.?

7. What part of 1 lb. is 8 oz.? 12 oz.? 14 oz.?

8. How many pounds are in $\frac{4}{5}$ cwt.? in $\frac{3}{4}$ cwt.? in .7 cwt.? in .9 cwt.?

9. What part of 1 T. is 200 lb.? 500 lb.? 250 lb.? 333$\frac{1}{3}$ lb.?

10. What fraction of 1 T. is 800 lb.? 1200 lb.? 1600 lb.? 1800 lb.? 750 lb.?

159. The process of reducing units of any given denomination to units of higher denomination is called **reduction ascending.**

160. The process of reducing units of a higher denomination or of higher denominations to units of lower denomination is called **reduction descending.**

161. Reduce 1,000,204 oz. to higher denominations.

16	1000204
100	62512 lb. 9 oz.
20	625 cwt. 12 lb.
	31 T. 5 cwt.

Divide by 16 to get the number of pounds. Divide by 100 to get the number of hundredweights. Divide by 20 to get the number of tons.

Ans. 31 T. 5 cwt. 12 lb., 9 oz.

Weights are generally expressed in tons or in pounds.

EXERCISE 75 (Written)

Reduce to higher denominations :

1. 7800 oz.	**2.** 9763 oz.	**3.** 75,497 oz.
4. 1,000,000 oz.	**5.** 7987 lb.	**6.** 32,721 lb.

7. How many ordinary or short tons are in 100 long tons?

8. Reduce 10,000 lb. to long tons.

9. Reduce 93,500 lb. to long tons.

162. Reduce 7 T. 3 cwt. 12 lb. 10 oz. to ounces.

T.	cwt.	lb.	oz.
7	3	12	10

20
143 cwt.
100
14312 lb.
16
85882
14312
229002 oz.

We reduce the 7 T. to hundredweights by multiplying by 20. Add 3 cwt. to the product and get 143 cwt. Multiply 143 by 100 and add 12 lb. to the product. This gives us 14,312 lb. Multiply this by 16, adding 10 oz., when we multiply by the first figure, 6.

EXERCISE 76 (Written)

1. Reduce 19 T. to ounces.

2. Reduce 14 T. 4 cwt. to pounds.

3. Reduce 17 T. 3 cwt. to pounds.

4. Reduce 25 T. 2 cwt. to ounces.

5. Reduce 3 T. 15 cwt. 2 lb. to pounds.

6. Reduce 4 T. 11 cwt. 58 lb. to pounds.

7. Reduce 8 T. 2 cwt. 73 lb. to pounds.

8. A dealer buys 50 long tons of coal and sells it by the short ton. How many short tons does he sell?

9. A dealer buys 100 long tons of coal at $6.75 per ton. He sells it by the short ton at $6.75 per ton. How much profit does he make?

10. Convert 784 short tons into long tons.

LINEAR OR LONG MEASURE

163. 12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
$5\frac{1}{2}$ yards = 1 rod (rd.)
320 rods = 1 mile (mi.)
6080 feet = 1 knot, geographical or nautical mile
3 knots = 1 league
1 mi. = 320 rd. = 1760 yd. = 5280 ft.

SURVEYORS' LINEAR MEASURE

100 links (l.) = 1 chain (ch.)
80 chains = 1 mile
1 ch. = 4 rd. = 22 yd. = 66 ft.

The **unit of length** is the **yard.** This is defined by law in the United States and in England.

The legal value of the yard in the United States (that of the Bureau of Standards) does not exactly agree with the legal value in England.

An English yard is the distance between two lines on a brass rod at 62° Fahr. This rod is kept in the Office of Standards, London. The yard, in the United States, used as a standard since 1893 is $\frac{3600}{3937}$ of the meter.

EXERCISE 77 (Oral)

1. How many feet are in 1 rd.? in 2 rd.? in 4 rd.?
2. How many inches are in 1 yd.? in 4 yd.? in 5 yd.?
3. How many inches are in 1 ch.? in 1 l.? in 2 l.?
4. How many feet are in $\frac{1}{2}$ mi.? in $\frac{1}{4}$ mi.? in $\frac{1}{11}$ mi.?
5. How many yards are in $\frac{1}{4}$ mi.? in $\frac{1}{5}$ mi.? in $\frac{1}{8}$ mi.?
6. How many feet are in $\frac{1}{3}$ ch.? in $\frac{1}{4}$ ch.? in $\frac{1}{6}$ ch.?
7. What fraction of 1 mile is 5 ch.? 16 ch.? 30 ch.?

8. What fraction of a chain is 2 rd.? 6 yd.? 14 yd.?
9. What fraction of a mile is 440 yd.? 176 yd.? 88 yd.?
10. Reduce to inches 2 ft. 8 in., 5 ft. 4 in.
11. Reduce to yards 3 rd., 4 rd., 5 rd.
12. Reduce to feet 1 rd., 2 rd., 4 rd., 5 rd.
13. Reduce to rods 2 mi., 3 mi., 5 mi., 6 mi.

164. Reduce 2 mi. 127 rd. 3 yd. 2 ft. 4 in. to inches.

mi.	rd.	yd.	ft.	in.
2	127	3	2	4

$$\begin{array}{r l}
320 & \\ \hline
640 & \\
127 & \\ \hline
767 & \text{rd.} \\
5\tfrac{1}{2} & \\ \hline
383\tfrac{1}{2} & \\
3838 & \\ \hline
4221\tfrac{1}{2} & \text{yd.} \\
3 & \\ \hline
12666\tfrac{1}{2} & \text{ft.} \\
12 & \\ \hline
152002 & \text{in.}
\end{array}$$

Reduce the miles to rods by multiplying the number of miles by 320. Add 127 to the product.

Next reduce the rods to yards by multiplying the number of rods, 767, by 5½. To do this, take ½ of 767, and to this add 5 times 767, and the 3 yd. we already have.

We next multiply by 3, adding in the 2 ft. at the same time. Finally we multiply by 12, adding at the same time the 4 in. Therefore 2 mi. 127 rd. 3 yd. 2 ft. 4 in. = 152,002 in.

A problem in reduction descending involves multiplication and addition. We invariably begin with the highest denomination and multiply the given number of units of that denomination by the number of units of the next lower denomination that makes one unit of the highest denomination. We treat the next denomination in the same way, and so on to the end.

NOTE. Distances are generally expressed in miles or in feet. We never hear of a length expressed in miles, yards, feet, and inches, all at the same time.

EXERCISE 78 (Written)

1. Reduce 4 yd. 2 ft. to inches.
2. Reduce 110 yd. 1 ft. to inches.
3. Reduce $5\frac{1}{2}$ mi. to yards.
4. Reduce 7 mi. 120 rd. to yards.
5. Reduce 10 mi. 110 rd. 4 yd. to yards.
6. Reduce $445\frac{3}{4}$ mi. to yards.
7. Reduce 7.74 mi. to yards.
8. Reduce 8.35 mi. to rods.
9. Reduce 238 rd. to feet.
10. Find the number of inches in 1 mi.
11. In 54 ch. how many inches are there?
12. In 15.54 ch. how many feet are there?
13. Find, in feet, the difference between 115 ch. and $1\frac{1}{2}$ mi.
14. Find, in yards, the difference between 134 ch. and $1\frac{3}{4}$ mi.

165. Reduce 300,000 in. to higher denominations.

$$\begin{array}{r|l}
12 & 300000 \\ \hline
3 & 25000 \text{ ft.} \\ \hline
 & 8333 \text{ yd. 1 ft.} \\
 & 2 \\ \hline
11 & 16666 \text{ half yd.} \\ \hline
320 & 1515 \text{ rd. 1 half yd.} \\ \hline
\multicolumn{2}{r}{4 \text{ mi. 235 rd.}}
\end{array}$$

Ans. 4 mi. 235 rd. $\frac{1}{2}$ yd. 1 ft.

Since $\frac{1}{2}$ yd. = 1 ft. 6 in., 4 mi. 235 rds. $\frac{1}{2}$ yd. 1 ft. =

$$\begin{array}{r}
4 \text{ mi. 235 rd. 0 yd. 1 ft.} \phantom{\text{ 6 in.}} \\
1 \text{ ft. 6 in.} \\ \hline
4 \text{ mi. 235 rd. 0 yd. 2 ft. 6 in.}
\end{array}$$

Therefore, 300,000 in. = 4 mi. 235 rd. 2 ft. 6 in.

A problem in reduction ascending involves only division.

EXERCISE 79 (Written)

1. Reduce 250 in. to higher denominations.

2. Reduce 384 ft. to higher denominations.

3. Reduce 64,000 in. to higher denominations.

4. Reduce 274,000 in. to yards, giving your answer correct to three decimal figures.

5. Reduce 10,240 rd. to miles.

6. Reduce 152 mi. to knots.

7. Reduce 165 knots to miles.

8. Reduce 1356 ch. to miles.

9. Reduce 7.7 mi. to chains.

10. Reduce 250 ch. to feet.

11. Reduce 484 yd. to chains.

12. Find how many inches are in 1 link. In 17 links.

13. Express in miles and the decimal of a mile correct to three figures the heights of the following mountains: Everest, 29,002 ft.; Aconcagua, 22,860 ft.; Ararat, 17,325 ft.; Blanc, 15,781 ft.; Fremont Peak, 13,790 ft.; McKinley, 20,464 ft.; Logan, 19,539 ft.; Orizaba, 18,314 ft.; Pikes Peak, 14,108 ft.

SQUARE MEASURE

166. 144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)
10 square chains = 1 acre
1 acre = 4840 square yards
1 section = 1 square mile
36 sections = 1 township

Square measure is used to measure the areas of surfaces.

EXERCISE 80 (Oral)

1. How many square inches are in $\frac{1}{2}$ sq. ft.? in $\frac{1}{3}$ sq. ft.? in $\frac{1}{4}$ sq. ft.? in $\frac{3}{4}$ sq. ft.?

2. How many square feet are in $\frac{2}{3}$ sq. yd.? in $\frac{3}{4}$ sq. yd.?

3. How many square rods are in $\frac{1}{4}$ of an acre? in $\frac{1}{5}$ of an acre? in $\frac{3}{5}$ of an acre? in $\frac{7}{8}$ of an acre? in .7 of an acre? in .9 A.?

4. How many square rods are in 1 sq. ch.? in $\frac{3}{4}$ sq. ch.? in $\frac{7}{8}$ of a square chain?

5. How many square yards are in $\frac{1}{4}$ A.? in $\frac{1}{2}$ A.? in $\frac{1}{3}$ A.?

6. Reduce .25 A. to square rods.

7. Reduce .75 sq. ft. to square inches.

8. What part of a township is 1 sq. mi.?

9. How many square links are in 1 sq. ch.?

10. How many square links are in 1 sq. rd.?

11. Reduce $\frac{4}{5}$ sq. ch. to square links.

12. Reduce 87 sq. ch. to acres.

13. Reduce 480 sq. rd. to acres.

14. Reduce $60\frac{1}{2}$ sq. yd. to square rods.

15. A ranch occupies 5 sections. How many acres does it contain?

16. Reduce 288 sq. ft. to square yards.

167. A **line** has length without width. A line has one dimension.

168. A **surface** has length and breadth. A surface has two dimensions.

169. A **plane surface**, or a **plane**, is one which is perfectly even.

170. A **solid** has length, breadth, and thickness. It has three dimensions.

171. A **cube** is a solid bounded by six plane surfaces, each of which is a square.

172. A solid having the shape of a box or of an ordinary room, *i.e.* one bounded by six plane surfaces each of which is a rectangle, is called a **rectangular solid.**

173. The **volume** of a solid means the amount of space it occupies. This is measured by the number of times the solid contains the unit of measurement.

174. The **unit of volume** is a cube having for edges equal linear units. The **cubic unit** from which all others are derived is the cubic yard.

CUBIC OR SOLID MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)

1. What is a cubic foot? What is a cubic inch?

2. How many cubic inches are in $\frac{1}{2}$ cu. ft.? in $\frac{1}{3}$ cu. ft.? in $\frac{1}{4}$ cu. ft.? in $\frac{1}{6}$ cu. ft.?

3. How many times longer is an edge of a cubic foot than an edge of a cubic inch?

4. How many times longer is an edge of a cubic yard than an edge of a cubic foot?

5. How many times longer is an edge of a cubic yard than an edge of a cubic inch?

MEASURES OF CAPACITY

175. There are two **measures of capacity** in general use, namely, Liquid Measure and Dry Measure.

Liquid Measure

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

A gallon contains 231 cu. in.

In some text-books the statement is made that a gallon of distilled water weighs 10 lb. The imperial Winchester gallon measure of England, when filled with distilled water, weighs 10 lb.

This is an entirely different gallon from that used in the United States.

DRY MEASURE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

One bushel contains 2150.42 cu. in. It is the volume of a cylindrical vessel 18½ in. in diameter and 8 in. deep.

EXERCISE 81 (Written)

1. How many pints are in 1 gal.?
2. How many gills are in 1 qt.?
3. How many gills are in 1 gal.?
4. What part of 1 gal. is 1 pt.? 3 pt.? 5 pt.?
5. How many pints are in 1 pk.? 2 pk.? 3 pk.?
6. How many quarts are in 1 bu.?
7. How many pints are in 1 bu.?
8. What part of 1 bu. is 1 pt.?
9. How many cubic inches are in 1 pt. dry measure?
10. How many cubic inches are in 1 pt. liquid measure?
11. What is the ratio of a pint liquid measure to a pint dry measure?
12. How many more cubic inches are there in a pint dry measure than in a pint liquid measure?
13. How many quarts are in $\frac{3}{4}$ pk.?

14. Reduce 2.75 bu. to quarts.

15. Reduce 4.25 gal. to quarts.

16. How many gallons would be required to fill 10 bushel measures?

ANGULAR MEASURE

176. A **point** has position, but no dimensions.

177. If two lines be such that they cannot intersect each other in more than one point, each is called a **straight line.**

It is not an easy matter to define a straight line. The following definitions have been given:

A straight line is a line which lies evenly between any two of its points.

The path of a point which moves without changing its direction is a straight line.

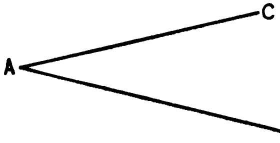
A straight line is the shortest line between two points.

All of these definitions are objectionable.

The following illustration, taken from Professor Henrici, may serve to give a clear idea of a straight line:

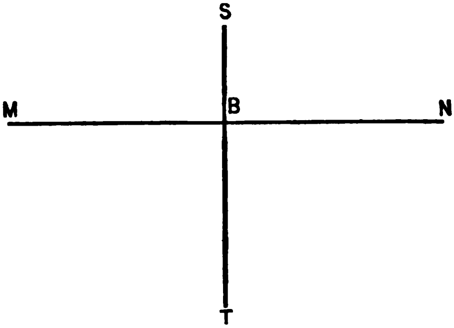
"If we suspend a weight by a string, the string becomes stretched, and we say it is straight, by which we mean to express that it has assumed a peculiar shape. If we mentally abstract from this string all thickness, we obtain the notion of the simplest of all lines, which we call a straight line."

178. The mutual inclination of two straight lines which proceed from the same point is called an **angle.**



The angle formed by the lines *AB*, *AC* is read angle *BAC*. The lines *AB*, *AC* are called the sides of the angle *BAC*, or arms of the angle *BAC*. The point *A* is called the vertex of the angle.

179. If one straight line meets another straight line so as to make the adjacent angles equal to each other, each angle is called a **right angle.**



If the lines *MN*, *ST* meet in *B* so as to make the angles *NBS*, *SBM* equal, then each angle is a right angle. Their sum is called a **straight angle.**

180. The **unit of angular measure** is 1 degree.

181.

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
90 degrees = 1 right angle
2 right angles = 1 straight angle.

EXERCISE 82 (Oral)

1. What part of 1′ is 1″?

2. How many seconds are in $\frac{1}{2}$ minute?

3. Reduce 1′ 30″ to seconds.

4. What part of a straight angle is a right angle?

5. What part of a right angle is an angle of 45°? 30°? 15°? 18°? 60°? 75°?

6. How many degrees are in 1 straight angle? in $\frac{3}{4}$ of a straight angle?

7. What part of a straight angle is an angle of 15°? 24°? 30°? 45°? 60°? 80°? 100°? 105°? 120°? 135°? 150°?

CIRCULAR MEASURE

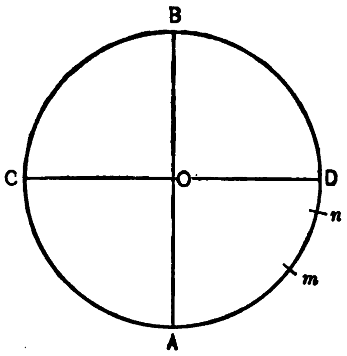
182. A **circle** is a plane figure bounded by a line called the **circumference**, every point of which is equally distant from a point within the figure called the **center**.

183. A straight line from the center to the circumference is called a **radius**.

184. A straight line drawn through the center and terminated by the circumference is called a **diameter**.

The lines *AB* and *CD* are diameters.

185. Any portion of a circumference is called an **arc**. *mn* is an arc.



186. An arc equal to one half of a circumference is called a **semicircumference**.

187. An arc equal to one fourth of a circumference is called a **quadrant**.

188.

60 seconds ($''$) = 1 minute ($'$)
60 minutes = 1 degree ($^\circ$)
360 degrees = 1 circumference.

The division of the circle into 360 equal parts is due to the Chaldean astronomers, who thought the year consisted of 360 days, and that the sun annually described a circle in the heavens among the fixed stars. The degree was taken as a unit to indicate the distance the sun traveled each day.

EXERCISE 83 (Oral)

1. How many degrees are in a quadrant? in a semicircumference?

2. If two diameters are drawn in a circle at right angles to each other, for example, *AB*, *CD*, Fig. 1, how many degrees are in the arcs *AD*, *DB*, *BC*, *CA*?

3. Through how many degrees does a bicycle wheel revolve in making $\frac{1}{2}$ of a revolution?

4. The earth revolves on its axis in 24 hours. Through how many degrees does it revolve in 1 hour?

5. Through how many degrees does the hour hand of a clock turn in 1 hour? in 3 hours? in 6 hours?

6. Through how many degrees does the minute hand of a clock pass in 1 hour? in 2 hours? in 6 hours?

7. An arc of 30° is what part of a quadrant? of a circumference?

8. What fraction of a circumference is an arc of 45°? 60°? 72°? 108°? 120°? 135°? 150°?

9. What fraction of a quadrant is an arc of 10°? 15°? 20°? 18°? 36°? 54°? 72°? 75°?

10. What fraction of a right angle is an angle of $7\frac{1}{2}$°? 18°? 40°? 36°? 72°? 75°?

11. How many degrees are in a straight angle? How many degrees are in a semicircumference?

TIME MEASURE

189. 60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
365 days = 1 common year (yr.)
366 days = 1 leap year (yr.)
100 years = 1 century.

There are twelve calendar months in a year. Their names and the number of days in each are:

January (Jan.)	31 days	July	31 days
February (Feb.)	28 or 29 days	August (Aug.)	31 days
March (Mar.)	31 days	September (Sept.)	30 days
April (Apr.)	30 days	October (Oct.)	31 days
May	31 days	November (Nov.)	30 days
June	30 days	December (Dec.)	31 days

The following lines will enable one to remember the number of days in each month:

"Thirty days hath September,
April, June, and November,
February twenty-eight alone,
And all the others thirty-one;
But leap year, coming once in four,
Gives February one day more."

190. A day is the **standard unit of time.** It is fixed by nature, and is of the same duration at all places between the Arctic and Antarctic circles. It represents the period of time that elapses between two successive passages of the sun across the meridian of any place.

Another natural standard of time is the period from one new moon to the next new moon, called a lunar month, and a third is the period of the revolution of the earth in its orbit, called a year.

191. The length of a year is 365.24222 days, or 365 da, 5 hr. 48 min. 47 sec., very nearly.

The common year has 365 days. This difference between the length of the common year and of the actual, or solar year, necessitates the introduction of *leap years.*

In 1 year there are 365.24222 days.

In 100 years there are 36524.222 days.

In 400 years there are 146096.888 days = 146,097 days, nearly.

In 400 years of 365 days each there are 146,000 days. Hence in 400 years 97 days have to be added in some way.

By adding 1 day to every fourth year, and designating as leap years those denoted by numbers exactly divisible by 4, we would have 100 leap years in 400 years. This would give us 3 days in excess every 4 centuries. To balance this excess, it is agreed that centennial years are not to be considered leap years, unless the number denoting the century is exactly divisible by 4. Thus the years 1700, 1800, 1900 were not leap years, because 17, 18, 19, the numbers denoting the centuries, are not exactly divisible by 4. The year 2000 will be a leap year, 20 being divisible by 4.

The present arrangement, known as the Gregorian Calendar (from Pope Gregory XIII) still contains a slight error. Adding 97 days to the common years in every 400 years makes the average length of a year $365\frac{97}{400}$ days, or expressed decimally, 365.2425 days.

The length of a solar year correct to five decimal figures is 365.24222 days. Taking the difference between these two, we have an error of .00028 day in a year. In about 3600 years this error will amount to 1 day.

Since the beginning and the end of a year are not marked by such obvious phenomena as mark the beginning and the end of a day, the question of determining the length of the year to a close degree of approximation is by no means a simple problem. Accordingly the civil year varied in length with different peoples. When the errors accumulated, corrections were made. One of the most important efforts to get the true length of the solar year was that of Julius Cæsar in 46 B.C. After much effort it was decided that the year contained 365¼ days.

By a decree of Cæsar, every fourth year was to have 366 days; the additional day was to be added to February, and the remaining months were to have 30 and 31 days alternately. This is what is known as the Julian Calendar. It made the year 365.25 days. Hence the year was too long by .00778 of a day. In 400 years this error would amount to 3.112.

When Pope Gregory XIII undertook, in 1582, the correction of the Julian Calendar, the error had amounted to 10 days. The vernal equinox came on March 11 instead of March 21. To even up matters, the day following the 4th of October, 1582, was called the 15th of October. This made the vernal equinox occur on March 21, the date of its occurrence in A.D. 325.

The method of reckoning time by the Julian Calendar is known as the Old Style; that by the Gregorian Calendar is called the New Style. Russia still adheres to the Old Style.

EXERCISE 84 (Oral)

1. How many weeks are in a common year?
2. How many minutes are in a day?
3. How many seconds are in 1 hour?
4. How many hours are in 1 week?
5. What part of a day is 6 hours?
6. What fraction of a week is 12 hours?
7. What fraction of a common year is 73 da.? 146 da.? 219 da.? 292 da.?

8. How many days are in the month of June? July? August?

9. How many days are in the three months, March, April, and May?

10. How many minutes are in $\frac{2}{3}$ of an hour? in $\frac{3}{4}$ of an hour? in $\frac{5}{6}$ of an hour?

11. What part of a day is 8 hr.? 12 hr.? 3 hr.?

12. What fraction of a minute is 15 sec.? 20 sec.? 36 sec.? 54 sec.? 55 sec.? 18 sec.? 16 sec.? 24 sec.? 36 sec.?

13. When will leap year next occur? When was the last leap year?

14. How many leap years will there be in this century? How many were in the last century? How many were in the eighteenth century?

15. The Gregorian Calendar was introduced into England in 1752. The day following Sept. 2 was called Sept. 14. How many days were in the English civil year 1752?

16. How many days will there be in the present century?

17. How many days are there between March 1, 1903, and March 1, 1907?

18. How many days were there in the last century?

19. How many days are there between Jan. 1, 1900, and Jan. 1, 1904?

20. How many days are there between Feb. 1, 1904, and Apr. 1, 1904?

21. How many days are there in 400 consecutive years?

MISCELLANEOUS MEASURE

192. 1 bushel of barley = 48 lb.
1 bushel of wheat = 60 lb.
1 bushel of oats = 32 lb.
1 bushel of rye = 56 lb.
1 bushel of potatoes (Irish) = 60 lb.
1 bushel of potatoes (sweet) = 55 lb.
1 bushel of buckwheat = 48 lb.
1 bushel of beans = 60 lb.
1 bushel of shelled corn = 56 lb.
1 bushel of peas = 60 lb.
1 bushel of clover seed = 60 lb.
1 barrel of flour = 196 lb.
1 barrel of pork or beef = 200 lb.
1 cental of grain = 100 lb.

NUMBERS

12 units = 1 dozen (doz.)
12 dozen = 1 gross
12 gross = 1 great gross
20 units = 1 score

PAPER MEASURE

24 sheets of paper = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

TROY WEIGHT

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)
1 pound Troy = 5760 grains

Troy weight is used in weighing precious metals and jewelry.

(The measures in this paragraph are inserted merely for reference.)

REDUCTION DESCENDING

EXERCISE 85 (Written)

1. Reduce 1 sq. mi. to square rods.
2. Reduce 2½ sq. mi. to acres.
3. Reduce 12 A. to square feet.
4. Reduce 27 sq. rd. to square feet.
5. Reduce 3 mi. 50 rd. to feet.
6. Reduce 8 mi. 40 rd. to feet.
7. Reduce 2¾ mi. to yards.
8. Reduce 3.75 mi. to yards.
9. Reduce 2.125 mi. to feet.
10. Reduce 25 cu. yd. to cubic feet.
11. Reduce 38 cu. yd. 20 cu. ft. to cubic feet.
12. Reduce 17½ cu. yd. to cubic inches.
13. Reduce 18.75 cu. yd. to cubic feet.
14. Reduce 20.25 cu. yd. to cubic inches.
15. Reduce 2 gal. 2 qt. to quarts.
16. Reduce 5 gal. 3 qt. to quarts.
17. Reduce 3 gal. 1 pt. to pints.
18. Reduce 7 gal. 1 pt. to pints.
19. Reduce 19.25 gal. to pints.
20. Reduce 14¾ gal. to pints.
21. Reduce 21.25 gal. to gills.
22. Reduce 10.75 gal. to pints.
23. Reduce 16.375 gal. to pints.
24. Reduce 3 gal. 2 qt. 1 pt. 3 gi. to gills.
25. Reduce 17.69 T. to pounds.
26. Reduce 17¼ T. to pounds.

27. Reduce 33.125 T. to pounds.

28. Reduce 14.375 T. to pounds.

29. How many pounds Troy measure are equivalent to 144 T. Avoirdupois?

30. Reduce 4 bu. to quarts.

31. Reduce $3\frac{7}{8}$ bu. to quarts.

32. Reduce 3.625 bu. to quarts.

33. Reduce 7 pk. to quarts.

34. Reduce 7.375 pk. to quarts.

35. Reduce $18\frac{7}{8}$ bu. to pints.

36. Reduce 13 bu. 3 qt. to pints.

37. Reduce 100 bu. to cubic inches.

38. Reduce 9 gal. to cubic inches.

39. How many gallons are equivalent to 1 bu.?

40. Reduce $2\frac{3}{5}$ mi. to feet.

41. Reduce 4.1875 mi. to feet.

42. Reduce 3.5625 mi. to yards.

43. Reduce 45 ch. to feet.

44. Reduce 125 ch. to feet.

45. Reduce 12 ch. to inches.

EXERCISE 86 (Written)

1. Reduce 18° 20′ 20″ to seconds.

2. Reduce a quadrant to seconds.

3. Reduce 12° 5′ 10″ to seconds.

4. Reduce $14\frac{1}{4}°$ to minutes.

5. Reduce $7\frac{3}{4}°$ to seconds.

6. Reduce 120.3° to seconds.

7. Reduce 75.75° to minutes.

8. Reduce 254.125° to minutes.

9. Reduce 45° 30′ 20″ to seconds.

10. Reduce $4\frac{2}{3}$° to minutes; $18\frac{5}{6}$° to minutes.

11. Reduce a common year to minutes.

12. Find the number of minutes in the years 1903, 1904, 1905.

13. Find the number of minutes in February, 1904.

14. Find the number of minutes in the first three months of the year 1903; also in the first three months of the year 1904.

15. Find the number of seconds in a solar year, consisting of 365 da. 5 hr. 48 min. 47 sec.

16. The pulse of a healthy person beats 70 times a minute. At this rate, how many times will it beat in a leap year? How many times will it beat in the four successive years, beginning 1904?

17. Reduce 30 wk. 6 da. 12 hr. to minutes.

18. Reduce $1\frac{3}{5}$ common years to days.

19. Reduce $5\frac{5}{7}$ wk. to hours.

20. Reduce 20.4 yr. to hours.

21. How many days are there between Jan. 30, 1902, and Jan. 30, 1910?

22. Reduce $\frac{2}{15}$ of a circumference to degrees.

23. Reduce $\frac{29}{30}$ of a straight angle to degrees.

24. Find the number of days in the first half of the twentieth century.

25. Find the number of days in the last half of the nineteenth century.

193. Reduce 2,347,854 sq. in. to higher denominations.

$$\begin{array}{r|rl}
12 & 2347854 & \text{sq. in.} \\ \hline
12 & 195654 \,..\, 6 & \multirow{2}{*}{$\left.\begin{matrix}\\ \\ \end{matrix}\right\}$ 78 sq. in.} \\ \hline
9 & 16304 \,..\, 6 & \\ \hline
 & 1811 & \text{sq. yd. 5 sq. ft.} \\
 & 4 & \\ \hline
11 & 7244 & \text{quarter sq. yd.} \\ \hline
11 & 658 \,..\, 6 & \left.\right\} \text{105 quarter sq. yd.} \\ \hline
 & 59 \,..\, 9 & \quad \text{or } 26\tfrac{1}{4} \text{ sq. yd.}
\end{array}$$

2,347,854 sq. in. =

59 sq. rd. 26¼ sq. yd. 5 sq. ft. 78 sq. in.
¼ sq. yd. = 2 sq. ft. 36 sq. in.

Hence 2,347,854 sq. in. =

59 sq. rd. 26 sq. yd. 7 sq. ft. 114 sq. in.

Step 1. Reduce the square inches to square feet by dividing by the two equal factors of 144.

Step 2. Reduce the square feet to square yards by dividing by 9.

Step 3. Reduce the square yards to square rods by dividing by 30¼. To divide by 30¼, multiply by 4 and divide by the two equal factors of 121.

Step 4. Reduce the quarter square rods to square yards and the fraction of a square yard.

Step 5. Convert the quarter of a square yard into square feet and square inches, and then substitute its value and add.

(Exercises like the above are inserted merely as disciplinary exercises.)

EXERCISE 87 (Written)

Reduce to higher denominations:

1. 5,308,298 sq. in.
2. 2,100,293 sq. in.
3. 17,463,000 sq. in.
4. 234 pt. liquid measure.
5. 2000 pt. dry measure.
6. 3456 pt. liquid measure.
7. 28,000 oz.
8. 47,385 cu. in.
9. 94,000 lb. of wheat to bu.
10. 33,000 lb. of oats to bu.
11. 1,283,496 sec.
12. 1,256,000″.
13. 2,000,000 sec.
14. 3,280,000 in.
15. 5,401,000 in.
16. 393,000 cu. in.
17. 3,200,500 oz.
18. 4985 ft.
19. 10,240 rd.
20. 20,000,000 A.
21. 300,100 sq. rd.
22. 490,000 gr. to lb. Avoirdupois.

23. The equatorial diameter of the earth is 41,852,404 ft. Express this distance in miles and the decimal of a mile, correct to two decimal figures. *Ans.* 7926.59 mi.

24. The polar diameter of the earth is 41,709,790 ft. What is the polar diameter of the earth in miles, correct to two decimal points? *Ans.* 7899.58 mi.

25. By how many miles does the equatorial diameter exceed the polar diameter?

26. The circumference of a circle is 3.1416 times the diameter. (*a*) From the data in Example 23 find the length of the equator. *Ans.* 24902.18 mi.

(*b*) Find the length of a degree on the earth's surface measured on the equator. *Ans.* 69.407 mi.

(*c*) Find the length of a minute of an arc on the equator.

27. Light takes 8 min. 18 sec. to come from the sun to the earth. The mean distance of the sun from the earth is 92,790,000 mi. Find the velocity of light per second.

28. The greatest depth of the Atlantic Ocean is estimated at 27,366 ft.; of the Pacific, 30,000 ft.; of the Southern Ocean, 25,200 ft.; of the Indian Ocean, 18,600 ft. Express these depths in miles correct to two decimal figures.

ADDITION

194. Add:

mi.	rd.	yd.	ft.	in.
3	27	4	2	7
4	98	3	1	5
7	129	2	0	9
8	211	4	2	11
9	168	2	2	5
32	316	$1\frac{1}{2}$	1	1
			1	6
32	316	1	2	7

Step 1. Adding the numbers in the right-hand column, we get 37 in. = 3 ft. 1 in. Write 1 in the column for inches, and carry 3 ft. to the next column.

Step 2. Adding the second column, we get 10 ft. = 3 yd. 1 ft. Write 1 ft. and carry the 3 yd.

Step 3. Adding the next column, we get 18 yd. = 36 half yd. = 3 rd. and 3 half yd. = 3 rd. $1\frac{1}{2}$ yd. Write $1\frac{1}{2}$ yd. and carry 3 rd. to the next column.

Step 4. Adding the rods, we get 636 rd. = 1 mi. 316 rd. Write 316 rd. and carry 1 mi. Adding the miles, we get 32 mi.

Step 5. In place of $\frac{1}{2}$ yd. we substitute its value, 1 ft. 6 in. Adding this quantity, we get the final result.

In **adding compound quantities**, proceed as follows:

Step 1. Arrange the quantities so that units of the same denomination stand in the same vertical column, the highest denomination being written first, the next to the highest second, and so on.

Step 2. Beginning with the right-hand column, add the numbers in it, and divide their sum by the number of units which makes one unit of the next higher denomination. Write the remainder in the right-hand column, and carry the quotient to the next column.

Step 3. Treat the next column to the left in the same manner. The remaining columns are dealt with in the same way.

EXERCISE 88 (Written)

LINEAR MEASURE

Add:

1.

yd.	ft.	in.
9	0	8
11	2	4
6	1	10
5	0	6
3	2	2
7	0	10

2.

rd.	yd.
306	5
234	1
137	4
249	4
198	3
105	3

3.

mi.	rd.
75	124
32	302
19	190
85	16
109	298
56	214

4.

yd.	ft.	in.
26	1	6
33	2	6
20	1	0
70	1	9
104	2	3
127	1	9

5.

yd.	ft.	in.
5	1	11
3	1	2
4	1	3
11	2	5
17	1	3
21	0	9

6.

rd.	yd.
291	4
77	2
208	3
96	3
264	4
193	3

SQUARE MEASURE

7.

A.	sq. rd.
76	144
85	131
37	33
63	99
54	77
48	136

8.

A.	sq. rd.
33	79
173	27
254	28
45	53
28	68
236	56

9.

A.	sq. rd.
127	88
192	99
238	77
413	25
92	84
127	63

10.

sq. yd.	sq. ft.	sq. in.
18	7	34
19	2	83
15	7	81
29	3	50
19	2	75
74	8	24

11.

sq. yd.	sq. ft.	sq. in.
15	7	72
28	3	92
17	3	75
18	2	14
17	3	74
73	8	21

12.

sq. rd.	sq. yd.	sq. ft.
18	14	4
9	2	7
8	5	4
29	29	8
17	4	9

CAPACITY

13. Add: 2 gal. 3 qt. 1 pt., 3 gal. 2 qt. 1 pt., 5 gal. 2 qt. 1 pt., 4 gal. 2 qt. 1 pt.

14. Add: 7 gal. 2 qt. 1 pt., 9 gal. 3 qt., 4 gal. 1 qt. 1 pt., 6 gal. 3 qt. 1 pt., 9 gal. 1 pt., 7 gal. 1 pt.

15. Add: 3 bu. 3 pk. 5 qt., 4 bu. 2 pk. 4 qt., 9 bu. 2 pk. 7 qt., 9 bu. 7 qt., 8 bu. 2 pk. 3 qt., 6 bu. 3 pk. 2 qt.

16. Add: 4 bu. 7 qt., 3 bu. 4 pk. 6 qt., 7 bu. 2 pk. 6 qt., 8 bu. 3 qt., 9 bu. 2 pk. 3 qt., 4 bu. 3 pk. 2 qt.

17. Add: 17 gal. 1 pt., 14 gal. 2 qt. 1 pt., 2 gal. 2 qt. 1 pt., 15 gal. 1 pt., 13 gal. 1 qt. 1 pt., 14 gal. 2 qt. 1 pt.

18. Add: 14 bu. 2 pk. 7 qt., 23 bu. 3 pk. 5 qt., 44 bu. 2 pk. 6 qt., 39 bu. 6 qt., 28 bu. 3 pk., 17 bu. 2 pk. 7 qt.

19. Add: 38 bu. 3 pk. 2 qt., 16 bu. 2 pk. 1 qt., 28 bu. 3 pk. 7 qt., 3 bu. 7 qt., 5 bu. 3 pk., 24 bu. 2 pk. 2 qt.

20. Add: 15 bu. 5 qt., 12 bu. 3 pk., 17 bu. 7 qt., 18 bu. 6 qt., 29 bu. 2 pk. 3 qt., 71 bu. 3 pk. 2 qt., 18 bu. 3 pk.

AVOIRDUPOIS WEIGHT

21. Add: 20 T. 215 lb., 18 T. 425 lb., 17 T. 328 lb., 92 T. 411 lb.

22. Add: 384 lb. 12 oz., 125 lb. 15 oz., 82 lb. 14 oz., 73 lb. 11 oz.

23. Add: 425 lb. 10 oz., 17 lb. 14 oz., 30 lb. 12 oz., 72 lb. 9 oz.

24. Add: 15 T. 290 lb., 17 T. 184 lb., 12 T. 127 lb., 15 T. 9 lb., 18 T. 18 lb.

25. Add: 18 lb. 8 oz., 64 lb. 7 oz., 82 lb. 6 oz., 90 lb. 5 oz., 16 lb. 13 oz.

26. Add: 16 T. 175 lb., 71 T. 29 lb., 28 T. 245 lb., 97 T. 159 lb., 13 T. 1300 lb.

TIME

27. Add: 5 da. 4 hr. 15 min., 17 da. 17 hr. 17 min., 92 da. 14 hr. 14 min., 27 da. 23 hr. 12 min., 29 da. 16 hr. 14 min., 45 da. 15 hr. 18 min.

28. Add: 4 wk. 5 da. 7 hr., 9 wk. 6 da. 11 hr., 18 wk. 5 da. 12 hr., 23 wk. 11 hr., 28 wk. 4 da. 4 hr., 73 wk. 6 da. 19 hr., 82 wk. 5 da. 21 hr.

29. Add: 20 hr. 30 min. 18 sec., 17 hr. 45 min. 37 sec., 14 hr. 18 min. 18 sec., 14 hr. 12 min. 12 sec., 9 hr. 48 min. 48 sec., 8 hr. 39 min. 39 sec.

30. Add: 12 da. 17 hr. 44 min., 15 da. 18 hr. 18 min., 31 da. 19 hr. 19 min., 33 da. 21 hr. 27 min., 12 da. 12 hr. 36 min., 34 da. 20 hr. 23 min.

31. Add: 3 wk. 5 da. 23 hr., 8 wk. 6 da. 16 hr., 9 wk. 5 da. 18 hr., 4 wk. 4 da. 14 hr., 10 wk. 5 da. 13 hr., 8 wk. 4 da. 14 hr.

32. Add: 14 hr. 14 min. 14 sec., 9 hr. 54 min. 38 sec., 11 hr. 12 min. 19 sec., 4 hr. 31 min. 27 sec., 5 hr. 45 min. 43 sec., 8 hr. 41 min. 42 sec.

VOLUME

33. Add: 4 cu. ft. 1421 cu. in., 9 cu. ft. 294 cu. in., 18 cu. ft. 998 cu. in., 7 cu. ft. 778 cu. in., 9 cu. ft. 499 cu. in., 15 cu. ft. 498 cu. in.

34. Add: 27 cu. yd. 19 cu. ft., 84 cu. yd. 24 cu. ft., 87 cu. yd. 19 cu. ft., 16 cu. yd. 22 cu. ft., 55 cu. yd. 17 cu. ft., 34 cu. yd. 16 cu. ft.

35. Add: 37 cu. yd. 13 cu. ft., 38 cu. yd. 26 cu. ft., 49 cu. yd. 25 cu. ft., 62 cu. yd. 26 cu. ft., 77 cu. yd. 17 cu. ft., 94 cu. yd. 28 cu. ft.

36. Add: 15 cu. ft. 578 cu. in., 18 cu. ft. 902 cu. in., 18 cu. ft. 978 cu. in., 15 cu. ft. 293 cu. in., 9 cu. ft. 1727 cu. in.

37. Add: 7 cu. yd. 21 cu. ft., 29 cu. yd. 24 cu. ft., 24 cu. yd. 18 cu. ft., 34 cu. yd. 9 cu. ft., 16 cu. yd. 19 cu. ft., 83 cu. yd. 17 cu. ft.

38. How many cubic inches are in 1 qt. dry measure? in 1 qt. liquid measure? in 1 pk.?

39. (*a*) The British imperial gallon contains 277.274 cu. in. This quantity of distilled water weighs 10 lb. Calculate in ounces the weight of 1 cu. ft. of water.

(*b*) How many of our gallons are equivalent to 100 imperial gallons?

40. How many liquid quarts would it take to fill a bushel measure?

SUBTRACTION

195. From 19 sq. yd. 5 sq. ft. 20 sq. in. take 14 sq. yd. 7 sq. ft. 45 sq. in.

sq. yd.	sq. ft.	sq. in.
19	5	20
14	7	45
4	6	119

Step 1. Write the quantities so that units of the same denomination are in the same column.

Step 2. Find what concrete quantity added to 45 sq. in. will give 1 sq. ft. 20 sq. in., *i.e.* 164 sq. in. Write the remainder, 119 sq. in., in the column for square inches. Carry 1 sq. ft.

Step 3. Find what concrete quantity added to 8 sq. ft. will give 1 sq. yd. 5 sq. ft., *i.e.* 14 sq. ft. Write the remainder, 6 sq. ft., in the column for square feet. Carry 1 sq. yd.

Step 4. Find the difference between 15 sq. yd. and 19 sq. yd. and write it in its proper place.

EXERCISE 89 (Written)

CIRCULAR OR ANGULAR MEASURE

1. Subtract 5° 12′ 13″ from 84° 14′ 30″.
2. Subtract 19° 14′ 14″ from 27° 15′ 10″.
3. Subtract 38° 15′ 45″ from 90° 10′ 10″.
4. Subtract 54° 14′ 54″ from 172° 0′ 19″.
5. Subtract 84° 5′ 15″ from 90°.
6. Subtract 113° 13′ 54″ from 180°.
7. Subtract 94° 53′ 50″ from 180°.

8. Subtract 87° 15′ from 133° 12′.

9. Subtract 119° 54′ 17″ from 180°.

10. Subtract 15° 14′ 17″ from 94° 14′ 7″.

11. The tropic of Cancer is 23° 27′ 6″ north of the equator; the Arctic circle is 23° 27′ 6″ south of the north pole. Find the width of the north temperate zone.

CAPACITY

12. From 3 bu. 2 pk. 4 qt. take 1 bu. 2 pk. 5 qt.

13. From 12 gal. 3 qt. 1 pt. take 4 gal. 3 qt.

14. From 11 gal. take 4 gal. 3 qt. 1 pt.

15. From 17 gal. take 11 gal. 1 qt. 1 pt.

16. From 37 bu. 2 pk. 4 qt. take 17 bu. 3 pk. 7 qt.

17. From 29 bu. 1 pk. 2 qt. take 19 bu. 3 pk. 5 qt.

18. From 37 gal. take 17 gal. 1 qt. 1 pt.

19. From 134 gal. take 112 gal. 3 qt. 1 pt.

20. From $1\frac{1}{2}$ gal. take $\frac{3}{4}$ gal. and express the result in quarts.

21. From $1\frac{3}{4}$ bu. take $\frac{7}{8}$ bu. and express the result in quarts.

22. From $5\frac{1}{2}$ bu. take $1\frac{3}{4}$ bu. and express the result in quarts.

TIME

23. From 3 da. 4 hr. 11 min. take 1 da. 7 hr. 14 min.

24. From 11 da. 5 hr. 10 min. take 4 da. 11 hr. 19 min.

25. Almanacs give the time of sunrise in Florida, Louisiana, and Texas on March 5 as 6.22 A.M., and that of sunset as 6.2 P.M. Find the length of the day.

26. (*a*) In southern New York the sun rises on June 22 at 4.29 A.M. and sets at 7.34 P.M. Find in hours and minutes the length of the longest day in southern New York. (*b*) On the same date the sun rises in places near the 32d parallel, at 4.59 A.M. and sets at 7.4 P.M. Find the time from sunrise to sunset.

27. The moon's phases for places near the 90th meridian are for June, 1903:

First Quarter	2 da.	7 hr.	24 min.	A.M.
Full Moon	9 da.	9 hr.	8 min.	P.M.
Last Quarter	18 da.	0 hr.	44 min.	A.M.
New Moon	25 da.	0 hr.	11 min.	A.M.

(*a*) Find the time from First Quarter to Full Moon.

(*b*) Find the time from Full Moon to Last Quarter.

(*c*) Find the time from Full Moon to New Moon.

28. On Aug. 1, 1903, the sun crosses the meridian at 6 min. 11 sec. past 12 o'clock; on Sept. 2, 1903, it crosses at 59 min. 52 sec. past 11 o'clock. How many minutes and seconds earlier does the sun come to the meridian on Sept. 2 than on Aug. 1?

29. The equation of time is the difference between the time indicated by a sundial and that of a clock keeping correct time. On Nov. 6, when it is noon by the clock, it is 43 min. and 42 sec. past 11 by the sundial. Find the equation of time Nov. 6.

30. The planet Venus rises in New Orleans on Nov. 1, 1903 at 8.14 P.M., and at New York City it rises on the same day at 7.41 P.M. How many minutes earlier is Venus visible in New York City than in New Orleans on that date?

31. On April 1, 1903, the moon rose at 10.28 P.M. On April 4 following, it rose at 12.28 A.M. How many hours and minutes earlier did it rise on April 1 than on April 4?

196. Time between events happening in two different years.

Example. How many years, months, and days were between Aug. 27, 1880, and Jan. 22, 1901?

yr.	mo.	da.
1901	1	22
1880	8	27
20	4	25

Since January is the first month of the year and August is the eighth month of the year, we write 1 instead of January and 8 instead of August. In finding the difference, a month is taken as 30 days. The work is then performed as in the subtraction of compound quantities.

EXERCISE 90 (Written)

1. The battle of New Orleans was fought on Jan. 8, 1815. Find the time from that date to the present day.

2. The first telegraph message was sent by Professor Morse on May 24, 1844. Find the time from that date to the present day.

3. The Spanish fleet under Cervera was destroyed near Santiago on July 3, 1898. Find the time from that date to Feb. 1, 1903.

4. The Mecklenberg Declaration of Independence was signed May 20, 1775. Find the time from this date to the surrender of Cornwallis, Oct. 19, 1781.

5. The following named men were born and died on the dates specified. Find how long each lived.

	Born	Died
John Milton	Dec. 9, 1608.	Nov. 8, 1674.
Alexander Pope . .	May 21, 1688.	May 30, 1744.
William Shakespeare .	April 23, 1564.	April 23, 1616.
Edmund Burke . . .	Jan. 12, 1730.	July 9, 1797.
Robert E. Lee . . .	Jan. 19, 1807.	Oct. 12, 1870.
U. S. Grant	April 27, 1822.	July 23, 1885.
Oliver Goldsmith . .	Nov. 10, 1728.	April 4, 1774.
Benjamin Franklin .	Jan. 17, 1706.	April 17, 1790.
Alexander Hamilton .	Jan. 11, 1757.	July 12, 1804.
H. W. Longfellow .	Feb. 27, 1807.	March 24, 1882.
J. H. Newman . . .	Feb. 21, 1801.	Aug. 11, 1890.
W. E. Gladstone . .	Dec. 9, 1809.	May 19, 1898.

MULTIPLICATION

197. Multiply 5 yd. 2 ft. 10 in. by 7.

yd.	ft.	in.
5	2	10
		7
41	1	10

Multiply 10 in. by 7 and we get 70 in. = 5 ft. 10 in. Write 10 in. and carry 5 ft. 7 times 2 ft. are 14 ft. 14 ft. and 5 ft. = 19 ft. = 6 yd. 1 ft. Write 1 ft. Carry 6 yd. 7 times 5 yd. are 35 yd. 35 yd. and 6 yd. = 41 yd.

EXERCISE 91 (Written)

1. Multiply 4 yd. 2 ft. 3 in. by 9.
2. Multiply 6 yd. 1 ft. 9 in. by 11.
3. Multiply 9 yd. 2 ft. 11 in. by 8.
4. Multiply 3 bu. 2 pk. 7 qt. by 7.
5. Multiply 4 bu. 1 pk. 6 qt. by 12.

6. Multiply 9 gal. 3 qt. 1 pt. by 6.

7. Multiply 6 gal. 2 qt. 1 pt. by 12.

8. Multiply 3 bu. 3 pk. 7 qt. by 7.

9. Multiply 12 T. 400 lb. by 12.

10. Multiply 13 T. 387 lb. by 9.

11. Multiply 17 T. 254 lb. by 10.

12. Multiply 5° 29′ 28″ by 16.

13. Multiply 16° 38′ 32″ by 15.

14. Multiply 15 da. 10 hr. 14 min. by 9.

15. Multiply 12 da. 9 hr. 18 min. by 8.

16. Multiply 16 sq. yd. 8 sq. ft. 135 sq. in. by 7.

17. Multiply 84 ch. 85 l. by 18.

18. Multiply 16 ch. 64 l. by 24.

19. Multiply 64 A. 150 sq. rd. by 12.

20. Multiply 15 A. 27 sq. rd. by 11.

21. Multiply 16 sq. yd. 7 sq. ft. 84 sq. in. by 9.

22. Multiply 16 T. 250 lb. by 8.

23. Multiply 18° 9′ 54″ by 14.

24. Multiply $\frac{5}{8}$ T. by 9 and express the result in pounds.

25. Multiply $\frac{3}{4}$ mi. by 19 and give the result in feet.

DIVISION

198. Divide 97° 10′ 50″ by 8.

$$\begin{array}{r|lll} 8 & 97^\circ & 10' & 50'' \\ \hline & 12^\circ & 8' & 51\frac{1}{4}'' \end{array}$$

The eighth part of 97° is 12° with a remainder of 1°. 1° 10′ = 70′. The eighth part of 70′ is 8′ with a remainder of 6′. 6′ 50″ = 410″. 8 into 410 goes 51$\frac{1}{4}$ times.

EXERCISE 92 (Written)

1 Divide 12 T. 385 lb. by 5.

2. Divide 93 T. 297 lb. by 9.

3. Divide 21 yd. 2 ft. 3 in. by 9.

4. Divide 93° 15′ 15″ by 7.

5. Divide 84° 14′ 14″ by 12.

6. Divide 98 ch. 46 l. by 9.

7. Divide 45 ch. 84 l. by 8.

8. Divide 34 yd. 2 ft. 8 in. by 6.

9. Divide 77 yd. 2 ft. 4 in. by 7.

10. Divide 13 bu. 3 pk. 4 qt. by 6.

11. If 4 oxen weigh 3 T. 15 cwt., find the average weight of each.

12. How often is 231 cu. in. contained in 1 cu. ft. 582 cu. in.?

13. How often is 7 yd. 1 ft. contained in a mile?

14. If a link is the unit of measurement, what number would represent 1 mi.?

15. If 16 sq. yd. is the unit of surface measure, what concrete quantity would represent 1 sq. mi.?

16. A meter is 39.37 in. nearly. How many meters equal 1 mi.?

17. The angular unit in analysis is 206265″ nearly. Find, correct to four decimal figures, the value of a straight angle expressed in terms of this unit.

18. The planet Mercury revolves around the sun in 88 da., in an orbit nearly circular. Find in degrees, minutes, and seconds its daily progress.

19. Civil engineers use a chain 100 ft. long. How many of these chains are in 5 mi.?

199. *Example* **1.** Reduce .875 yd. to feet and inches.

$$\begin{array}{r} .875 \text{ yd.} \\ 3 \\ \hline 2.625 \text{ ft.} \\ 12 \\ \hline 7.500 \text{ in.} \end{array}$$

$$.875 \text{ yd.} = 2 \text{ ft. } 7.5 \text{ in.}$$

Example **2.** Reduce $\frac{7}{12}$ bu. to lower denominations.

$$\tfrac{7}{12} \text{ bu.} = \tfrac{7}{12} \text{ of } 4 \text{ pk.} = \tfrac{7}{3} \text{ pk.} = 2\tfrac{1}{3} \text{ pk.}$$
$$\tfrac{1}{3} \text{ pk.} = \tfrac{1}{3} \text{ of } 8 \text{ qt.} = \tfrac{8}{3} \text{ qt.} = 2\tfrac{2}{3} \text{ qt.}$$
$$\tfrac{2}{3} \text{ qt.} = \tfrac{2}{3} \text{ of } 2 \text{ pt.} = \tfrac{4}{3} \text{ pt.} = 1\tfrac{1}{3} \text{ pt.}$$

Hence, $\frac{7}{12}$ bu. = 2 pk. 2 qt. $1\frac{1}{3}$ pt.

EXERCISE 93 (Written)

1. Reduce $\frac{7}{8}$ T. to pounds.

2. Reduce .15° to minutes.

3. Reduce $\frac{7}{9}$ da. to hours and minutes.

4. Reduce .2345 T. to pounds.

5. Reduce .95 da. to hours and minutes.

6. Reduce .375 bu. to quarts.

7. Reduce 18.4 mi. to feet.

8. Reduce $\frac{7}{11}$ mi. to yards.

9. Reduce .1875 mi. to rods.

10. Reduce $15\frac{3}{4}$° to minutes.

11. Reduce $\frac{2}{3}$ of a common year to days and hours.

12. Reduce .3125 of a common year to days, hours, and minutes.

13. Reduce .45 bu. to lower denominations.

14. Reduce $\frac{2}{3}$ of a gallon to lower denominations.

15. Reduce .85 of an acre to square rods.

16. Reduce .273 sq. yd. to square feet and square inches.

17. Reduce .328 mi. to lower denominations.

18. Reduce .756 mi. to lower denominations.

19. Reduce $\frac{9}{16}$ of a common year to lower denominations.

20. Reduce 216 da. to the fraction of a common year.

21. Reduce 292 da. to the decimal of a common year.

22. Reduce .575 of a leap year to lower denominations.

23. Reduce .345 of a mile to feet.

24. Reduce $\frac{17}{22}$ of an acre to square yards.

25. Reduce $\frac{39}{44}$ of an acre to square yards.

26. Reduce $\frac{13}{16}$ bu. to quarts.

27. Reduce $\frac{7}{16}$ gal. to pints.

REDUCTION DESCENDING

28. Reduce $\frac{11}{16}$ of a nautical mile to feet.

29. Reduce $\frac{5}{16}$ cwt. to pounds and ounces.

30. Reduce $\frac{41}{48}$ of a day to hours and minutes.

31. Reduce .265 da. to lower denominations.

32. Reduce $\frac{3}{4}$ of a right angle to degrees and minutes.

33. Reduce .56 of a straight angle to degrees and minutes.

EXERCISE 94 (Oral)

1. What part of 1 yd. is 1 in.? What part of 1 yd. is 6 in.?

2. What part of 1 yd. is 18 in.? What part of 1 yd. is 1 ft. 6 in.?

3. What fraction of 1 yd. is 2 ft. 3 in.? 2 ft. 6 in.? 1 ft. 9 in.?

4. What part of 1 A. is 80 sq. rd.? 32 sq. rd.? 16 sq. rd.?

5. What part of 1 bu. is 16 qt.? 4 qt.?

6. What fraction of 1 bu. is 2 pk. 4 qt.?

7. What fraction of 1° is 15′? 20′? 25′? 36′? 48′?

8. What fraction of 1° is $7\frac{1}{2}'$? $8\frac{4}{7}'$? $6\frac{2}{3}'$? 18′?

9. What fraction of 1 da. is $4\frac{1}{2}$ hr.? $7\frac{1}{2}$ hr.? $2\frac{1}{3}$ hr.?

10. What fraction of 1 da. is 6 hr.? 18 hr.? 20 hr.?

11. What fraction of a square mile is 160 A.? 120 A.? 480 A.?

12. What fraction of a gallon is 1 qt.? 1 qt. 1 pt.? 2 qt. 1 pt.?

13. What fraction of a mile is 10 ch.? 15 ch.? 25 ch.? 50 ch.?

14. Reduce $3\frac{1}{2}$ da. to the fraction of 1 wk.?

15. What fraction of 1 wk. is $2\frac{1}{2}$ da.? $4\frac{1}{2}$ da.? $5\frac{1}{2}$ da.? $2\frac{1}{3}$ da.?

16. What fraction of 1 yr. is 9 mo.? $4\frac{1}{2}$ mo.? $2\frac{2}{5}$ mo.? $1\frac{1}{2}$ mo.?

17. What fraction of 1 cu. yd. is 6 cu. ft.? $6\frac{3}{4}$ cu. ft.? $3\frac{3}{8}$ cu. ft.?

18. What fraction of 1 sq. yd. is $2\frac{1}{4}$ sq. ft.? 2 sq. ft.? 36 sq. in.? $4\frac{1}{2}$ sq. ft.? 4 sq. ft.? 72 sq. in.? $2\frac{1}{3}$ sq. ft.? 2 sq. ft.? 48 sq. in.?

200. *Example* 1. Express 27 rd. 4 yd. 2 ft. as a fraction of 1 mi.

27 rd. 4 yd. 2 ft.
$5\frac{1}{2}$
$13\frac{1}{2}$
139
$152\frac{1}{2}$ yd.
3
$459\frac{1}{2}$ ft.

$$1 \text{ mi.} = 5280 \text{ ft.}$$

$$27 \text{ rd. } 4 \text{ yd. } 2 \text{ ft.} = \frac{459\frac{1}{2}}{5280} \text{ of } 1 \text{ mi.}$$

$$= \frac{919}{10560} \text{ of } 1 \text{ mi.}$$

To express one compound quantity as a fraction of another compound quantity, or as a fraction of a simple concrete quantity, reduce both to simple concrete quantities of the same denomination and then divide the quantity to be expressed in terms of the other quantity, by the other quantity.

Example 2. Express 2 yd. 2 ft. 8 in. as a decimal of 1 mi.

Step 1. Express 2 yd. 2 ft. 8 in. as a fraction of 1 mi.

Step 2. Reduce this fraction to a decimal.

EXERCISE 95 (Written)

1. Reduce 4400 ft. to the decimal of 1 mi.
2. Reduce 293 yd. 1 ft. to the decimal of 1 mi.
3. Reduce 117 yd. 1 ft. to the decimal of 1 mi.
4. Reduce 1 qt. 1 pt. to the decimal of a gallon.
5. Reduce 1 pk. 6 qt. to the decimal of 1 bu.
6. Reduce 2 pk. 2 qt. to the decimal of 1 bu.
7. Reduce 3 pk. 1 qt. 1 pt. to the decimal of 1 bu.
8. Reduce $1\frac{1}{8}$ in. to the decimal of 1 ft.
9. Reduce 2.34 in. to the decimal of 1 ft.
10. Reduce 3° 15′ to the fraction of a right angle.
11. Express in degrees 4° 16′ 30″.
12. What fraction of 1 mi. is 83 ft. 4 in.?
13. What fraction of 1 mi. is 660 ft.?
14. What fraction of 1 T. is 125 lb.?
15. What fraction of 1 T. is $437\frac{1}{2}$ lb.?
16. What decimal of 1 T. is 1312 lb.?
17. Reduce to the decimal of a ton 3 cwt. 12 lb. 8 oz.

18. Reduce 2 cwt. 22 lb. 3$\frac{5}{9}$ oz. to the decimal of a ton.

19. What decimal of a long ton is 560 lb.?

20. Express decimally 1 long ton in terms of a short ton.

21. Express decimally a short ton in terms of a long ton.

22. What fraction of 1 T. is the weight of 5$\frac{1}{2}$ bu. of wheat?

23. Give the weights of 1 bu. of wheat, of 1 bu. of barley, and of 1 bu. of oats, as decimals of a ton.

24. Express, in its simplest form, 22′ 30″ as a fraction of 1°.

25. Express, in its simplest form, 5′ 37$\frac{1}{2}$″ as a fraction of 1°.

26. Reduce 4 hr. 48 min. to the decimal of 1 da.

27. Reduce 1 hr. 36 min. to the decimal of 1 da.

28. Reduce 6 cu. ft. 1296 cu. in. to the decimal of 1 cu. yd.

29. Reduce 4 cu. ft. 864 cu. in. to the decimal of 1 cu. yd. Give the result also as a common fraction.

30. Reduce 1 da. 18 hr. to the decimal of 1 wk.

31. Reduce 1 da. 9 hr. 36 min. to the decimal of 1 wk.

32. Reduce 4 da. 4 hr. 48 min. to the decimal of 1 wk.

33. Reduce 18 wk. 1 da. 6 hr. to the decimal of 1 common year.

34. Reduce 2$\frac{4}{5}$ mo. to the fraction of a year.

35. The planets, Mercury, Venus, Mars, and Jupiter, revolve around the sun in periods of 87.97, 224.7, 686.95, 4332.58 days respectively. Express each of these periods in terms of our common year.

AREAS OF RECTANGLES

EXERCISE 96 (Written)

1. Find the number of acres in the area of a rectangle whose dimensions are 360 ft. and 121 ft.

2. Find the area, in acres, of a rectangle 1321 ft. by 720 ft.

3. Find, in acres, the area of a rectangular plot of ground 201 yd. by 10 rd. 5 yd.

6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

FIG. 1.

4. Find the acreage of a rectangular farm 240 rd. long by 51 rd. 1 yd. 2 ft. 6 in. wide.

5. A railway company acquires the right of way through a territory 154 mi. long, and fences in a strip

80 ft. wide. How many acres does it thus inclose, and how much does it pay for the land at $25 an acre?

6. Find the area of a park 396 ft. by 396 ft. Give your answer in acres.

7. How many acres are in a rectangular farm 1.5 mi. long by $1\frac{1}{4}$ mi. wide?

201. A **township** is a tract of land 6 mi. square, and it is divided into 36 **sections** each 1 mi. square. The sections are numbered as in Fig. 1.

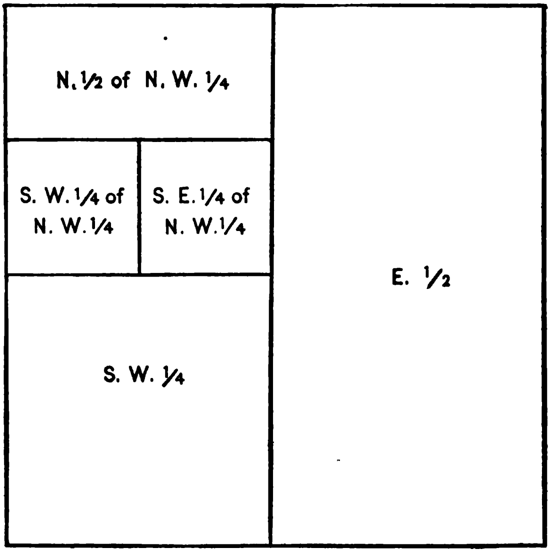


FIG. 2.

A section is subdivided as indicated in Fig. 2. There are two divisions, E. $\frac{1}{2}$ and W. $\frac{1}{2}$. The E. $\frac{1}{2}$ is divided into two equal squares called N.E. $\frac{1}{4}$ and S.E. $\frac{1}{4}$. The W. $\frac{1}{2}$ is divided into two equal squares called N.W. $\frac{1}{4}$ and S.W. $\frac{1}{4}$. These are again subdivided as shown.

EXERCISE 97 (Written)

1. Draw a figure and locate S.W. $\frac{1}{4}$ of S.E. $\frac{1}{4}$. N.W. $\frac{1}{4}$ of S.E. $\frac{1}{4}$. N.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$. S.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$.

2. Locate N.E. $\frac{1}{4}$ of N.E. $\frac{1}{4}$. S.W. $\frac{1}{4}$ of N.E. $\frac{1}{4}$. How many acres are in N.W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$? How many acres are in N.W. $\frac{1}{4}$? In S.E. $\frac{1}{4}$? In S. $\frac{1}{2}$ of S.W. $\frac{1}{4}$?

3. A man buys $\frac{1}{2}$ of S. $\frac{1}{2}$ of N.W. $\frac{1}{4}$ and also $\frac{1}{2}$ of the N.W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ at the rate of $20 an acre. Find the cost of his purchase.

4. Find the value of E. $\frac{1}{2}$ at $35.75 per acre.

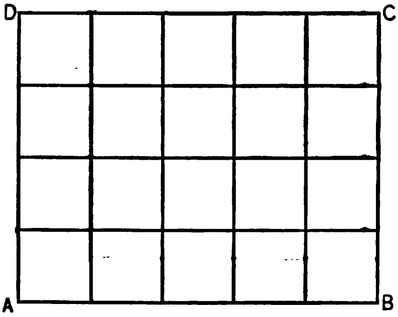
5. A man buys S.W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ and S.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ at $84 an acre. How much does he pay?

6. What is the value of S. $\frac{1}{2}$ of N.E. $\frac{1}{4}$ at $45 an acre?

7. Find the combined values of S.E. $\frac{1}{4}$ and S.W. $\frac{1}{4}$ of N.E. $\frac{1}{4}$ at $25 an acre.

VOLUMES OF RECTANGULAR SOLIDS

202. Let *ABCD* be a rectangle 5 in. long by 4 in. wide. Its area is 20 sq. in. If we take 20 cubes each 1 in. on an edge, we can place them on this rectangle so as to make a rectangular solid 5 in. by 4 in., and 1 in. in height. Its volume is 20 cu. in.



If we take 20 more cubes each 1 in. on an edge, and place them on top of our rectangular solid, we get a second rectangular solid 5 in. by 4 in. and 2 in. in height. Its volume is 2 times 20 cu. in.

If we take 20 more cu. in. and place them on top of our second rectangular solid, we get a third rectangular solid 5 in. by 4 in. and 3 in. in height. Its volume is 3 times 20 cu. in.

If we add another layer of these cubes, we shall have a rectangular solid 5 in. by 4 in. and 4 in. in height. Its volume is 4 times 20 cu. in.

If we add, once more, another layer of these cubes, we shall have a rectangular solid 5 in. by 4 in. and 5 in. in height. Its volume is 5 times 20 cu. in.

If we start out with another rectangle, it matters not what its dimensions are, and form, in the same way, a rectangular solid, we can find its volume by multiplying the number of units in the area of the rectangle by the number of units in the height of the rectangular solid.

The **area of a rectangle** is obtained by taking the product of its dimensions. To get the volume of a rectangular solid we multiply this product by the height of the solid.

The **volume of a rectangular solid** is obtained by taking the product of its three dimensions expressed in units of the same denomination.

EXERCISE 98 (Written)

1. Find the volume of a box 12 ft. by 7 ft. by 6 ft.

2. Find the cubical contents of a room 18 ft. by 12 ft. and 9 ft. high.

3. Find the number of cubic feet in a room 32 ft. by 24 ft. and 12 ft. high.

4. How many cubic yards of earth must be removed for the foundation of a house 75 ft. by 54 ft., if the earth to the depth of 21 ft. is removed?

5. A cistern, in the shape of a rectangular solid, is 22 ft. by 14 ft. and 6 ft. deep. How many gallons of water does it contain?

6. A bin is 8 ft. by 3 ft. and 6 ft. high. How many bushels does it hold?

7. In order to build a concrete wall, earth is removed to the depth of 6 ft. If the wall is 210 ft. long and 12 ft. wide, how many cubic yards of earth must be removed?

8. How many cubic yards of gravel will fill, to the depth of 6 in., a street 1 mi. long and 36 ft. wide?

9. How many cubical boxes 2 ft. each way would a storeroom 18 ft. by 12 ft. and 10 ft. high hold?

10. The Sault Ste. Marie Canal is 1.6 mi. long, 160 ft. wide, and 25 ft. deep. Express in cubic yards the volume of water required to fill it.

11. A block of marble is 4 ft. by 3 ft. and 24 ft. long. How many tons does it weigh, if a cubic foot of marble weighs 170 lb.?

12. How many pounds does a cedar beam 14 in. by 10 in. and 40 ft. long weigh, if a cubic foot of cedar wood weighs 38.1 lb.?

13. A cubic foot of clay weighs 75 lb. Find, in tons, the weight of a clay bank 10 ft. by 4 ft. and 80 ft. long.

14. A box 9 in. by 8 in. and 6 in. deep is filled with mercury. Find its weight in pounds, if a cubic foot of mercury weighs 13,570 oz.

15. How many 3-in. cubes are required to fill a cubical box each of whose edges is 1 yd.?

16. A pile of 4-ft. wood 8 ft. long and 4 ft. high contains a cord. How many cords of wood are in a pile of 4-ft. wood 120 ft. long and 12 ft. high?

REVIEW QUESTIONS

1. Define simple quantity. Compound quantity.
2. What is the unit in Avoirdupois weight?
3. How many pounds are in a short ton? in a long ton?

4. What is reduction ascending? What is reduction descending?

5. Give a rule for reduction ascending.

6. Give a rule for reduction descending.

7. Define long measure. Unit of length.

8. What is the legal value of a yard in the United States?

9. How are yards reduced to miles?

10. How many yards are in a chain?

11. Define square mile. Nautical mile. League.

12. What is a township? What is a section?

13. What is a solid?

14. How is the volume of a rectangular solid obtained?

15. How does the quart liquid measure differ from the quart dry measure?

16. Define angle. Give the table of angular measure.

17. What is a straight angle?

18. What is the unit of angular measure?

19. What is the origin of the division of the circle into 360 equal parts?

20. How many days are in the common year? Name the calendar months and the number of days in each.

21. What is a lunar month? the length of a year?

22. What is the Julian calendar? the Gregorian Calendar? When was the latter introduced into England?

23. What was the error at that time in the Julian calendar?

24. How are the sections of a township numbered?

CHAPTER XI

MEASURES AND MULTIPLES

NOTE. In this chapter we shall give several statements requiring proof. These statements are necessary to a clear understanding of G.C.M. and L.C.M., and should therefore be the permanent possession of every teacher. We shall not, however, undertake to give a proof for any of the statements. We shall merely furnish illustrations to make clear the truth of each of the statements. This is entirely different from giving a formal proof. Should some of the matter be thought too difficult for the first presentation of the subject, that portion of it may be omitted until arithmetic is gone over in review.

203. A number which measures each of two numbers measures their sum and also their difference.

ILLUSTRATION. If 7 measures 84, and measures 63, then it measures 84 + 63, and also measures 84 – 63. If 7 apples are contained an exact number of times in some given number of apples, then this given number of apples can be put into bags each containing 7 apples. We shall then have an exact number of bags. If 7 apples are contained an exact number of times in some other number of apples, then this other number of apples likewise can be put into bags each containing 7 apples. We shall then have an exact number of bags. Now if we sum the first number of bags and the second number of bags, we get a number of bags each containing 7 apples. In other words, we have a multiple of 7. If we take the difference between the first number of bags and the second number of bags, we again have a multiple of 7.

204. A number which measures a second number measures any multiple of that second number.

ILLUSTRATION. If 11 measures 154, it measures 5 times 154. If 11 measures a number, then that number contains an exact number of groups of 11 each. If we multiply this number by some other number, we get a product which contains the same number of single objects as the product arising from the multiplication of our groups of 11 each by the same number. But multiplying a number of groups of 11 each gives an exact number of groups of 11 each. In other words, it gives a multiple of 11.

A number which measures a second number measures any power of the second number. This follows at once from previous statements.

205. By combining the statements in §§ 203 and 204, we have the following statement: **A number which measures two numbers measures the sum and also the difference of any multiples of these two numbers.**

ILLUSTRATION. If 3 measures 21 and 15, then 3 measures 9 times 21 + 4 times 15.

206. *Two* measures a number if it measures the units' digit of the number. For example, 2 measures 56 because the units' digit, 6, is measured by 2. The reason for this is, 2 measures 6 and 10, therefore (§ 205) it measures 5 tens + 6.

Five is a measure of any number ending in 5 or 0. For example, 5 measures 3285. The reason for this is, 5 measures 10 and 5, hence it measures 328 tens + 5.

207. A number is measured by *nine* if the sum of its digits is measured by 9; for example, 69453.

Since *three* is a measure of 9, 3 is a measure of any number the sum of whose digits is measured by 3.

208. *Eleven* is a measure of any number the difference of the sums of whose digits in the even places and in the odd places is 0 or a multiple of 11. Thus, 38962 is a multiple of 11, since the sum of the digits in the even places, 6, 8, equals 14, the sum of the digits in the odd places, 2, 9, 3, equals 14; the difference of the sums is 0.

209. *Example* 1. Resolve 1001 into its prime factors.*

$$\begin{array}{r} 7\underline{)1001} \\ 11\underline{)143} \\ 13 \end{array}$$

1001 is not divisible by 2, because its units' figure is not exactly divisible by 2. It is not divisible by 3, because the sum of the digits, 1, 1, is not divisible by 3. It is not divisible by 5, because its units' figure is not 0 or 5. Trying 7, we find 7 is contained in 1001, 143 times. We see at once that 143 is divisible by 11, because the sum of the digits, 1, 3, equals 4, the digit in the even place. Hence the prime factors of 1001 are 7, 11, 13. Hence, $1001 = 7 \times 11 \times 13$.

Example 2. Resolve 5040 into its prime factors, and express 5040 as the product of prime numbers.

$$\begin{array}{r|r} 2 & \underline{5040} \\ 2 & \underline{2520} \\ 2 & \underline{1260} \\ 2 & \underline{630} \\ 5 & \underline{315} \\ 3 & \underline{63} \\ 3 & \underline{21} \\ \multicolumn{1}{r}{} & 7 \end{array}$$

Divide by 2 as often as possible. Since 315 ends in 5, 5 is a factor of 315. Divide next by 3 as often as possible.

The prime factors of 5040 are 2, 2, 2, 2, 3, 3, 5, 7.

$$\begin{aligned} 5040 &= 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 \\ &= 2^4 \times 3^2 \times 5 \times 7. \end{aligned}$$

---

* When we speak of the factors of a number we have in mind only the *integral* factors of the number.

EXERCISE 99 (Written)

Resolve into prime factors and express each number as the product of its prime factors:

1. 8, 12, 16, 18, 20, 24, 27, 28, 30, 32, 36, 39, 40, 42.

2. 45, 48, 49, 50, 56, 60, 65, 69, 72, 75, 77, 80, 84, 88, 92.

3. 98, 99, 111, 117, 119, 120, 124, 128, 132, 133, 135, 140, 144.

4. 240, 720, 343, 512, 216, 729, 736, 608, 544.

5. 1331, 11,011, 1309, 858, 1274, 891, 3575.

6. Write all the measures of each of the following numbers: 36, 360, 200, 567, 576, 448.

7. Write all the common measures of: (*a*) 36, 24; (*b*) 18, 27; (*c*) 48, 72; (*d*) 21, 63; (*e*) 32, 96; (*f*) 18, 72.

210. When several numbers are to be taken as a whole and made the subject of an operation, they are inclosed in a sign or symbol known as a *parenthesis* ( ). Thus, $3 + (9 - 2)$ signifies that 3 is to be added to the difference of 9 and 2. A number written immediately to the left of a parenthesis denotes multiplication. Thus, $7 \times 4 + 3(8 + 5)$ means 7 times 4 is to be added to 3 times the sum of 8 and 5.

211. A composite number can be resolved into only one set of prime factors. Thus, the prime factors of 36 are 2, 2, 3, 3. $36 = 2^2 \times 3^2$. The product of no other prime numbers will give 36.

212. If a number is prime to each of two other numbers, it is prime to their product.

ILLUSTRATION. If 7 is prime to 207 and to 8, then 7 is prime to 8×207. Resolving 8 and 207 into their prime factors, we have $8 \times 207 = 2^3 \times (3^2 \times 23) = 2^3 \times 3^2 \times 23$

(Associative Law). Now, 7 does not appear among the prime factors of the product, and as there is only one set of factors of a number, hence 7 does not measure the product.

From the above statement it follows that if a number is prime to a second number, it is prime to any power of that second number.

GREATEST COMMON MEASURE

213. The **G.C.M. of two or more numbers can be found** by finding all the measures of each of the numbers and then selecting the greatest measure they have in common.

For example, take 24, 36, 60.

The measures of 24 are 2, 3, 4, 6, 8, 12.

The measures of 36 are 2, 3, 4, 6, 9, 12, 18.

The measures of 60 are 2, 3, 4, 5, 6, 12, 15, 20, 30.

Hence, the common measures are 2, 3, 4, 6, 12, and the G. C. M. is 12. This method is generally too long to be practical.

214. *Example* **1.** Find the G. C. M. of 48, 120, 168.

Expressing these numbers as products of their prime factors, we have

$$48 = 2^4 \times 3.$$

$$120 = 2^3 \times 3 \times 5.$$

$$168 = 2^3 \times 3 \times 7.$$

2 is contained 3 times as a factor in 168, 3 times as a factor in 120, and 4 times as a factor in 48; 3 is contained once as a factor in each of the numbers. Hence, the G. C. M. $= 2^3 \times 3 = 24$.

To find the G. C. M. of two or more numbers, express each of the numbers as the product of its prime factors;

then take the product of the prime factors common to all the numbers, each factor being taken the least number of times it occurs in any of the numbers.

EXERCISE 100 (Written)

Find the G. C. M. of:

1. 16, 24.
2. 24, 32.
3. 18, 27.
4. 24, 36.
5. 45, 60.
6. 75, 90.
7. 54, 72.
8. 108, 180.
9. 84, 96.
10. 120, 156.
11. 91, 105.
12. 26, 117.
13. 57, 76.
14. 115, 161.
15. 144, 264.
16. 140, 252.
17. 20, 30, 40.
18. 30, 75, 105.
19. 36, 60, 84.
20. 39, 65, 91.
21. 60, 84, 132.
22. 54, 90, 108.
23. 64, 80, 96.
24. 63, 84, 105.
25. 64, 96, 224.
26. 72, 108, 180.
27. 88, 132, 220.
28. 126, 189, 252.
29. 144, 240, 336.
30. 162, 270, 378.
31. 168, 224, 392.
32. 252, 420, 588.
33. 264, 360, 600.

EXERCISE 101 (Written)

Find the common measures of:

1. 24, 36.
2. 32, 56.
3. 36, 63.
4. 45, 54.
5. 60, 90.
6. 72, 132.
7. 36, 54.
8. 90, 126.
9. 54, 126.
10. 56, 140.
11. 112, 84.
12. 42, 105.
13. 44, 132.
14. 75, 125.
15. 48, 80.
16. 32, 128.
17. 108, 144.
18. 78, 117.
19. 90, 135.
20. 28, 70.
21. 60, 84.

What other common measures have two or more numbers whose G. C. M. is:

22. 27.
23. 36.
24. 21.
25. 24.
26. 18.
27. 15.
28. 48.
29. 56.
30. 65.
31. 72.
32. 84.
33. 96.

EXERCISE 102 (Written)

1. A boy has 21 marbles of one kind and 35 marbles of another kind. He puts the marbles into sacks, the same number of marbles being in each sack. What is the largest number of marbles he can put into each sack (without mixing the marbles), so as to have none left over?

2. A rectangular plot of ground, 220 ft. by 180 ft., has a board fence around it, each board being the same length. What is the greatest length a board can have so that an exact number of boards may go once around the plot?

3. A dealer has two barrels of sugar, 180 lb. being in one barrel, and 238 lb. in the other barrel. He puts each kind of sugar into packages of the same weight. What is the greatest number of pounds he can put into each package?

4. A farmer has 1560 lb. of oats and 1800 lb. of barley. He puts the oats and the barley each into sacks of the same weight. What is the greatest number of pounds he can put into each sack? How many sacks will he have in all?

5. What is the greatest number of pounds which measures exactly a short ton and a long ton?

6. A train runs at a uniform rate per hour. It makes two runs, one a distance of 495 mi., and the other a distance of 627 mi., and performs each journey in an exact number of hours. What is the greatest speed it can have?

7. A clerk saved the same amount for each of 20 consecutive weeks. In a number of these weeks he saved

$440, and in a number of weeks following he saved $280. What is the greatest amount he could have saved each week?

8. A carriage wheel makes an exact number of revolutions in going a distance of 91 ft., and also in going a distance of 156 ft. Find the circumference of the wheel if the number of revolutions is the minimum in each case.

215. Find the G. C. M. of 289, 391. In this instance the prime factors of the numbers cannot be discovered by inspection. To get the G. C. M. we proceed as follows:

```
289)391(1
    289
    ---
    102)289(2
        204
        ---
         85)102(1
             85
            ---
             17)85(5
                85
                --
```

Divide the greater number by the less. Divide the remainder into the less number. Divide the next remainder into the last divisor. Continue the process, always dividing the last remainder into the last divisor. The divisor which gives no remainder will be the G. C. M. of the numbers.

The above method is known as the **Euclidean Method of finding the G. C. M.** of two numbers. It is about 2200 years old. It never fails to produce the G. C. M. In our time we rarely have to find the G. C. M. of two large numbers.

The **reason for the above rule** for finding the G. C. M. of two numbers is: (1) Any measure of two numbers is a measure of the less of the two numbers and of the remainder obtained by dividing the greater number by the less. (2) If this holds for any measure, it holds for the G. C. M. Statement (1) follows at once from §§ 203 and 204. Let us see how our rule gives us the G. C. M.

Any measure of 289 and 391 is also a measure of 289 and 102.

Any measure of 289 and 102 is a measure of 102 and 85.

Any measure of 102 and 85 is also a measure of 85 and 17.

Hence, any measure of 289 and 391 is also a measure of 85 and 17; and therefore the G. C. M. of 289 and 391 is the G. C. M. of 85 and 17.

EXERCISE 103 (Written)

Find the G. C. M. of:

1. 777, 1887.
2. 429, 975.
3. 361, 760.
4. 529, 690.
5. 338, 403.
6. 1313, 2929.
7. 1441, 2227.
8. 1781, 2877.
9. 2567, 4681.
10. 6601, 8211.
11. 1369, 1739.
12. 3481, 5310.

13. Write the prime numbers less than 120.

14. Show that the G. C. M. of two or more numbers is measured by every common measure of those numbers.

15. By how many numbers can 43 be divided so as to give 1 for a remainder? Write those numbers.

16. By how many numbers can 59 be divided so as to give 4 as a remainder? Write the numbers.

17. How many numbers divided into 89 give 17 as a remainder? Write the numbers.

18. How many numbers, when divided into 100, give 16 for a remainder? Write those numbers.

19. Show that the difference of any two even numbers is divisible by 2.

LEAST COMMON MULTIPLE

First Method

216. *Example.* Find the L. C. M. of 12, 16, 24.

Writing a number of the multiples of these numbers, we have 12, 24, 36, **48**, 60, 72, 84, **96**, 108, 120, 132, **144**.

16, 32, **48**, 64, 80, **96**, 112, 128, **144**.

24, **48**, 72, **96**, 120, **144**.

The first three multiples common to 12, 16, 24, are 48, 96, 144. The least of those multiples is 48. Therefore the L. C. M. of 12, 16, 24, is 48.

This method of finding the L. C. M. is the simplest, but it is not the shortest.

217. Since the L. C. M. of two or more numbers is exactly divisible by each of the numbers, it follows that the **L. C. M. contains all the prime factors of each of the given numbers.**

This fact suggests another method of finding the L. C. M. of two or more numbers.

Second Method

218. *Example.* Find the L. C. M. of 48, 60, 72.

$$48 = 2^4 \times 3.$$
$$60 = 2^2 \times 3 \times 5.$$
$$72 = 2^3 \times 3^2.$$

Any multiple of 48 must contain 2 four times as a factor. Any multiple of 72 must contain 3 twice as a factor. Hence, the number $2^4 \times 3^2 \times 5 = 720$, contains all the factors of the three numbers, 48, 60, 72. Therefore the L. C. M. of 48, 60, 72, is 720.

To find the L. C. M. of two or more numbers, resolve each of the numbers into its prime factors, then find the product of all the prime factors of the given numbers, taking each factor the greatest number of times it occurs in any of the numbers.

Third Method

219. *Example* 1. Find the L. C. M. of 48, 60, 72.

$$\begin{array}{r|rrr}
2 & 48 & 60 & 72 \\ \cline{2-4}
2 & 24 & 30 & 36 \\ \cline{2-4}
2 & 12 & 15 & 18 \\ \cline{2-4}
3 & 6 & 15 & 9 \\ \cline{2-4}
\multicolumn{1}{r}{} & 2 & 5 & 3
\end{array}$$

L. C. M. = 3 × 5 × 2 × 3 × 2 × 2 × 2 = 720.

Example 2. Find the L. C. M. of 18, 27, 30, 45.

$$\begin{array}{r|rrrr}
2 & 18 & 27 & 30 & 45 \\ \cline{2-5}
3 & 9 & 27 & 15 & 45 \\ \cline{2-5}
3 & & 9 & & 15 \\ \cline{2-5}
\multicolumn{1}{r}{} & & 3 & & 5
\end{array}$$

L. C. M. = 5 × 3 × 3 × 3 × 2 = 270.

Step 1. Arrange the numbers in a horizontal row.

Step 2. Divide by a prime factor common to two or more of the numbers. Set down the quotients and the undivided numbers.

Step 3. Treat the second horizontal row in the same manner, and so on until a horizontal row is obtained which contains numbers prime to one another. If, at any stage of the process, a horizontal row contains a number which is a factor of some other number in that row, then we may strike out such factor.

The continued product of the numbers in the last row and of the divisors will be the L. C. M.

220. Why should the Third Method give the L. C. M.?

The L. C. M. of 18, 27, 30, 45, is 2 times the L. C. M. of 9, 27, 15, 45. Why?

2 times the L. C. M. of 9, 27, 15, 45, is 2 × 3 times the L. C. M. of 9, 15. Why?

2 × 3 times the L. C. M. of 9, 15, is 2 × 3 × 3 times the L. C. M. of 3, 5. Why?

EXERCISE 104 (Oral)

Find the L. C. M. of:

1. 4, 5.
2. 3, 7.
3. 4, 6.
4. 6, 8.
5. 5, 6.
6. 7, 14.
7. 7, 8.
8. 8, 6.
9. 8, 9.
10. 4, 10.
11. 4, 14.
12. 3, 5, 10.
13. 4, 5, 10.
14. 6, 8, 12.
15. 5, 6, 12.
16. 7, 14, 21.
17. 8, 16, 24.
18. 12, 18, 24.
19. 3, 4, 5.
20. 6, 7, 21.

EXERCISE 105 (Written)

Find the L. C. M. of:

1. 16, 20.
2. 21, 14.
3. 18, 60.
4. 18, 45.
5. 21, 49.
6. 28, 70.
7. 42, 56.
8. 36, 54.
9. 34, 51.
10. 48, 72.
11. 96, 120.
12. 28, 30.
13. 32, 80.
14. 26, 39.
15. 48, 84.
16. 60, 96.
17. 84, 108.
18. 55, 77.
19. 54, 90.
20. 72, 108.
21. 75, 125.
22. 36, 54, 72.
23. 36, 90, 60.
24. 48, 64, 36.
25. 12, 15, 18.
26. 14, 21, 35.
27. 30, 35, 21.
28. 28, 42, 70.
29. 32, 35, 150.
30. 30, 45, 48.
31. 15, 20, 25.
32. 12, 18, 20.
33. 45, 63, 70.
34. 14, 35, 40.
35. 12, 16, 18.
36. 14, 24, 40.
37. 15, 24, 25.
38. 77, 143, 22.
39. 18, 20, 45.
40. 30, 70, 105.
41. 30, 40, 48.
42. 21, 28, 35.
43. 12, 18, 27.
44. 12, 15, 16, 18.
45. 36, 40, 45.

221. Find the L. C. M. of 291 and 1261.

$$\begin{array}{r}
291)1261(4 \\
\underline{1164} \\
97)291(3 \\
\underline{291}
\end{array}$$

$291 = 3 \times 97.$

$1261 = 97 \times 13.$

Therefore, L. C. M. of 291 and $1261 = 3 \times 97 \times 13 = 3 \times 1261 = 13 \times 291$.

To find the L. C. M. of two numbers, divide either of the numbers by their G. C. M. and multiply the quotient by the other number.

This rule is to be recommended only when the numbers cannot be readily resolved into their prime factors by inspection.

222. The **L. C. M. of three or more numbers** can always be obtained by finding first the L. C. M. of two of the numbers, then by finding the L. C. M. of this result and the third number, then of the latter result and the fourth number, and so on.

EXERCISE 106 (Written)

1. Write the first ten multiples of 6, of 9, of 12, of 15, of 16, of 24.

2. Write the first five common multiples of: (*a*) 8 and 12; (*b*) 10 and 15; (*c*) 12 and 18; (*d*) 16 and 24; (*e*) 18 and 27; (*f*) 20 and 30.

3. Show that the L. C. M. of two numbers equals the product of the two numbers divided by their G. C. M.

4. Find the least number which, divided by 6, 8, 10, and 12, gives an integral quotient and 4 for a remainder.

5. Find the L. C. M. of two numbers whose product is 960 and G. C. M. 4.

6. Find the G. C. M. of two numbers whose product is 864 and L. C. M. 72.

7. Find the L. C. M. of three numbers, one of which is 18, and the L. C. M. of the other two is 45.

8. A man earns $15 a week. Find the smallest number of weeks he must work to earn an exact number of twenty-dollar gold pieces.

9. What is the greatest number to divide 128 and 217, leaving remainders of 8 and 1 respectively?

10. Three men walk round a circular path 1200 yd. in circumference at the rate of 80 yd., 100 yd., 120 yd. per minute respectively. If they start together, when will they be together again?

11. The L. C. M. of two numbers is 120, and one of them is 15. Write all possible values of the other number.

12. The L. C. M. of two numbers is 84, and one of the numbers is 28. Write all possible values of the other number.

13. Find the prime factors of 1980, 5250, 7350, and 16,170. Find their G. C. M. and L. C. M.

14. What numbers of two figures each have 17 for their G. C. M. and 204 for their L. C. M.?

15. Two trains run at the rate of 22 mi. and 33 mi. an hour respectively. Find the least distance that can be run in an exact number of hours by both trains.

16. The L. C. M. of 319 and of another number is 957 and their G. C. M. is 11. Find the other number.

17. The numbers 961 and 1271 have a factor in common. Resolve these numbers into their prime factors.

18. Find the number of prime numbers greater than 100 and less than 150.

19. Two surveyors use two different chains, one chain being 100 ft. long and the other 66 ft. long. Find the shortest distance that is exactly measured by both chains.

REVIEW QUESTIONS

1. Will 3 divide each of the following numbers exactly: 7281, 31,926,358, 910,638, 74,823? Give reason for your answer.

2. Will 4 exactly measure each of the following numbers: 886,674, 23,456, 9,887,746? Give reason for your answer.

3. Show that 8 is a measure of any number if it measures the number formed by the last three digits.

4. What is the test of divisibility by 11? by 9?

5. What is a parenthesis?

6. In how many ways can a composite number be resolved into its prime factors?

7. How is the G.C.M. of two or more numbers obtained? Mention three ways.

8. Give the Euclidean Method of finding the G.C.M. of two or more numbers.

9. Upon what principles is the Euclidean Method based?

10. Show that the difference between any two odd numbers is divisible by 2.

11. Show that the product of three consecutive numbers, the first of which is even, is divisible by 24.

12. Give three ways of finding the L.C.M. of two or more numbers.

13. Show that the product of the L.C.M. and the G.C.M. of two numbers equals the product of the two numbers.

CHAPTER XII

FRACTIONS AND DECIMALS

223. Reduce to lowest terms, $\frac{216}{324}$.

$$\frac{216}{324} = \frac{108}{162} = \frac{54}{81} = \frac{18}{27} = \frac{2}{3}.$$

224. A number is exactly divisible by 4, or by 25, if the number expressed by its two right-hand figures is exactly divisible by 4 or by 25. For example:

764 contains 4 as a factor, since 64 contains 4 as a factor.
775 contains 25 as a factor, since 75 contains 25 as a factor.

A number is exactly divisible by 8 or by 125 if the number expressed by its three right-hand figures is exactly divisible by 8 or by 125, *e.g.* 11,128, 3125; the first number is divisible by 8 because 8 is a factor of 128. The second number is divisible by 125 because 125 is a factor of 125.

Both statements follow from §§ 203, 204.

EXERCISE 107 (Written)

Reduce to lowest terms:

1. $\frac{48}{72}$.
2. $\frac{120}{168}$.
3. $\frac{90}{126}$.
4. $\frac{198}{324}$.
5. $\frac{144}{192}$.
6. $\frac{54}{135}$.
7. $\frac{66}{154}$.
8. $\frac{84}{231}$.
9. $\frac{100}{325}$.
10. $\frac{180}{288}$.
11. $\frac{891}{990}$.
12. $\frac{198}{330}$.
13. $\frac{144}{216}$.
14. $\frac{343}{392}$.
15. $\frac{392}{504}$.
16. $\frac{405}{729}$.
17. $\frac{65}{156}$.
18. $\frac{216}{468}$.
19. $\frac{132}{168}$.
20. $\frac{648}{864}$.
21. $\frac{176}{220}$.
22. $\frac{91}{169}$.
23. $\frac{111}{148}$.
24. $\frac{146}{219}$.
25. $\frac{320}{512}$.
26. $\frac{495}{630}$.
27. $\frac{126}{567}$.
28. $\frac{182}{495}$.
29. $\frac{294}{686}$.
30. $\frac{1024}{5632}$.
31. $\frac{405}{972}$.
32. $\frac{875}{1625}$.

EXERCISE 108 (Written)

Reduce to common fractions in their lowest terms:

1. .25.
2. .75.
3. .68.
4. .375.
5. .625.
6. .35.
7. .65.
8. .85.
9. .3125.
10. .1875.
11. .4375.
12. .5625.
13. .6875.
14. .8125.
15. .9375.

16. Reduce 1′ 30″ to the fraction of 1°.

17. Reduce 5 da. 6 hr. to the fraction of 1 wk.

18. Reduce 2 yd. 2 ft. 3 in. to the fraction of 1 rd.

19. Reduce 7 sq. yd. 5 sq. ft. 9 sq. in. to the fraction of 1 sq. rd.

20. Reduce 480 ft. to the fraction of 1 mi.

21. What fraction of 1 A. is a rectangular plot 120 ft. by 33 ft.?

22. What fraction of 1 section is S. $\frac{1}{2}$ of N.W. $\frac{1}{4}$ of the section?

23. What fraction of a cubic yard is the volume of a rectangular box whose dimensions are 1 ft. 6 in. by 9 in. and 6 in. deep?

24. What fraction of a common year is 18 da. 6 hr.?

25. What part of a circumference is an arc of 45°?

26. What fraction of 1 sq. mi. is 360 A.?

27. What fraction of 1 bu. is 1 pk. 2 qt.?

EXERCISE 109 (Oral)

Reduce to mixed numbers:

1. $\frac{15}{4}$.
2. $\frac{37}{4}$.
3. $\frac{25}{8}$.
4. $\frac{39}{8}$.
5. $\frac{94}{8}$.
6. $\frac{85}{12}$.
7. $\frac{72}{7}$.
8. $\frac{65}{7}$.
9. $\frac{87}{10}$.
10. $\frac{91}{9}$.
11. $\frac{93}{11}$.
12. $\frac{97}{12}$.

13. $\frac{85}{13}$.
14. $\frac{78}{11}$.
15. $\frac{99}{14}$.
16. $\frac{88}{15}$.
17. $\frac{29}{18}$.
18. $\frac{27}{17}$.
19. $\frac{35}{18}$.
20. $\frac{45}{14}$.
21. $\frac{54}{13}$.
22. $\frac{64}{15}$.
23. $\frac{67}{16}$.
24. $\frac{69}{17}$.
25. $\frac{58}{19}$.
26. $\frac{71}{11}$.
27. $\frac{73}{14}$.
28. $\frac{92}{16}$.
29. $\frac{101}{11}$.
30. $\frac{102}{13}$.
31. $\frac{110}{13}$.
32. $\frac{107}{15}$.
33. $\frac{145}{8}$.
34. $\frac{127}{9}$.
35. $\frac{119}{18}$.
36. $\frac{188}{15}$.
37. $\frac{120}{23}$.
38. $\frac{120}{29}$.
39. $\frac{150}{29}$.
40. $\frac{190}{37}$.
41. $\frac{184}{45}$.
42. $\frac{199}{33}$.
43. $\frac{203}{21}$.
44. $\frac{147}{19}$.

EXERCISE 110 (Oral)

Reduce to improper fractions :

1. $4\frac{2}{5}$.
2. $3\frac{3}{7}$.
3. $5\frac{5}{6}$.
4. $4\frac{2}{9}$.
5. $8\frac{3}{7}$.
6. $5\frac{3}{8}$.
7. $6\frac{7}{8}$.
8. $9\frac{7}{9}$.
9. $8\frac{5}{9}$.
10. $7\frac{3}{11}$.
11. $11\frac{2}{11}$.
12. $3\frac{3}{13}$.
13. $4\frac{5}{13}$.
14. $9\frac{4}{13}$.
15. $8\frac{7}{15}$.
16. $7\frac{11}{15}$.
17. $19\frac{2}{3}$.
18. $23\frac{1}{4}$.
19. $24\frac{2}{5}$.
20. $18\frac{5}{6}$.
21. $17\frac{3}{7}$.
22. $24\frac{7}{8}$.
23. $17\frac{5}{9}$.
24. $11\frac{3}{13}$.
25. $14\frac{1}{12}$.
26. $12\frac{5}{12}$.
27. $11\frac{5}{18}$.
28. $27\frac{3}{5}$.
29. $24\frac{1}{6}$.
30. $16\frac{7}{9}$.
31. $17\frac{1}{7}$.
32. $19\frac{2}{9}$.
33. $14\frac{5}{11}$.
34. $9\frac{7}{15}$.
35. $12\frac{11}{12}$.
36. $23\frac{2}{5}$.
37. $27\frac{1}{4}$.
38. $34\frac{2}{5}$.
39. $31\frac{1}{6}$.
40. $22\frac{3}{7}$.
41. $23\frac{4}{7}$.
42. $31\frac{1}{10}$.
43. $36\frac{2}{3}$.
44. $19\frac{4}{5}$.
45. $16\frac{5}{8}$.
46. $15\frac{5}{9}$.
47. $13\frac{2}{11}$.
48. $10\frac{5}{11}$.

EXERCISE 111 (Oral)

1. Reduce 5 to a fraction having 8 for its denominator.
2. Reduce 4 to a fraction having 9 for its denominator.
3. Reduce 6 to a fraction having 7 for its denominator.

4. Reduce 11 to a fraction having 10 for its denominator.
5. Reduce 13 to a fraction having 4 for its denominator.
6. Reduce 15 to a fraction having 3 for its denominator.
7. Reduce 14 to a fraction having 6 for its denominator.
8. Reduce $\frac{3}{4}$ of a foot to the fraction of 1 yd.
9. Reduce $2\frac{1}{2}$ pk. to the fraction of 1 bu.
10. Reduce $5\frac{2}{5}$ cu. ft. to the fraction of 1 cu. yd.
11. Reduce $1\frac{1}{2}$ hr. to the fraction of 1 da.
12. Reduce $6\frac{3}{4}$ sq. ft. to the fraction of 1 sq. yd.
13. Express 80 rd. in miles.
14. Express $\frac{3}{4}$ of an inch in feet.
15. Express $\frac{2}{3}$ bu. in quarts.
16. Express $\frac{2}{3}$ of a dozen as the fraction of a score.

225. Reduce to equivalent fractions of the same denomination, $\frac{3}{10}$, $\frac{5}{12}$, $\frac{11}{14}$, $\frac{9}{16}$.

Step 1. Find the L. C. M. of the denominators, 10, 12, 14, 16. The L. C. M. of 10, 12, 14, 16 = 1680.

Step 2. Reduce each fraction to 1680ths.

$$\frac{3}{10} = \frac{504}{1680},$$
$$\frac{5}{12} = \frac{700}{1680},$$
$$\frac{11}{14} = \frac{1320}{1680},$$
$$\frac{9}{16} = \frac{945}{1680}.$$

Ans. $\frac{504}{1680}$, $\frac{700}{1680}$, $\frac{1320}{1680}$, $\frac{945}{1680}$.

Upon what property of fractions does Step 2 depend?

How would you reduce $\frac{3}{4}$ to 24ths?

How would you reduce $\frac{5}{6}$ to an equivalent fraction having 18 for a denominator?

EXERCISE 112 (Oral)

1. Reduce $\frac{2}{3}$ to 12ths.
2. Reduce $\frac{2}{5}$ to 25ths.
3. Reduce $\frac{3}{8}$ to 24ths.
4. Reduce $\frac{5}{9}$ to 27ths.
5. Reduce $\frac{3}{11}$ to 55ths.
6. Reduce $\frac{7}{12}$ to 84ths.
7. Reduce $\frac{11}{12}$ to 84ths.
8. Reduce $\frac{13}{15}$ to 75ths.
9. Reduce $\frac{2}{3}$ to 48ths.
10. Reduce $\frac{4}{5}$ to 70ths.
11. Reduce $\frac{7}{8}$ to 96ths.
12. Reduce $\frac{5}{6}$ to 90ths.
13. Reduce $\frac{4}{9}$ to 63ds.
14. Reduce $\frac{3}{4}$ to 92ds.
15. Reduce $\frac{2}{7}$ to 119ths.
16. Reduce $\frac{11}{13}$ to 104ths.

EXERCISE 113 (Written)

Reduce to fractions of the same denomination :

1. $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}$.
2. $\frac{1}{2}, \frac{1}{3}, \frac{1}{7}$.
3. $\frac{5}{6}, \frac{3}{4}, \frac{7}{8}$.
4. $\frac{2}{5}, \frac{1}{3}, \frac{4}{25}$.
5. $\frac{2}{3}, \frac{1}{2}, \frac{8}{9}$.
6. $\frac{1}{8}, \frac{1}{12}, \frac{3}{30}$.
7. $\frac{5}{9}, \frac{3}{4}, \frac{1}{5}$.
8. $\frac{1}{4}, \frac{4}{7}, \frac{8}{9}$.
9. $\frac{2}{3}, \frac{1}{2}, \frac{8}{9}$.
10. $\frac{7}{11}, \frac{2}{3}, \frac{8}{13}$.
11. $\frac{3}{4}, \frac{1}{12}, \frac{3}{8}$.
12. $\frac{2}{3}, \frac{3}{4}, \frac{8}{9}$.
13. $\frac{5}{8}, \frac{5}{9}, \frac{7}{10}$.
14. $\frac{1}{12}, \frac{1}{18}, \frac{5}{18}$.
15. $\frac{1}{4}, \frac{3}{5}, \frac{5}{6}, \frac{7}{8}$.

Arrange in order of magnitude:

16. $\frac{13}{24}, \frac{1}{2}, \frac{7}{12}$.
17. $\frac{3}{4}, \frac{2}{3}, \frac{5}{6}$.
18. $\frac{3}{4}, \frac{5}{6}, \frac{4}{5}$.
19. $\frac{5}{6}, \frac{11}{12}, \frac{8}{9}$.
20. $\frac{7}{12}, \frac{6}{11}, \frac{9}{13}$.
21. $\frac{2}{5}, \frac{4}{7}, \frac{1}{2}$.
22. $\frac{7}{10}, \frac{29}{40}, \frac{6}{9}$.
23. $\frac{1}{12}, \frac{2}{25}, \frac{33}{400}$.
24. $\frac{3}{40}$, .07, $\frac{9}{125}$.
25. $\frac{1}{8}$, .124, $\frac{3}{25}$.
26. $\frac{2}{9}$, .21, $\frac{7}{34}$.
27. $\frac{4}{17}$, .253, $\frac{47}{129}$.

ADDITION

EXERCISE 114 (Oral)

Add :

1. $\frac{1}{2}, \frac{1}{4}$.
2. $\frac{1}{2}, \frac{3}{4}$.
3. $\frac{1}{2}, \frac{1}{3}$.
4. $\frac{1}{2}, \frac{2}{3}$.
5. $\frac{1}{3}, \frac{1}{4}$.
6. $\frac{1}{3}, \frac{3}{4}$.
7. $\frac{2}{3}, \frac{1}{4}$.
8. $\frac{1}{3}, \frac{1}{6}$.
9. $\frac{2}{3}, \frac{1}{6}$.
10. $\frac{2}{3}, \frac{5}{6}$.
11. $\frac{1}{3}, \frac{5}{6}$.
12. $\frac{1}{2}, \frac{5}{6}$.
13. $\frac{1}{4}, \frac{1}{6}$.
14. $\frac{1}{4}, \frac{5}{6}$.
15. $\frac{3}{4}, \frac{1}{6}$.

16. $\frac{3}{4}$, $\frac{5}{6}$.
17. $\frac{3}{4}$, $\frac{2}{3}$.
18. $\frac{3}{4}$, $\frac{1}{8}$.
19. $\frac{3}{4}$, $\frac{5}{8}$.
20. $\frac{3}{4}$, $\frac{7}{8}$.
21. $\frac{1}{4}$, $\frac{5}{8}$.
22. $\frac{1}{2}$, $\frac{7}{8}$.
23. $\frac{1}{5}$, $\frac{1}{10}$.
24. $\frac{3}{5}$, $\frac{1}{10}$.
25. $\frac{4}{5}$, $\frac{7}{10}$.
26. $\frac{3}{5}$, $\frac{1}{2}$.
27. $\frac{4}{5}$, $\frac{1}{2}$.
28. $\frac{2}{5}$, $\frac{9}{10}$.
29. $\frac{2}{5}$, $\frac{1}{4}$.
30. $\frac{3}{5}$, $\frac{3}{4}$.
31. $\frac{4}{5}$, $\frac{1}{4}$.
32. $\frac{3}{4}$, $\frac{5}{12}$.
33. $\frac{1}{4}$, $\frac{11}{12}$.
34. $\frac{1}{3}$, $\frac{5}{12}$.
35. $\frac{2}{3}$, $\frac{11}{12}$.
36. $\frac{3}{4}$, $\frac{11}{12}$.
37. $\frac{2}{3}$, $\frac{11}{12}$.
38. $\frac{1}{6}$, $\frac{11}{12}$.
39. $\frac{5}{6}$, $\frac{5}{12}$.
40. $\frac{5}{6}$, $\frac{7}{12}$.
41. $\frac{5}{6}$, $\frac{11}{12}$.
42. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.
43. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{1}{4}$.
44. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$.
45. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$.
46. $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$.
47. $\frac{1}{3}$, $\frac{3}{4}$, $\frac{1}{6}$.
48. $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$.

EXERCISE 115 (Written)

Add:

1. $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{50}$.
2. $\frac{1}{6}$, $\frac{1}{54}$.
3. $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{42}$, $\frac{1}{126}$.
4. $\frac{1}{30}$, $\frac{1}{330}$.
5. $\frac{1}{6}$, $\frac{1}{11}$, $\frac{1}{22}$, $\frac{1}{66}$.
6. $\frac{1}{3}$, $\frac{3}{10}$, $\frac{7}{18}$, $\frac{2}{15}$.
7. $\frac{1}{9}$, $\frac{1}{12}$, $\frac{5}{24}$.
8. $\frac{1}{8}$, $\frac{2}{15}$, $\frac{7}{30}$.
9. $\frac{5}{9}$, $\frac{13}{15}$, $\frac{2}{45}$.
10. $\frac{1}{6}$, $\frac{9}{60}$, $\frac{9}{40}$, $\frac{5}{16}$.
11. $\frac{1}{9}$, $\frac{1}{14}$, $\frac{3}{20}$, $\frac{7}{12}$.
12. $\frac{1}{4}$, $\frac{7}{18}$, $\frac{13}{24}$, $\frac{7}{10}$.
13. $\frac{4}{15}$, $\frac{5}{16}$, $\frac{17}{120}$.
14. $\frac{3}{8}$, $\frac{3}{14}$, $\frac{1}{10}$.
15. $1\frac{3}{4}$, $1\frac{5}{6}$, $1\frac{8}{9}$, $1\frac{11}{12}$, $1\frac{13}{27}$.
16. $1\frac{11}{16}$, $1\frac{5}{12}$, $\frac{7}{24}$, $1\frac{13}{18}$.
17. $5\frac{3}{4}$, $12\frac{5}{7}$, $1\frac{7}{12}$.
18. $1\frac{5}{12}$, $3\frac{9}{10}$, $2\frac{3}{5}$.
19. $1\frac{2}{3}$, $3\frac{1}{4}$, $1\frac{5}{16}$.
20. $\frac{3}{8}$, $\frac{7}{24}$, $\frac{5}{16}$.
21. $1\frac{3}{5}$, $1\frac{5}{6}$, $1\frac{6}{7}$, $\frac{7}{8}$.
22. $\frac{3}{10}$, $\frac{7}{100}$, $\frac{5}{1000}$.
23. $\frac{9}{10}$, $\frac{6}{100}$, $\frac{4}{1000}$.
24. $\frac{3}{20}$, $\frac{3}{10}$, $\frac{8}{15}$.
25. $2\frac{8}{9}$, $3\frac{19}{24}$, $4\frac{11}{15}$.
26. $1\frac{7}{9}$, $\frac{11}{27}$, $\frac{7}{24}$, $1\frac{1}{12}$.
27. $2\frac{13}{24}$, $2\frac{29}{36}$, $4\frac{5}{27}$.
28. $\frac{7}{8}$, $3\frac{11}{12}$, $1\frac{11}{16}$, $6\frac{8}{9}$.
29. $\frac{7}{10}$, $3\frac{2}{15}$, $\frac{17}{20}$, $5\frac{23}{45}$.
30. $2\frac{1}{12}$, $3\frac{13}{14}$, $2\frac{1}{3}$, $\frac{15}{16}$.

SUBTRACTION

Example 1. From $7\frac{3}{4}$ take $2\frac{9}{10}$.

SOLUTION. $7\frac{3}{4} - 2\frac{9}{10} = (7 - 2\frac{9}{10}) + \frac{3}{4} = 4\frac{1}{10} + \frac{3}{4} = 4\frac{17}{20}$.

EXERCISE 116 (Oral)

Find the difference between :

1. $\frac{3}{4}$ and $\frac{1}{2}$.
2. $\frac{7}{8}$ and $\frac{1}{2}$.
3. $\frac{7}{8}$ and $\frac{1}{4}$.
4. $\frac{7}{8}$ and $\frac{3}{4}$.
5. $\frac{5}{8}$ and $\frac{1}{4}$.
6. $\frac{5}{8}$ and $\frac{1}{2}$.
7. $\frac{1}{2}$ and $\frac{1}{3}$.
8. $\frac{2}{3}$ and $\frac{1}{2}$.
9. $\frac{3}{4}$ and $\frac{2}{3}$.
10. $\frac{3}{4}$ and $\frac{1}{3}$.
11. $\frac{9}{10}$ and $\frac{1}{5}$.
12. $\frac{9}{10}$ and $\frac{4}{5}$.
13. $\frac{7}{10}$ and $\frac{2}{5}$.
14. $\frac{3}{10}$ and $\frac{1}{5}$.
15. $\frac{3}{5}$ and $\frac{1}{2}$.
16. $\frac{3}{5}$ and $\frac{1}{3}$.
17. $\frac{4}{5}$ and $\frac{2}{3}$.
18. $\frac{4}{5}$ and $\frac{3}{4}$.
19. $\frac{4}{5}$ and $\frac{1}{4}$.
20. $\frac{11}{15}$ and $\frac{2}{5}$.
21. $\frac{13}{15}$ and $\frac{4}{5}$.
22. $\frac{14}{15}$ and $\frac{3}{5}$.
23. $\frac{14}{15}$ and $\frac{9}{10}$.
24. $\frac{5}{6}$ and $\frac{1}{2}$.
25. $\frac{5}{6}$ and $\frac{2}{3}$.
26. $\frac{5}{6}$ and $\frac{3}{4}$.
27. $\frac{5}{6}$ and $\frac{1}{4}$.
28. $\frac{7}{8}$ and $\frac{5}{6}$.
29. $\frac{7}{8}$ and $\frac{2}{3}$.
30. $\frac{5}{8}$ and $\frac{3}{4}$.
31. $1\frac{1}{2}$ and $\frac{3}{4}$.
32. $1\frac{2}{3}$ and $\frac{3}{4}$.
33. $2\frac{1}{6}$ and $\frac{1}{2}$.
34. $2\frac{1}{3}$ and $\frac{5}{6}$.
35. $3\frac{3}{4}$ and $\frac{7}{8}$.
36. $4\frac{1}{2}$ and $1\frac{7}{8}$.
37. $5\frac{1}{2}$ and $3\frac{2}{3}$.
38. $2\frac{1}{2}$ and $\frac{7}{12}$.
39. $1\frac{3}{4}$ and $\frac{11}{12}$.
40. $5\frac{1}{3}$ and $\frac{1}{6}$.
41. $7\frac{3}{7}$ and $2\frac{9}{14}$.
42. $9\frac{1}{5}$ and $6\frac{1}{3}$.

EXERCISE 117 (Written)

Find the difference between :

1. $7\frac{2}{3}$ and $4\frac{11}{12}$.
2. $9\frac{5}{6}$ and $4\frac{3}{4}$.
3. $7\frac{3}{11}$ and $2\frac{1}{2}$.
4. $9\frac{5}{12}$ and $3\frac{7}{8}$.
5. $10\frac{3}{11}$ and $7\frac{1}{4}$.
6. $12\frac{7}{15}$ and $9\frac{9}{10}$.
7. $18\frac{5}{9}$ and $7\frac{5}{6}$.
8. $21\frac{2}{3}$ and $11\frac{13}{18}$.
9. $8\frac{3}{16}$ and $5\frac{9}{10}$.
10. $6\frac{9}{16}$ and $3\frac{2}{3}$.
11. $19\frac{2}{3}$ and $8\frac{5}{8}$.
12. $23\frac{11}{14}$ and $12\frac{16}{21}$.

13. $4\frac{5}{14}$ and $1\frac{3}{4}$.
14. $1\frac{38}{100}$ and $\frac{78}{100}$.
15. $1\frac{27}{100}$ and $\frac{7}{10}$.
16. $\frac{1}{11}$ and $\frac{1}{12}$.
17. $\frac{1}{9}$ and $\frac{1}{12}$.
18. $\frac{1}{8}$ and $\frac{1}{18}$.
19. $\frac{9}{10}$ and $\frac{8}{9}$.
20. $\frac{5}{6}$ and $\frac{4}{5}$.
21. $\frac{11}{12}$ and $\frac{10}{11}$.
22. $\frac{17}{24}$ and $\frac{13}{20}$.
23. $\frac{17}{18}$ and $\frac{14}{15}$.
24. $7\frac{5}{7}$ and $3\frac{11}{14}$.
25. $8\frac{5}{9}$ and $7\frac{11}{12}$.
26. $9\frac{5}{12}$ and $6\frac{13}{18}$.
27. $5\frac{7}{16}$ and $2\frac{17}{24}$.
28. $6\frac{7}{18}$ and $5\frac{11}{24}$.
29. $8\frac{17}{20}$ and $6\frac{29}{30}$.
30. $12\frac{5}{22}$ and $8\frac{13}{33}$.

31. What number must be added to $3\frac{3}{4}$ to give $5\frac{2}{3}$?

32. A man's capital amounts to \$$1727\frac{1}{2}$. By how much must he increase his capital so that it may amount to \$3000?

33. What number must be taken from $30\frac{7}{8}$ to leave $14\frac{2}{3}$?

34. By how much does $84\frac{1}{6}$ exceed $17\frac{3}{4}$?

35. What number must be added to $19\frac{4}{5}$ to give $29\frac{1}{2}$ for the result?

36. Of the weight of the earth's atmosphere $\frac{231}{1000}$ is oxygen. What fraction of the weight of the earth's atmosphere do its other constituents aggregate?

MULTIPLICATION

Example 1. Simplify $\frac{2}{3}$ of $\frac{3}{4}$ of $1\frac{7}{9}$.

SOLUTION. $\frac{2}{3}$ of $\frac{3}{4}$ of $1\frac{7}{9} = \frac{2}{3} \times \frac{3}{4} \times \frac{16}{9} = \frac{1 \times 1 \times 8}{1 \times 1 \times 9} = \frac{8}{9}$.

Example 2. Multiply $84\frac{2}{7}$ by 9.

SOLUTION. $84\frac{2}{7} \times 9 = (84 \times 9) + (\frac{2}{7} \times 9) = 756 + 2\frac{4}{7} = 758\frac{4}{7}$.

EXERCISE 118 (Written)

Simplify:

1. $\frac{2}{3}$ of $\frac{5}{6}$ of $2\frac{4}{7}$.
2. $\frac{3}{4}$ of $\frac{9}{10}$ of $4\frac{4}{9}$.
3. $\frac{7}{8}$ of $1\frac{2}{5}$ of $\frac{20}{49}$.
4. $\frac{4}{11}$ of $\frac{2}{3}$ of $4\frac{1}{8}$.
5. $\frac{3}{8} \times \frac{3}{11} \times 4\frac{8}{9}$.
6. $\frac{9}{10} \times 1\frac{1}{2} \times 1\frac{1}{18}$.
7. $\frac{3}{13}$ of $4\frac{7}{8}$ of $1\frac{1}{7}$.
8. $\frac{11}{14} \times 5\frac{1}{3} \times 1\frac{10}{11}$.
9. $\frac{7}{16}$ of $1\frac{7}{8}$ of $1\frac{1}{15}$.
10. $2\frac{1}{7} \times 3\frac{1}{7} \times 2\frac{32}{33}$.
11. $5\frac{1}{2} \times 5\frac{1}{2} \times \frac{4}{121}$.
12. $2\frac{3}{4} \times 2\frac{3}{4} \times 1\frac{1}{15}$.
13. $7\frac{1}{8} \times \frac{16}{19} \times \frac{3}{8}$.
14. $9\frac{2}{7} \times 1\frac{8}{13} \times 1\frac{1}{6}$.
15. $1\frac{2}{3} \times 1\frac{2}{3} \times 1\frac{2}{3} \times 1\frac{2}{25}$.
16. $(\frac{3}{4} + \frac{1}{2}) \times (\frac{3}{4} - \frac{1}{2})$.
17. $(\frac{5}{6} + \frac{1}{3}) \times (\frac{5}{6} - \frac{1}{3})$.
18. $(\frac{9}{10} + \frac{1}{4}) \times (\frac{9}{10} - \frac{1}{4})$.
19. $\frac{2}{5}(3\frac{1}{3} + 1\frac{1}{4}) \times \frac{3}{8}$.
20. $\frac{2}{3}$ of $4\frac{1}{2} - \frac{1}{8}$ of $3\frac{1}{4} \times 3\frac{1}{5}$.
21. $3\frac{9}{11} \times (3\frac{1}{3} + \frac{5}{6} - \frac{20}{21})$.
22. $2\frac{1}{2} \times (1\frac{3}{4} + 2\frac{1}{2} + 1\frac{7}{8})$.
23. $7\frac{1}{2} \times 1\frac{7}{8} \times \frac{16}{225} \times (\frac{1}{2} + \frac{1}{3} + \frac{1}{4})$.
24. $1\frac{11}{12} \times 1\frac{12}{17} \times \frac{101}{58} \times (\frac{3}{4} + \frac{2}{3} - 1\frac{5}{12})$.
25. $2\frac{2}{3} \times 5\frac{1}{4} \times 1\frac{1}{2} \times (\frac{9}{10} + \frac{1}{4} - 1\frac{3}{20})$.
26. $3\frac{1}{2} \times 1\frac{1}{14} + 7\frac{1}{2} - (1\frac{1}{4} \div \frac{2}{3} \times \frac{5}{21})$.

ANALYTICAL METHOD OF SOLUTION

226. *Example* 1. A man invested a sum of money in business. He gained $\frac{1}{12}$ of his investment and then had \$15,600. How much did he invest?

SOLUTION. The investment + $\frac{1}{12}$ of the investment = \$15,600. But the investment + $\frac{1}{12}$ of the investment = $\frac{12}{12}$ of the investment + $\frac{1}{12}$ of the investment = $\frac{13}{12}$ of the investment. Therefore, $\frac{13}{12}$ of the investment = \$15,600. Therefore, $\frac{1}{12}$ of the investment $= \frac{\$15,600}{13}$. Therefore, $\frac{12}{12}$ of the investment $= \frac{\$15,600}{13} \times 12 = \$14,400$. Therefore, the investment is \$14,400.

Instead of the word *therefore*, we shall in future use the symbol $\therefore$.

Example 2. A farmer sells $\frac{1}{4}$ of his wheat, and then has 261 bu. How many bushels had he originally?

SOLUTION. The number of bushels $-\frac{1}{4}$ of the number of bushels $=$ 261 bu. But the number of bushels $-\frac{1}{4}$ of the number of bushels $=\frac{4}{4}$ of the number of bushels $-\frac{1}{4}$ of the number of bushels $=\frac{3}{4}$ of the number of bushels.

$\therefore$ $\frac{3}{4}$ of the number of bushels $=$ 261 bu.
$\therefore$ $\frac{1}{4}$ of the number of bushels $=\frac{261}{3}$ bu.
$\therefore$ $\frac{4}{4}$ of the number of bushels $=\frac{261}{3}$ bu. $\times 4 = 348$ bu.
$\therefore$ The number of bushels is 348.

EXERCISE 119 (Written)

1. If $\frac{5}{4}$ of a man's capital is $2950, what is the man's capital?

2. If $\frac{19}{20}$ of a number is 361, what is the number?

3. If $\frac{13}{25}$ of the distance by rail from New York City to Mexico City is 1950 mi., find the distance between these two cities.

4. If $\frac{3}{4}$ of the distance between New York City and Liverpool is 2655 mi., how many miles are there between the two cities?

5. Find the postal distance between New York City and Cheyenne, $\frac{7}{9}$ of such distance being 1477 mi.

6. Given $\frac{85}{100}$ of the distance between New York City and Chicago as 765 mi., how far is the one city from the other?

7. Knowing that $\frac{7}{8}$ of the distance from Chicago to Shanghai is 7973 mi., compute how far Chicago is from Shanghai.

8. Given that $\frac{3}{5}$ of the distance from San Francisco to Melbourne is 5397 mi., find the distance between these ports.

9. If $\frac{11}{200}$ of the number of miles from Paris to Turin is $27\frac{1}{2}$, what is the entire distance separating the cities?

10. If $\frac{11}{15}$ of the number of miles from New York City to Panama is 1727, how far is Panama from New York?

11. Given .9 of the distance from London to Constantinople as 1827 mi., how many miles is it from the former to the latter?

12. If $\frac{13}{55}$ of the distance from Hamburg to Vienna is 143 mi., find the distance between these cities.

13. In the year 1902, $\frac{11}{25}$ of the United States internal revenue receipts from tobacco amounted to $22,852,687. Find the total internal revenue receipts from tobacco for that year.

14. In the year 1902, $\frac{13}{81}$ of the excise tax in the United States on gross receipts under the War Revenue Law of 1898 amounted to $117,221. Find the total tax on gross receipts in 1902.

15. In the year 1902, $\frac{9}{20}$ of the United States internal revenue receipts from the tax on oleomargarine amounted to $1,325,021.40. Find the total receipts from this source.

16. The mark is the unit of money in Germany; $\frac{3}{17}$ of its value in our currency is 42 mills. Express the value of a mark in dollars.

17. The yen is the standard of value in Japan; $\frac{7}{83}$ of its value is equivalent to 4 cents and 2 mills. Express in dollars the value of the yen.

18. In Venezuela, the monetary unit is the Bolivar; $\frac{3}{5}$ of its value is equivalent to $.1158. Find its value in cents.

19. Thirty-two thirty-fifths of a meter is very nearly equivalent to 1 yd. Express the value of a meter in yards.

20. In 1901 Louisiana produced $1\frac{1}{61}$ times as much sugar as Hawaii, and $\frac{3}{5}$ of the sugar production of Louisiana was 186,000 tons. Find the number of tons of sugar Hawaii produced in 1901.

DIVISION

227. The quotient obtained by dividing unity by a number is called the **reciprocal** of the number. In other words, if the product of two numbers is unity, either one of these numbers is called the reciprocal of the other number. Thus, 8 is the reciprocal of $\frac{1}{8}$, and $1\frac{1}{2}$ is the reciprocal of $\frac{2}{3}$.

228. *Example* 1. Divide $7\frac{7}{8}$ by $4\frac{1}{5}$.

SOLUTION. $$7\tfrac{7}{8} \div 4\tfrac{1}{5} = \frac{\overset{3}{\cancel{63}}}{8} \times \frac{5}{\underset{1}{\cancel{21}}} = \frac{3 \times 5}{(8 \times 1)} = \frac{15}{8} = 1\tfrac{7}{8}.$$

Step 1. Reduce mixed numbers to improper fractions.

Step 2. Invert the terms of the divisor.

Step 3. Proceed then as in multiplication of fractions.

The following **rule for division** is sometimes given: Multiply the dividend by the reciprocal of the divisor.

NOTE. The art of dividing by a fraction by using its reciprocal is old. It is found, for instance, in the "Arithmetica Integra" of Stifel (1544).

Example 2. Divide $234\frac{7}{8}$ by 5.

SOLUTION. $$\begin{array}{r} 5\underline{)234\tfrac{7}{8}} \\ 46\tfrac{39}{40} \end{array}$$

In a case where the divisor is an integer, and the dividend is a mixed number, the integral part of the dividend being larger than the divisor, it is best to perform the division as indicated in Example 2 above.

Example 3. Simplify $\frac{2}{3}$ of $3\frac{1}{2} \times (1\frac{1}{8} - \frac{5}{6}) \div 4\frac{1}{12}$.

SOLUTION. $\frac{2}{3}$ of $3\frac{1}{2} \times \left(1\frac{1}{8} - \frac{5}{6}\right) \div 4\frac{1}{12} = \frac{2}{3} \times \frac{7}{2} \times \frac{7}{\underset{2}{\cancel{24}}} \times \frac{\cancel{12}}{\underset{7}{\cancel{49}}} = \frac{1}{6}$

Step 1. Find the value of the fractions in the parenthesis. Then proceed as in previous examples.

EXERCISE 120 Oral

Divide:

1. $\frac{1}{2}$ by 2.
2. $1\frac{1}{2}$ by 2.
3. $\frac{3}{4}$ by 2.
4. $\frac{7}{8}$ by 3.
5. $\frac{7}{9}$ by 4.
6. $\frac{7}{11}$ by 5.
7. $\frac{10}{13}$ by 5.
8. $\frac{3}{4}$ by 6.
9. $\frac{5}{6}$ by 10.
10. $\frac{4}{5}$ by 5.
11. $\frac{5}{12}$ by 15.
12. $\frac{5}{8}$ by 6.
13. $\frac{12}{17}$ by 6.
14. $\frac{14}{19}$ by 7.
15. $\frac{14}{23}$ by 21.
16. $\frac{8}{9}$ by 14.
17. $\frac{8}{9}$ by 16.
18. $1\frac{1}{2}$ by 4.
19. $1\frac{2}{3}$ by 5.
20. $1\frac{3}{4}$ by 7.
21. $2\frac{1}{6}$ by 13.
22. $1\frac{5}{9}$ by 7.
23. $3\frac{3}{8}$ by 9.
24. $4\frac{4}{9}$ by 20.
25. $4\frac{4}{11}$ by 12.
26. $5\frac{5}{11}$ by 15.
27. $3\frac{1}{13}$ by 5.
28. $9\frac{1}{2}$ by 3.
29. $11\frac{1}{4}$ by 3.
30. $14\frac{2}{3}$ by 6.
31. $12\frac{3}{4}$ by 7.
32. $8\frac{1}{4}$ by 11.
33. $8\frac{3}{4}$ by 7.
34. $17\frac{1}{2}$ by 5.
35. $18\frac{2}{7}$ by 8.
36. 2 by $\frac{1}{2}$.
37. 9 by $\frac{1}{2}$.
38. 5 by $\frac{1}{3}$.
39. 7 by $\frac{2}{3}$.
40. 6 by $\frac{3}{4}$.
41. 5 by $\frac{1}{5}$.
42. 11 by $\frac{1}{6}$.
43. 13 by $\frac{5}{6}$.
44. $1\frac{1}{2}$ by $\frac{1}{3}$.
45. $1\frac{3}{4}$ by $\frac{7}{8}$.
46. $1\frac{2}{9}$ by $\frac{7}{9}$.
47. $1\frac{3}{10}$ by $\frac{3}{10}$.
48. $1\frac{1}{6}$ by $\frac{5}{12}$.
49. $8\frac{2}{3}$ by $1\frac{1}{6}$.
50. $2\frac{1}{2}$ by $\frac{3}{4}$.
51. $7\frac{3}{5}$ by $8\frac{1}{10}$.
52. 14 by $3\frac{1}{2}$.

EXERCISE 121 (Written)

Divide:

1. 21 by $1\frac{3}{11}$.
2. 48 by $2\frac{2}{7}$.
3. 42 by $1\frac{5}{16}$.
4. 72 by $3\frac{3}{8}$.
5. $72\frac{1}{2}$ by $4\frac{1}{7}$.
6. $93\frac{3}{7}$ by $62\frac{2}{7}$.
7. $17\frac{3}{4}$ by $9\frac{31}{36}$.
8. $2\frac{4}{5}$ by $3\frac{9}{11}$.
9. $8\frac{1}{10}$ by $4\frac{8}{25}$.
10. $2\frac{11}{20}$ by $2\frac{32}{35}$.

11. $25\frac{2}{3}$ by $1\frac{5}{6}$.

12. $3\frac{1}{5}$ by $2\frac{2}{5}$.

13. $10\frac{1}{8}$ by $1\frac{35}{64}$.

14. $7\frac{5}{16}$ by $292\frac{1}{2}$.

15. $52\frac{1}{2}$ by $131\frac{1}{4}$.

16. $16\frac{1}{5}$ by $\frac{6}{7}$ of $6\frac{3}{4}$.

17. $\frac{3}{8}$ by $\frac{4}{9}$ of $1\frac{13}{32}$.

18. $\frac{1}{6}$ by $\frac{2}{7}$ of $1\frac{1}{4}$.

19. $\frac{37}{60}$ by $\frac{5}{14}$ of $1\frac{1}{6}$.

20. $2\frac{15}{16}$ by $1\frac{13}{72}$ of $1\frac{3}{5}$.

21. $1\frac{2}{3}$ by $\frac{3}{8}$ of $1\frac{3}{7}$.

22. $3\frac{8}{14}$ by $1\frac{8}{17}$ times $73\frac{4}{15}$.

23. $(\frac{9}{16}-\frac{1}{4})$ by $(\frac{3}{4}+\frac{1}{2})$.

24. $(1\frac{24}{25}-1\frac{21}{100})$ by $(1\frac{2}{5}+1\frac{1}{10})$.

25. $(3\frac{1}{16}-2\frac{1}{4})$ by $3\frac{1}{4}$.

26. $11\frac{1}{9}$ by $(1\frac{5}{6}+1\frac{1}{2})$.

229. A **complex fraction** is a fraction one or both of whose terms contain one, or more than one, fraction.

Thus, $\dfrac{2\frac{1}{2}}{7}$, $\dfrac{3}{4\frac{1}{5}}$, $\dfrac{1+\frac{1}{2}-\frac{1}{3}}{\frac{2}{5}+\frac{2}{3}}$ are complex fractions.

230. *Example* 1. Simplify $\dfrac{2\frac{2}{3}}{1\frac{5}{6}}$.

SOLUTION. $2\frac{2}{3} \div 1\frac{5}{6} = \frac{8}{3} \times \frac{6}{11} = 1\frac{5}{11}$.

Or, multiply numerator and denominator by any number which will make the terms of the fraction integers.

$$\frac{2\frac{2}{3}}{1\frac{5}{6}} = \frac{2\frac{2}{3}\times 6}{1\frac{5}{6}\times 6} = \frac{16}{11} = 1\frac{5}{11}.$$

Example 2. Simplify $\dfrac{3\frac{6}{25}-2\frac{1}{4}}{1\frac{4}{5}-1\frac{1}{2}} = \dfrac{\frac{99}{100}}{\frac{3}{10}} = 3\frac{3}{10}$.

Step 1. Simplify the numerator.

Step 2. Simplify the denominator.

Step 3. Divide the result of Step 1 by the result of Step 2.

Or, multiply numerator and denominator by 100, the L.C.M. of 2, 5, 4, 25. We then have

$$\frac{324-225}{180-150} = \frac{99}{30} = 3\frac{3}{10}.$$

EXERCISE 122 (Written)

1. $\frac{4\frac{1}{2}}{39}$.

2. $\frac{3\frac{2}{3}}{22}$.

3. $\frac{4\frac{4}{5}}{12}$.

4. $\frac{3\frac{3}{8}}{18}$.

5. $\frac{5\frac{3}{8}}{9\frac{5}{9}}$.

6. $\frac{7\frac{2}{3}}{9\frac{1}{5}}$.

7. $\frac{11\frac{7}{8}}{6\frac{10}{11}}$.

8. $\frac{13\frac{5}{7}}{9\frac{3}{5}}$.

9. $\frac{14\frac{2}{3}}{26\frac{2}{5}}$.

10. $\frac{19\frac{1}{4}}{46\frac{1}{5}}$.

11. $\frac{25\frac{1}{4}}{40\frac{2}{5}}$.

12. $\frac{1\frac{7}{9}-\frac{49}{64}}{2\frac{5}{24}}$.

13. $\frac{2\frac{14}{25}-\frac{4}{9}}{2\frac{4}{15}}$.

14. $\frac{1\frac{24}{25}-\frac{1}{9}}{2\frac{2}{15}}$.

15. $\frac{\frac{4}{7}-\frac{1}{3}+\frac{3}{4}}{\frac{8}{35}-\frac{2}{15}+\frac{6}{20}}$.

16. $\frac{\frac{6}{7}+1\frac{1}{2}-1\frac{7}{8}}{1\frac{4}{7}-1\frac{1}{4}}$.

17. $\frac{\frac{5}{8}+\frac{3}{4}\text{ of }1\frac{1}{5}}{\frac{18}{25}+\frac{1}{2}}$.

18. $\frac{1\frac{4}{5}-\frac{2}{3}\text{ of }1\frac{1}{4}\text{ of }1\frac{1}{5}}{1\frac{1}{20}-\frac{2}{9}\text{ of }2\frac{5}{8}}$.

19. $\frac{\frac{27}{64}-\frac{8}{27}}{\frac{5}{12}}\div 12\frac{1}{18}$.

20. $\frac{\frac{3}{4}\text{ of }\frac{5}{6}\div 1\frac{2}{3}}{2\frac{1}{4}\times\frac{5}{18}\text{ of }\frac{3}{5}}\times 1\frac{1}{4}\times 1\frac{1}{5}$.

EXERCISE 123 (Oral)

1. If 7 yd. of silk cost \$8¾, what is the cost of 1 yd. of silk?

2. How long will it take a person to walk 10½ mi. at the rate of 3 mi. an hour?

3. If coal costs \$5 per ton, how many tons can I buy for \$11.25?

4. If oranges sell for 3 cents each, how many can I buy for 75 cents?

5. If \$1.61 is divided equally among 7 boys, how much does each receive?

6. A train ran 60 mi. at the rate of 24 mi. an hour. How many hours did the train take to go the 60 mi.?

7. A boy rides on his bicycle 15 mi. in 2½ hr. What is his rate per hour?

8. How many pounds of sugar can be bought for 54 cents, at 4½ cents per pound?

9. When chairs sell for \$1.50, how many of them can be bought for \$12?

10. If a boy loses $\frac{5}{12}$ of his marbles, what fraction of his marbles has he left?

11. If 4 times a number is $19\frac{1}{2}$, what is the number?

12. If $\frac{2}{3}$ of a number is 12, what is the number?

13. If $\frac{3}{4}$ of a loaf of bread cost $13\frac{1}{2}$ cents, what is the cost of one loaf of bread?

14. Three-fifths of a boy's money is 24 cents. How much money has he?

15. A boy had a sum of money. He earned $\frac{1}{4}$ as much more and then had \$10. How many dollars had he at first?

16. How long would it take a person to save \$9 if he saved 75 cents a day?

17. Two-sevenths of Henry's age is $\frac{2}{3}$ of 6 yr. How old is Henry?

18. How many turns will a wheel $11\frac{1}{2}$ ft. in circumference make in going 69 ft.?

19. B gained an amount equal to $\frac{1}{2}$ of his money, and then had \$900. How many dollars had he at first?

20. After paying $\frac{1}{4}$ of his debt, a man owed \$60. What was his debt?

21. At \$25 an acre, how many acres can be bought for \$112.50?

22. How many pounds of beef can be bought for \$1.35, at 15¢ per pound?

23. If A can do a piece of work in 4 days, how much can he do in 1 day? If B can do the same work in 3 days, how much can B do in 1 day? How much can A and B, working together, do in 1 day?

24. If A can do a piece of work in 4 days, and B can do this same work in 6 days, what part of the work can both do in one day?

SOLUTIONS BASED ON INDUCTION AND DEDUCTION

231. *Example* 1. If .95 of a man's money is \$92.85, how much money has he?

SOLUTION. .95 of the money = \$92.85.

$$\therefore \text{ The money} = \frac{\$92.85}{.95} = \$97.$$

In this example \$92.85 is the product of two factors; one of the factors is .95.

Example 2. A farmer sold $\frac{3}{20}$ of his wheat, and then had 306 bu. How many bushels had he at first?

SOLUTION. After selling $\frac{3}{20}$ of his wheat, he had $\frac{17}{20}$ of his wheat left.

$$\therefore \tfrac{17}{20} \text{ of the quantity of wheat} = 306 \text{ bu.}$$

$$\therefore \text{ The quantity of wheat} = 306 \text{ bu.} \div \tfrac{17}{20} = 360 \text{ bu.}$$

Example 3. A man gained $\frac{1}{8}$ of his money, and then had \$3420. How many dollars had he originally?

SOLUTION. The man's money + $\frac{1}{8}$ of the man's money = $\frac{9}{8}$ of the man's money.

$$\therefore \tfrac{9}{8} \text{ of the man's money} = \$3420.$$

$$\therefore \text{ The man's money} = \$3420 \div \tfrac{9}{8} = \$3040.$$

EXERCISE 124 (Written)

1. How long will it take a person to earn \$44.40, if he earns \$1.85 a day?

2. If $\frac{5}{8}$ of a quantity of coal is $71\frac{1}{4}$ tons, how many tons of coal are there in all?

3. A train runs at the rate of $33\frac{1}{3}$ mi. per hour. How many hours will it take the train to run 350 mi. at this rate?

4. A man being asked his age replied, "Three elevenths of my age is $13\frac{1}{2}$ years." Find the man's age.

5. A and B engage in business. A's share is $\frac{2}{5}$ of the joint capital. If A contributes $2540, what is the joint capital? How much does B invest?

6. A dealer sells a piano for $240, thereby gaining $\frac{1}{5}$ of the cost of the piano. What did the piano cost?

7. If a dealer sold a piano for $240 and lost thereby $\frac{1}{5}$ of the cost price what did the piano cost?

8. If the dividend is $18\frac{3}{4}$ and the quotient is $10\frac{1}{2}$, what is the divisor?

9. The sum of two numbers is 25, and one of the numbers is $1\frac{1}{2}$ times the other number. Find the numbers.

10. A farmer sells $\frac{2}{9}$ of his cattle to one jobber and $\frac{4}{9}$ to another jobber. If he keeps the remainder, what part does he keep? Suppose the number remaining is 27, how many head of cattle had the farmer originally?

11. A dealer sells $\frac{1}{2}$ of his coal, and then $\frac{1}{3}$ of it, and has $12\frac{1}{2}$ tons left. How many tons had he originally?

12. If the $\frac{1}{5}$ part of a number and the $\frac{1}{6}$ part of the same number together make $60\frac{1}{2}$, what is the number?

13. If .85 of a man's money is $289, how much money has he?

14. By gaining .15 of his outlay a man's property amounts to $7245. What was his outlay?

15. Five sevenths of a farm is sold for $1385. What is the value of the farm?

16. How many times will a wheel 12 ft. $4\frac{1}{2}$ in. in circumference revolve in going $3\frac{3}{4}$ miles?

17. How long will it take a person to go $10\frac{2}{5}$ mi. at the rate of $6\frac{1}{2}$ mi. per hour?

18. A train travels at the rate of $52\frac{1}{2}$ mi. in $1\frac{3}{4}$ hr. Find its rate per hour. How many hours and minutes will it take this train to travel 208 mi.?

19. From the ground to the first floor of a house is $\frac{4}{15}$ of the height of the house; if the first floor is 10 ft. above the ground, how high is the house?

20. What part of $4\frac{5}{6}$ is $\frac{1}{9}$ of $14\frac{1}{2}$?

21. If .625 of a gallon of maple sirup cost $1.25, what will 1 gal. of maple sirup cost?

22. When 3 oranges sell for 5¢, how many oranges can I buy for 80¢?

23. If $8\frac{1}{4}$ yd. of calico cost 66¢, how many yards can I buy for $3.30?

24. A merchant, by selling tea at 69¢ per pound, gains $\frac{3}{20}$ of the cost of the tea. Find the cost per pound of the tea.

25. By selling cloth at 78¢ per yard a clothier loses $\frac{1}{7}$ of the cost of the cloth. Find the cost price per yard of the cloth.

26. The rent of a house for 17 mo. is $225. Find the rent of this house for 12 mo.

27. Lead pencils cost $3\frac{1}{3}$¢ each. At this price, how many can be bought for $1.40?

RATIO IDEA OF NUMBER

EXERCISE 125 (Oral)

1. If 1 in. is the unit, what number measures 1 yd.?

2. If 6 in. is the unit, how is a yard expressed?

3. Express a dollar, a dime, and an eagle, each in terms of 1¢. Also in terms of 1 mill.

4. If 25¢ were the unit of value, what integer would represent $10? $13? $14.75?

5. What is the ratio of 1 gal. to 1 pt.? Express 3 gal. in pints.

6. What is the ratio of a square yard to a square foot? of a square yard to a square inch?

7. If 1000 is the unit, what number expresses the distance of the moon from the earth, *i.e.* 238,000 miles?

8. Express $10.50 as an integer in which 50¢ is the unit.

9. If the linear unit were 6 in., what number would express the value of 1 sq. ft.? of 1 sq. yd.?

10. If 30° were the unit of circular measure, what number would measure 1 circumference?

11. Express 30 twenty-dollar bills as an integer where the unit is a five-dollar bill.

12. Express $\frac{1}{4}$ mi. as an integer in which the chain is the standard of measurement.

13. What is the ratio of 1 bu. to 1 qt.? of 1 sq. rd. to $\frac{1}{4}$ sq. yd.?

14. How many square yards are in 1 A.? How many quarter square yards are in 1 A.? What is the ratio of 1 A. to 1 sq. rd.? of 1 A. to 1 sq. yd.?

15. What is the ratio of 1 cu. yd. to 1 cu. ft.? If a cubical box 6 in. high were the unit of cubic measure, what number would express 1 cu. ft.? 1 cu. yd.?

16. What is the ratio of 1 gal. to the volume of a cubical box 11 in. by 7 in. and 1 in. in height?

EXERCISE 126 (Written)

1. If the unit of surface is a square having for its side 11 yd., how many of these units does an acre contain?

2. If a square link is the unit of surface, find, in terms of this measure, the number which corresponds to 1 A.

3. If flour is put up in 10-lb. sacks, find the number of these sacks in 1 long ton of flour.

4. The English pound sterling is equivalent, in our currency to $4.8665. What is the value of $1 in pounds sterling? How many shillings are equivalent to $1? (20 shillings = £1 sterling.)

5. Express decimally 1 rd. in chains. How many chains are in 1 knot or geographical mile?

6. Express 1 sq. mi. in square chains.

7. How many square chains are in the N.W. $\frac{1}{4}$ of S.E. $\frac{1}{4}$ of a section of land?

8. If a square upon a line 394 in. long is the unit of surface measure, how many of these units are in 1 A.?

9. How many cubic yards are in a cube having 1 mi. for one of its dimensions? How many are in a cube having 1 rd. for one of its dimensions?

10. How many railway cars of 20,000 lb. capacity each will be required to load a ship whose cargo consists of 5470 T. of grain?

11. Express decimally in tons 7280 bu. of wheat.

12. Express decimally in tons 2890 bu. of oats.

13. Express decimally in tons 3845 bu. of barley.

14. What is the unit of measurement when $\frac{1}{4}$ mi. is expressed by the number 44?

15. What is the unit of measurement when $\frac{1}{2}$ mi. equals 15,840 units?

16. There are 3.1416 radians in 360°. Find in degrees, minutes, and seconds the value of a radian.

17. What is the unit of measurement when 1 rd. is expressed by the integer 25?

18. If 100 ft. were the unit, how would the distance from New York to Galveston be expressed? Galveston is 1789 mi. from New York City.

19. A cubic centimeter is a cube nearly .4 of an inch on an edge. How many cubic centimeters are in 1 cu. ft.?

20. How many square yards are in the walls and ceiling of a room 24 ft. by 18 ft. and 15 ft. high? What is the cubical contents of the room?

21. How many acres are in a rectangular plot 480 yd. long by $338\frac{1}{2}$ yd. wide? What decimal of 1 mi. is the length of its four sides?

22. Find the area in square yards of a rectangular plot 73 ft. 6 in. by 54 ft. 2 in. Express the result decimally.

232. Find a number contained an exact number of times in $3\frac{3}{4}$ and $4\frac{1}{6}$.

SOLUTION. $3\frac{3}{4} = \frac{15}{4} = \frac{45}{12}$.

$4\frac{1}{6} = \frac{25}{6} = \frac{50}{12}$.

Since $3\frac{3}{4} = \frac{45}{12}$, and $4\frac{1}{6} = \frac{50}{12}$, $\therefore \frac{1}{12}$ is a unit of measure contained an integral number of times in $3\frac{3}{4}$ and $4\frac{1}{6}$. Since 5 is a common measure of 45 and 50, $\frac{5}{12}$ is a common measure of $3\frac{3}{4}$ and $4\frac{1}{6}$; $\frac{5}{12}$ is also the G. C. M. of $3\frac{3}{4}$ and $4\frac{1}{6}$. The number $3\frac{3}{4}$ contains $\frac{5}{12}$, 9 times, and the number $4\frac{1}{6}$ contains $\frac{5}{12}$, 10 times. By subdividing the common measures, $\frac{5}{12}$ and $\frac{1}{12}$, we can get any number of common measures of $3\frac{3}{4}$ and $4\frac{1}{6}$.

By taking any other pair of numbers, we can get, in a similar manner, a third number which is contained an exact number of times in each of the given numbers. Hence we can express any two of the numbers we have thus far met, as integers in terms of a common unit.

Later we shall have to consider numbers which cannot be thus expressed.

Numbers which can be exactly measured by a third number are said to be **commensurable.**

NOTE. A clear conception of § 232 is indispensable in the study of elementary geometry. The authors have yet to meet the pupil who, in taking up the subject of measurement, had the faintest conception of how to express fractional number values as whole numbers in terms of a common unit of measurement.

EXERCISE 127 (Written)

Find a common unit of measurement of the following numbers, and express the numbers as integers in terms of the common unit:

1. $2\frac{2}{3}$, $2\frac{1}{6}$.
2. $4\frac{1}{2}$, .4.
3. $7\frac{7}{8}$, $14\frac{1}{2}$.
4. $3\frac{1}{4}$, $3\frac{5}{7}$.
5. $2\frac{2}{5}$, $3\frac{2}{3}$.
6. $4\frac{4}{5}$, $3\frac{5}{9}$.
7. $5\frac{5}{8}$, $2\frac{7}{9}$.
8. $1\frac{9}{11}$, $3\frac{1}{13}$.
9. $5\frac{3}{5}$, $3\frac{3}{8}$.
10. $2\frac{11}{12}$, $4\frac{1}{11}$.
11. $4\frac{2}{3}$, $5\frac{1}{4}$.
12. $1\frac{7}{9}$, $2\frac{2}{11}$.

Multiply $.04583\frac{1}{3}$ by $\frac{7}{24}$.

SOLUTION. $.04583\frac{1}{3} = \frac{.13750}{3} = \frac{.1375}{3}$. $\therefore .04583\frac{1}{3} \times \frac{7}{24} = \frac{.1375}{3} \times \frac{7}{24} = \frac{.9625}{72} = .013368$.

To divide .9625 by 72, proceed as follows:

$$\begin{array}{r|l} 8 & .9625 \\ \cline{2-2} 9 & .1203125 \\ \cline{2-2} & .013368055 \end{array}$$

EXERCISE 128 (Written)

Multiply:

1. $.0416\frac{2}{3}$ by $3\frac{3}{5}$.
2. $.00583\frac{1}{3}$ by $11\frac{1}{2}$.
3. $.0016\frac{2}{3}$ by $5\frac{1}{2}$.
4. $.00416\frac{2}{3}$ by $7\frac{2}{5}$.
5. $.00416\frac{2}{3}$ by $1\frac{11}{30}$.
6. $.00583\frac{1}{3}$ by $3\frac{13}{30}$.
7. $.0066\frac{2}{3}$ by $4\frac{7}{30}$.
8. $.00666\frac{2}{3}$ by $5\frac{17}{30}$.
9. $.00833\frac{1}{3}$ by $7\frac{7}{10}$.
10. $.009166\frac{2}{3}$ by $11\frac{4}{5}$.
11. $.0166\frac{2}{3}$ by $5\frac{1}{2}$.
12. $.004166\frac{2}{3}$ by $41\frac{2}{3}$.
13. $.33\frac{1}{3}$ by $245\frac{1}{6}$.
14. $.0244\frac{4}{9}$ by $12\frac{2}{3}$.
15. $.00583\frac{1}{3}$ by $\frac{19}{30}$.
16. $.1255\frac{5}{9}$ by $1\frac{1}{2}$.
17. $.64\frac{1}{2}$ by $57\frac{3}{4}$.
18. $.53\frac{3}{8}$ by $37\frac{1}{2}$.
19. $.29\frac{3}{4}$ by 4.5.
20. $.06\frac{5}{16}$ by 445.
21. $.06\frac{5}{8}$ by 238.
22. $.05\frac{1}{4}$ by $121\frac{1}{2}$.
23. $.06\frac{2}{3}$ by $13\frac{1}{2}$.
24. $.08\frac{1}{3}$ by $19\frac{3}{4}$.

EXERCISE 129 (Written)

REVIEW

1. Multiply the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, by $1\frac{2}{13}$, and divide the product by $\frac{2\frac{1}{2}}{3\frac{3}{4}}$.

2. What number taken from $20\frac{1}{2}$ leaves as a remainder $4\frac{2}{3}$?

3. The difference between two numbers is $1\frac{1}{2}$, and the less is $4\frac{5}{6}$. Find the other number.

4. Which is the greater, $\frac{3}{4}$ of $4\frac{4}{9}$, or $\frac{2}{3}$ of $4\frac{3}{4}$?

5. From $\frac{7}{8}$ of the sum of $\frac{1}{2}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{3}{40}$, take the sum of $\frac{7}{16}$, $\frac{7}{40}$, $\frac{7}{64}$.

6. From $\frac{3}{4}$ of $(\frac{1}{3} - \frac{1}{6} + \frac{8}{9})$ take $(\frac{1}{4} + \frac{2}{3} - \frac{1}{8})$.

7. **Express in pounds the difference between .0125 of a ton and $\frac{3}{4}$ cwt.**

8. What fraction having 48 for denominator is equivalent to .1875?

9. Find in feet the value of $\frac{15}{16}$ of a mile.

10. Express $\frac{2}{3}$ yd. as the decimal of a rod.

11. Express $\frac{3}{5}$ of 95 lb. as the decimal of $1\frac{1}{4}$ cwt.

12. Express $27\frac{3}{4}$ lb. as the decimal of a ton.

13. Take $\frac{4}{5}$ T. from $1\frac{1}{2}$ T., and express your result in pounds.

14. From $\frac{7}{8}$ of a right angle take 33° 45′.

15. From $\frac{1}{5}$ of a circumference take $\frac{1}{6}$ of the circumference, and express your answer in degrees.

16. There are two numbers in the ratio of 3 to 5. What fraction of their sum is their difference?

17. An estate is left to A, B, and C. A gets $\frac{1}{2}$ of the estate, B, $\frac{1}{5}$ of the estate, and C, the remainder. What part of the estate does C get? If C's share is $450, what is the value of the estate? Find A's share and B's share.

18. A man spends $\frac{2}{5}$ of his salary on board, $\frac{1}{3}$ on clothing, $\frac{1}{5}$ on rent. He saves the remainder, amounting to $125. Find his salary.

19. Subtract $\frac{1}{8}$ from 2.1, and divide the remainder by .25.

20. The dividend is 3.562 and the quotient is .3125. Find the divisor.

21. Express $5.24 as a decimal of $100.

22. After giving away $\frac{1}{9}$, $\frac{1}{12}$, and $\frac{17}{36}$ of his money, a man has left $392.95. How much money had he at first?

23. The third part of a number exceeds the fifth part of the same number by 15. What is the number?

24. The sixth part and the eighth part of a number together make $66\frac{1}{2}$. What is the number?

25. If $\frac{1}{3}$ and $\frac{2}{5}$ of a farm are together worth $1650, what is $\frac{1}{4}$ of the remainder worth?

26. Subtract the product of $\frac{1}{2}$ and $\frac{2}{9}$ from their sum.

27. What number divided by $3\frac{3}{5}$ gives $1\frac{1}{12}$ for the quotient?

28. Find the value of $\frac{7}{9}$ of $1\frac{3}{14}$ of $19.80?

29. A man owns $\frac{2}{3}$ of a boat and sells $\frac{3}{4}$ of his share for $750. At this rate, find the value of the boat.

30. If 6 men do a piece of work in 9 days, how long will it take 4 men to do the same work?

31. If 15 men pave a street in 16 days, how long will it take 40 men to pave the same street?

32. If 18 men remove an embankment in 12 da., how long will it take 24 men to remove the embankment?

33. Two trains start at the same time from two stations 840 mi. apart, and travel toward each other, one train going at the rate of 35 mi. an hour, and the other train at the rate of 25 mi. an hour. In how many hours will they meet?

34. If a man performs $\frac{3}{7}$ of a piece of work in 15 da., in how many days more will he complete the work?

35. Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{5}{16}$, and $\frac{7}{20}$. Express the sum as a decimal. Verify your work by reducing each fraction to a decimal and then adding.

36. What is the smallest number which, added to the sum of $\frac{1}{3}$, $\frac{1}{5}$, and $\frac{5}{9}$, will make the final result an integer?

37. If $\frac{3}{5}$ of a barrel of sugar is sold and afterward 40 lb. are sold, how many pounds of sugar were in the barrel originally, supposing it still contains 90 lb.?

38. By selling a piano at $\frac{7}{10}$ of its cost, a dealer loses $98. Find the cost of the piano.

39. If 45 sq. rd. of land cost $18, find the cost of 1 A. Find also the cost of $3\frac{3}{4}$ A.

40. If $\frac{3}{4}$ of a clerk's salary per year is $675, find his salary per month.

41. If $3\frac{1}{3}$ A. of land are worth $119, find the value of $2\frac{1}{2}$ A.

42. How much is a plot of ground 80 ft. by 60 ft. worth, if an acre is worth $55?

43. Find the weight of a piece of coal in the shape of a rectangular solid, if its dimensions are $1\frac{1}{2}$ ft. by $1\frac{1}{4}$ ft. by 10 in. A cubic foot of coal weighs $81\frac{1}{4}$ lb.

44. A man's property is assessed at $4550. If he pays 40¢ on every $100 for school tax, how many dollars school tax does he pay?

45. How much taxes will be paid on real estate worth $9580, if the tax is at the rate of $1.27 on $100?

46. A man invests $4800 and gains $540. How much does he gain on every dollar invested? How much does he gain on every $100?

47. If an investment of $9600 produces a gain of $1056, find the gain on $1; also on $100.

48. When $8400 produces a profit of $1092, how much does $1 produce? $100?

49. The school tax in a city is 2 mills on the dollar. The assessed valuation of the property is $17,294,000. Find the total tax levied for school purposes. If .95 of this total is collectible, find the amount collected.

50. Find the tax on $33,254,000 at 4 mills on the dollar.

51. A man insures his dwelling for $5450. If he pays $11.50 on every $1000, how much does he pay altogether?

52. Find the insurance on a house valued at $7840, if the rate of insurance is $1½ on every $100?

53. If $47.50 is paid to insure a boat valued at $9500, how much is paid on $1? on $100?

54. The distance from New York City to Plymouth, England, is 2962 knots. The steamship *Deutschland* sailed from Plymouth to New York in July, 1900, in 5 da. 15 hr. 45 min. Find, in knots, its rate per hour. Find also its rate in miles per hour.

55. In March, 1902, the steamship *La Savoie* made the voyage from Havre to New York in 6 da. 10 hr. The distance from Havre to New York is 3170 knots nearly. Find, in knots, the rate per hour. Express the rate also in miles per hour.

56. The steamer *Kronprinz Wilhelm* made, in September, 1902, a voyage from Cherbourg to New York in 5 da. 11 hr. 57 min. Calculate its rate per hour, the distance from Cherbourg to New York being 3184 knots.

57. In May, 1900, a passenger train made the distance from Burlington to Chicago, 205.8 mi., in 3 hr. 8 min. 30 sec. Calculate its rate per hour.

58. The fastest time on record by a passenger train for a distance over 450 miles was made in October, 1895, on the Lake Shore and Michigan Southern Railroad, from Chicago to Buffalo, a distance of 510 mi., in 8 hr. 1 min. Calculate its rate per hour.

59. The run from London to Edinburgh, 393½ mi., has been made in 7 hr. 45 min. Find the speed per hour.

60. The market quotations, Feb. 19, 1903, were: wheat, $78\frac{3}{4}$¢ per bushel; corn, $45\frac{5}{8}$¢ per bushel; oats, 34¢ per bushel. Find the price of 100 lb. of each of these commodities.

REVIEW QUESTIONS

1. What is the test of divisibility by 4? by 8? by 25? by 125?

2. What effect on the value of a fraction has multiplying both its terms by the same number?

3. What effect upon the value of the fraction has the addition of the same number to both of its terms?

4. What is the reciprocal of a number?

5. How do you reduce a decimal to a common fraction?

6. How do you reduce a concrete quantity to the fraction of another concrete quantity of the same kind?

7. How do you express a concrete quantity in terms of another concrete quantity of the same kind?

8. How do you find a common measure of two fractions?

9. How do you express two fractions as integers in terms of a common unit of measure?

10. Explain what is meant by the analytical method of solving problems.

11. How would you arrange three fractions in order of magnitude?

12. What effect upon the value of the fraction has dividing the denominator by an integer?

CHAPTER XIII

LONGITUDE AND TIME

233. A **meridian** is an imaginary line running due north and south from pole to pole.

234. Longitude is the distance, expressed in circular measure, east or west from the **prime** or **standard meridian.**

The meridian through any particular place may be used as the prime meridian. The meridians through the observatories of Greenwich, Washington, Paris, Madrid, Rome, Stockholm, Pulkova, and Lisbon, have been used as prime meridians by the nations to which these cities belong. The International Geodetic Congress, which met at Washington in 1884, recommended that the meridian passing through the observatory at Greenwich, a suburb of London, be the prime meridian. This recommendation is now generally adopted by the great nations of the world. The meridian of Greenwich is taken as prime meridian in this book.

The **longitude of a place** is the number of degrees the place is east or west of the prime meridian.

Longitude is reckoned in either direction halfway around the earth from the prime meridian. The greatest longitude a place can have is 180° E. or 180° W. The meridian 180° E. or 180° W. of the prime meridian is a continuation of the prime meridian on the other side of the earth, and forms with the prime meridian what is called a **great circle**, passing through the poles.

235. That the sun rises in the east, sets in the west, and passes once every 24 hr. over the meridian of every point on the earth's surface are obvious phenomena. The rising of the sun in the east, and its setting in the west, are, however, merely *apparent* phenomena due to the rotation of the earth on its axis once every 24 hr.

This rotation of the earth on its axis accounts for the fact which everybody who travels a considerable distance east or west experiences; namely, that the local time in one place is not the same as the local time in every other place.

Consider two places, one being east of the other. As the sun apparently travels from east to west, it will be directly over the meridian of the eastern place before it is directly over the meridian of the western place. In other words, it will be noon in the eastern place before it is noon in the western place. When it is noon in the western place, it will be past noon in the eastern place. Hence, as we go to the east, local time is later. To give an illustration, when it is noon in New York City it is 56 min. past 4 o'clock P.M. in London.

Is there a connection between the difference of the longitudes of two places and the difference between the local times of the two places? Let us see. Since the rate of motion of the earth on its axis is uniform, and furthermore, since,

in 24 hr. the earth rotates 360°,
∴ in 1 hr. the earth rotates 15°;
∴ in 1 min. the earth rotates $15'$;
∴ in 1 sec. the earth rotates $15''$.

A difference of 15° of longitude corresponds to a difference of 1 hr. of time. A difference of $15'$ of longitude corresponds to 1 min. of time. A difference of $15''$ of longitude corresponds to 1 sec. of time.

We have, therefore, a connection between the difference of longitudes and the difference of local times of two places.

The sun rises at almost the same instant, local time, at all places on the same parallel of latitude. Two places on the same meridian have the same local time.

East longitude and west longitude are designated by E. and W. respectively.

EXERCISE 130 (Oral)

1. When it is noon at London, what is the time at New Orleans, 90° W.?

2. When it is 9 o'clock A.M. on the meridian 75° W., what is the time on the meridian 90° W.?

3. The longitude of Denver is 105° W. When it is 3 o'clock P.M. in Denver, what is the time in London?

4. Two places differ in longitude by 20°. What is their difference in time?

5. A person travels east 15°. What change must he make in the time indicated by his watch so that it may indicate local time? Supposing he goes the same distance west, what change must be made in the time indicated by his watch?

6. When it is noon in London, what is the longitude of the places in which it is 4 o'clock P.M.? 5 o'clock A.M.?

7. When it is 2 o'clock P.M. in Washington, what is the time in places 30° W. of Washington? in places 75° E. of Washington?

8. What is the difference in longitude between two places which differ in time by 2 hr. 30 min.? by 4 hr. 10 min? by 5 hr. 20 min.?

9. If a person travels from Denver to New York, will his watch be fast or slow when he reaches New York, and how much?

10. To how many hours does a difference of 80° in longitude correspond?

11. What difference in longitude corresponds to a difference of 4 hr. 20 min. in time?

12. A person living on the 90th meridian W. wishes to send a telegram to a bank in New York City, directing the bank to pay on the same day a sum of money. Up to what hour in the afternoon may he do this, allowing 30 minutes for the transmission of the telegram, taking the longitude of New York as 74° W.? (New York banks close at 3 P.M.)

13. At places on the same parallel of latitude the sun rises at the same instant local time. How many minutes earlier does the sun appear to a person who travels 1° E.?

LONGITUDES OF CITIES REFERRED TO IN THIS CHAPTER

Austin, Tex.	97° 44′ W.
Baltimore, Md.	76° 37′ W.
Bangor, Me.	68° 47′ W.
Bismarck, N.D.	100° 47′ W.
Boston, Mass.	71° 3′ 50″ W.
Brisbane, Australia	153° 2′ E.
Buenos Ayres, Argentina	58° 22′ 14″ W.
Charleston, S.C.	79° 52′ 58″ W.
Chicago, Ill.	87° 40′ W.
Cincinnati, O.	84° 24′ W.
Constantinople, Turkey	29° 0′ 50″ E.
Detroit, Mich.	83° 3′ W.

Dublin, Ireland	6° 20′ 30″ W.
Galveston, Tex.	94° 47′ W.
Havana, Cuba	82° 21′ 30″ W.
Honolulu, Hawaii	157° 52′ W.
Louisville, Ky.	85° 46′ W.
Melbourne, Australia	144° 58′ 32″ E.
Manila, Philippine Is.	120° 58′ 3″ E.
Mexico City, Mexico	99° 6′ 39″ W.
Montreal, Canada	73° 33′ 4″ W.
New Orleans, La.	90° 3′ 28″ W.
New York, N.Y.	74° 0′ 24″ W.
Norfolk, Va.	76° 17′ 22″ W.
Paris, France	2° 20′ 15″ E.
Pekin, China	116° 29′ E.
Pensacola, Fla.	87° 16′ 6″ W.
Philadelphia, Pa.	75° 9′ 3″ W.
Portland, Ore.	122° 40′ W.
Providence, R.I.	71° 24′ 20″ W.
Rome, Italy	12° 28′ 40″ E.
St. Louis, Mo.	90° 16′ W.
St. Petersburg, Russia	30° 19′ 40″ E.
San Francisco, Cal.	122° 24′ 32″ W.
Savannah, Ga.	81° 5′ 25″ W.
Tien-tsin, China	117° 11′ 44″ E.
Tokio, Japan	139° 44′ 30″ E.
Washington, D.C.	77° 0′ 36″ W.

236. *Example* 1. Find the difference between the longitudes of Austin and Honolulu.

SOLUTION. Honolulu, 157° 42′ W.
Austin, 97° 44′ W.
59° 58′

∴ Honolulu is 59° 58′ farther west than Austin.

Example 2. Find the difference between the longitudes of Galveston and Constantinople.

SOLUTION. Galveston, 94° 47′ W.
Constantinople, 29° 0′ 50″ E.

Here, the places are on opposite sides of the prime meridian. By going east from Galveston, 94° 47′ we arrive at the prime meridian, and by going 29° 0′ 50″ still farther east, we arrive at Constantinople. Hence, the difference between the longitudes is (94° 47′ + 29° 0′ 50″) = 123° 47′ 50″.

To find the difference in the longitudes of two places: (1) Subtract their longitudes, if the places are on the same side of the prime meridian. (2) Add their longitudes, if the places are on opposite sides of the prime meridian.

EXERCISE 131 (Written)

Find the difference in longitude between:

1. Baltimore and Bismarck.
2. Bangor and Detroit.
3. Boston and Havana.
4. Buenos Ayres and Chicago.
5. Charleston and Constantinople.
6. Cincinnati and Honolulu.
7. Cincinnati and Melbourne.
8. Havana and Rome.
9. Louisville and St. Petersburg.
10. Constantinople and Tien-tsin.
11. Paris and Pekin.
12. Norfolk and Paris.
13. Montreal and Mexico City.
14. Pensacola and Portland.
15. St. Louis and St. Petersburg.
16. Savannah and Dublin.

17. San Francisco and Dublin.
18. New Orleans and Pekin.
19. Galveston and Tokio.
20. Austin and Manila.

237. *Example* **1.** Find the difference in local time between Boston and Portland, Ore.

SOLUTION.

$$\begin{array}{rrrrl} \text{Portland,} & 122^\circ & 40' & & \text{W.} \\ \text{Boston,} & 71^\circ & 3' & 50'' & \text{W.} \\ \cline{2-4} & 50^\circ & 36' & 10'' & \text{W.} \end{array}$$

∴ The difference between the longitudes is 50° 36′ 10″.

360° of longitude corresponds to 24 hours.

∴ 1° of longitude corresponds to $\frac{24}{360}$ hr. = 4 min.

∴ 1′ of longitude corresponds to $\frac{4}{60}$ min. = 4 sec.

∴ 50° of longitude corresponds to 4 min. × 50 = 3 hr. 20 min.

∴ 36′ of longitude corresponds to 4 sec. × 36 = 2 min. 24 sec.

∴ 10″ of longitude corresponds to 4 sec. × $\frac{10}{60} = \frac{2}{3}$ sec.

Ans. 3 hr. 22 min. $24\frac{2}{3}$ sec., or, 3 hr. 22 min. 25 sec., nearly.

Example **2.** Find the difference in local time between Washington and Manila.

$$\begin{array}{rrrrl} \text{Washington,} & 77^\circ & 0' & 36'' & \text{W.} \\ \text{Manila,} & 120^\circ & 58' & 3'' & \text{E.} \\ \cline{2-4} & 197^\circ & 58' & 39'' & \end{array}$$

The difference between the longitudes is 197° 58′ 39″.

197° of longitude corresponds to 4 min. × 197 = 13 hr. 8 min.

58′ of longitude corresponds to 4 sec. × 58 = 3 min. 52 sec.

39″ of longitude corresponds to 4 sec. × $\frac{39}{60} = 2\frac{3}{5}$ sec.

Ans. 13 hr. 11 min. $54\frac{3}{5}$ sec., or, 13 hr. 12 min., nearly.

EXERCISE 132 (Written)

Find the difference between the local times of:

1. Mexico City and Montreal.
2. Philadelphia and San Francisco.
3. Philadelphia and Dublin.
4. Norfolk and Tien-tsin.
5. Chicago and Tokio.
6. St. Louis and Rome.
7. Austin and St. Petersburg.
8. Savannah and Paris.
9. Washington and Brisbane.
10. Cincinnati and Manila.
11. Havana and Louisville.
12. Rome and Manila.
13. New Orleans and Portland.
14. Providence and St. Petersburg.
15. Montreal and Tokio.
16. Bangor and Melbourne.
17. Baltimore and Buenos Ayres.
18. Charleston and Constantinople.
19. Boston and Austin.
20. Galveston and Pekin.

238. *Example* 1. When it is noon in Bismarck, it is 31 min. past 6 o'clock P.M. in Liverpool. Find the longitude of Liverpool.

SOLUTION. 1 hr. corresponds to 15° of longitude.

$\therefore$ 1 min. of time corresponds to $15'$ of longitude.

$\therefore$ 6 hr. corresponds to $15^\circ \times 6 = 90^\circ$ of longitude.

∴ 31 min. corresponds to $15' \times 31 = 7^\circ 45'$ of longitude.

∴ 6 hr. 31 min. corresponds to 97° 45′ of longitude.

∴ Liverpool is 97° 45′ east of Bismarck. Longitude of Bismarck is 100° 47′ W.

∴ Longitude of Liverpool is $(100^\circ 47' - 97^\circ 45')$ W. $= 3^\circ 2'$ W.

Example 2. When it is noon Feb. 22 in St. Louis, it is 15.1 min. past 3 o'clock A.M. Feb. 23, in Adelaide, Australia. Calculate the longitude of Adelaide.

SOLUTION. Time difference between St. Louis and Adelaide = 15 hr. 15.1 min.

15 hr. corresponds to $15^\circ \times 15 = 225^\circ$ of longitude.

15.1 min. corresponds to $15' \times 15.1 = 226.5'$ of longitude $= 3^\circ 46' 30''$ of longitude.

∴ 15 hr. 15.1 min. corresponds to 228° 46′ 30″ of longitude.

∴ Adelaide is 228° 46′ 30″ E. of St. Louis. Longitude of St. Louis is 90° 16′ W. ∴ Longitude of Adelaide $= (228^\circ 46' 30'' - 90^\circ 16')$ E. $= 138^\circ 30' 30''$ E.

EXERCISE 133 (Written)

Calculate the longitude of each of the following cities, the time difference between New York City and each of them being given:

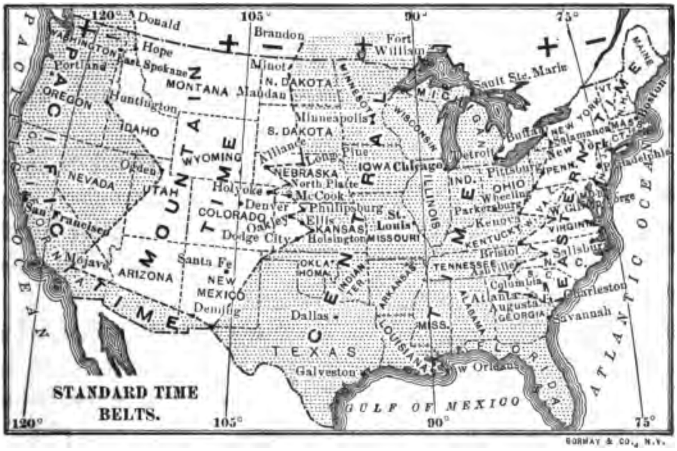
1. Berlin, 5 hr. 49.5 min.
2. Brussels, 5 hr. 13.4 min.
3. Calcutta, 10 hr. 49.2 min.
4. Edinburgh, 4 hr. 43.2 min.
5. Hamburg, 5 hr. 35.8 min.
6. London, 4 hr. 55.9 min.
7. Madrid, 4 hr. 41.1 min.
8. Vienna, 6 hr. 1.2 min.

9. When it is 10 o'clock A.M. in Philadelphia, it is 37.6 min. past 2 o'clock P.M. in Belfast. Find the longitude of Belfast.

10. When it is 9 o'clock P.M. Feb. 22, in Washington, what is the time in Tokio? in Tien-tsin? in Rome? in St. Petersburg? in Melbourne?

STANDARD TIME

239. If every place used its own true local time, there would be endless confusion in carrying on the commerce of the country. To obviate this, what is known as **Standard time** was introduced by the railroads and business circles in 1883.



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By Standard time is meant the time of the nearest meridian which is a multiple of 15°. The United States is divided into four belts, known as the *Eastern Belt*, *Central Belt*, *Mountain Belt*, *and Pacific Belt.* Places situated in the Eastern Belt use 75th meridian time, places in the Central Belt use 90th meridian time, places in the

Mountain Belt use 105th meridian time, and places in the Pacific Belt use 120th meridian time. The time belts are shown in the accompanying map. Standard time is now generally used throughout the civilized world.

EXERCISE 134 (Written)

1. The following places use Eastern time: Charleston, New York, Norfolk, Providence, Philadelphia, and Washington. Find the difference between the Standard time and the local time for each of these cities.

2. Austin, Bismarck, Chicago, Cincinnati, Detroit, Galveston, Louisville, New Orleans, Pensacola, Savannah, and St. Louis use Central time. Find the difference between the Standard time and the local time for each of these cities.

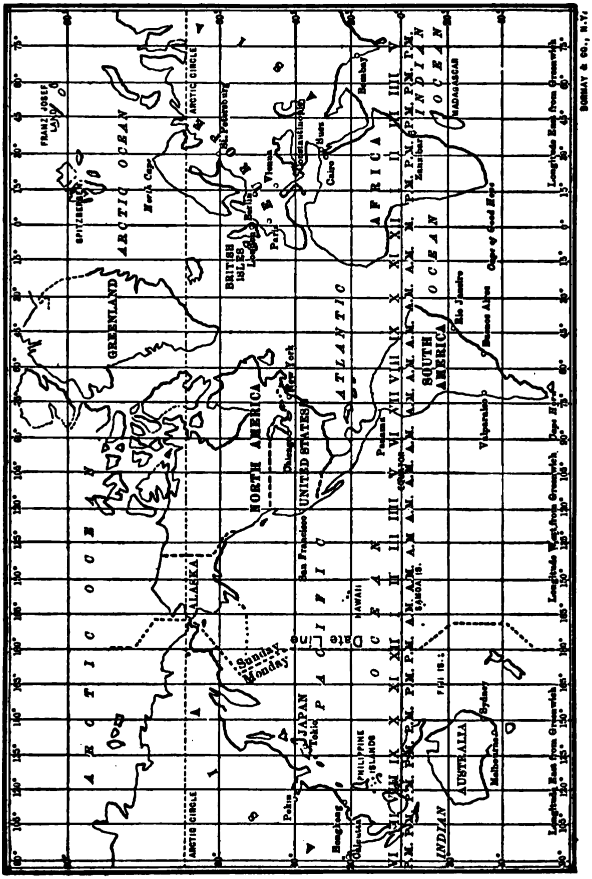
3. San Francisco and Portland, Oregon, use Pacific time. How many minutes, in each case, does this differ from local time?

4. Find the difference between the Standard time of San Francisco and Nashville; of San Francisco and Quincy, Ill.; of Santa Fe, New Mexico, and Cleveland; of Milwaukee and Denver; of Burlington, Iowa, and New Haven, Conn.

5. If a person travels from Rochester to Memphis, what change should he make in the time of his watch? If he travels from Richmond, Va., to Salt Lake City, what change should he make in his watch?

6. If a person travels from Fort Smith, Ark., to Portland, Me., what change should he make in his watch?

7. A person living in Omaha, Neb., finds that 24 min. must be added to the time of sunrise as given in the almanac to get the Standard time of sunrise. Determine the longitude of Omaha.



MAP SHOWING THE INTERNATIONAL DATE LINE

8. Clock time in Indianapolis is 16 min. earlier than local time. Determine the longitude of Indianapolis.

9. The sun rises March 6, in Kansas City, at 26 min. past 6 o'clock, local time. The Kansas City clocks indicate at that moment 45 min. past 6 o'clock. What is the longitude of Kansas City?

INTERNATIONAL DATE LINE

240. Consider next the local time of the following meridians: 15° E., 30° E., 45° E., 60° E., 75° E., and so on up to 180° E.

When it is noon in Greenwich, it is . . .	1 o'clock P.M. on the meridian,	15° E.
	2 o'clock P.M. on the meridian,	30° E.
	3 o'clock P.M. on the meridian,	45° E.
	4 o'clock P.M. on the meridian,	60° E.
	12 o'clock P.M. on the meridian,	180° E.

When it is noon in Greenwich, it is .	11 o'clock A.M. on the meridian,	15° W.
	10 o'clock A.M. on the meridian,	30° W.
	9 o'clock A.M. on the meridian,	45° W.
	8 o'clock A.M. on the meridian,	60° W.
	11 o'clock A.M. on the meridian,	165° W.
	12 o'clock A.M. on the meridian,	180° W.

The meridian 180° W. and the meridian 180° E. constitute one and the same meridian. Take any day, for example, March 2, 1903. When it is noon March 2, 1903, in Greenwich, it is, in places near the 180th meridian and just east of the 180th meridian, near the beginning of the day, March 2, 1903; and in places just west of the 180th meridian, it is near the end of the day, March 2, 1903, *i.e.* places on opposite sides of the 180th meridian differ in time by one day. This line of demarkation is known as the **International Date Line.** When it is Monday contiguous to this line and to the east, it is Tuesday across this line to the west.

A captain sailing in an easterly direction, on crossing the line is obliged to count one day twice. If his calendar says it is Monday when he reaches the 180th meridian, after he crosses the 180th meridian he is in territory where it is Sunday.

In sailing westward, a captain has to omit a day from his calendar. If it is Sunday when he comes to the 180th meridian, after he crosses it, he is in territory where it is Monday.

The International Date Line does not exactly coincide with the 180th meridian. The accompanying map shows the International Date Line.

EXERCISE 135 (Written)

1. Mariners carry on board ships chronometers which keep Greenwich time. When it is noon, local time, the chronometer indicates 4 hr. 48 min. P.M. What is the longitude of the ship?

2. When it is 10 o'clock P.M. March 2, in Washington, what is the Standard time in Manila? Melbourne? Berlin?

3. When it is 2 o'clock A.M., Standard time, in Denver, what is the Standard time of London? Manchester? Glasgow? Tien-tsin? Constantinople?

4. A telegram is sent from Madrid to Washington at 9 o'clock A.M. Allowing 1 hr. for transmission, when will it reach Washington? (See Example 7, Exercise 133.)

5. At noon, local time, a chronometer indicates 11 o'clock P.M. What is the longitude?

6. A telegram is sent from Galveston to London at 10 o'clock P.M. When will it be received, allowing 2 hr. for transmission?

FACTS ABOUT THE EARTH

Latitude is the distance in degrees north or south from the equator, measured on a meridian.

The following table directs attention to measurements on the earth's surface, of which everybody should have a clear conception :

	Length of 1° of Latitude	Length of 1° of Longitude		Length of 1° of Latitude	Length of 1° of Longitude
0°	362,746 ft.	365,230 ft.	50°	364,930 ft.	235,240 ft.
10°	362,860 ft.	359,720 ft.	60°	365,540 ft.	183,085 ft.
20°	363,180 ft.	343,340 ft.	70°	366,040 ft.	125,295 ft.
30°	363,675 ft.	316,570 ft.	80°	366,365 ft.	63,630 ft.
40°	364,280 ft.	280,180 ft.	90°	366,480 ft.	0 ft.

EXERCISE 136 (Written)

1. Express in miles and the decimal of a mile the length of 1° of latitude at the equator; at 20° N.; at 40° S.; at 60° N.; at 90°.

2. What is the length, in miles and the decimal of a mile, of 1° of longitude on the 20th parallel of latitude? on the 30th parallel of latitude? on the 40th parallel of latitude? on the 50th parallel of latitude? on the 60th parallel of latitude?

3. Find approximately the length of 1° of longitude in your own locality; of 1° of latitude.

4. Where does a degree of latitude attain its maximum length? Where its minimum?

5. Where is a degree of longitude of greatest length?

6. A knot is an arc of 1 min. on a meridian of the earth's surface. Calculate the length of 1 knot near the equator; at 40° N.; at 60° N.; at 80° N.; at 90° N.

7. What is the length in feet of an arc of 1″ on the 40th parallel of latitude? What is the length in feet of an arc of 5″ on the 50th parallel of latitude?

8. Two places on the 30th parallel of latitude are 3° 30′ apart. How many miles is one place from the other on a line due E.?

REVIEW QUESTIONS

1. Define meridian, longitude, latitude.

2. Do people who live east of you have noon earlier or later than you do?

3. The common schools are generally dismissed about three o'clock. At what places are they dismissed about the time school begins in your locality?

4. How would you find the number of degrees difference in longitude between two places on opposite sides of the first meridian?

5. How would you find the difference in time between two places, the difference in longitude being given?

6. How would you find the difference in longitude between two places, the difference in time being given?

7. If you are given the difference in time between two places and the longitude of one of them, how do you find the longitude of the other place?

8. What is Standard time?

9. Name the Standard time belts of the United States.

10. How are you situated with reference to a standard meridian, if your local time is faster than your Standard time?

11. When it is noon on May 1 on the Fiji Islands, what day of the month and hour of the day, Standard time, is it in Samoa?

CHAPTER XIV

MEASUREMENTS AND VALUES

241. A **triangle** is a portion of a plane bounded by three straight lines. *ABC* is a triangle.

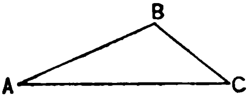


FIG. 1.

242. Two straight lines are **parallel** if they can never meet no matter how far they may be produced.

243. A **quadrilateral** is a portion of a plane bounded by four straight lines. Figure 2 represents a quadrilateral.

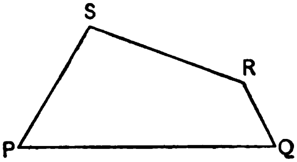


FIG. 2.

244. A quadrilateral having its opposite sides parallel is called a **parallelogram.** Figure 3 is a parallelogram.

The sides *AB*, *CD* are parallel. Also the sides *AD*, *BC* are parallel.

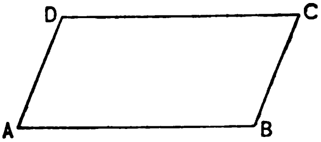


FIG. 3.

245. Consider next the parallelogram *ABCD* (Fig. 4) and the rectangle *ABKM*. They have a common side *AB*. By actual trial we find that the triangle *BKC* is equal to the triangle *AMD*. If we take

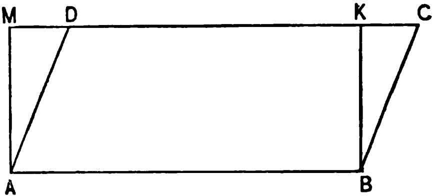


FIG. 4.

the triangle *AMD* from the figure *ABCM*, the parallelogram remains. If we take the triangle *BKC* from the figure *ABCM*, the rectangle remains. Hence, the rectangle equals the parallelogram in area. But the area of the rectangle is obtained by taking the product of its two dimensions, *i.e.* *AB* and *BK*. Therefore, the area of the parallelogram is equal to *AB* × *BK*.

AB is called the base of the parallelogram; *BK*, *i.e.* the distance between the parallel sides, is called the *altitude*, or *height*, of the parallelogram.

Consequently, the *area of a parallelogram* equals the product of its base by its altitude.

246. A quadrilateral having two sides parallel is called **a trapezoid.** *ABCD* (Fig. 5) is a trapezoid, having *AB* parallel to *DC*. *AB* and *CD* are respectively the lower and upper bases of the trapezoid.

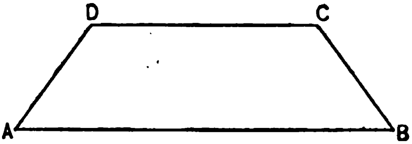


FIG. 5.

247. Take a piece of paper and make a trapezoid. Make another trapezoid just equal to it.

Place them as shown in Fig. 6. You then have a parallelogram *KBCL*. Its area is equal to (*KA* + *AB*) × height of the parallelogram, *i.e.* equal to (*DC* + *AB*) × height of the parallelogram.

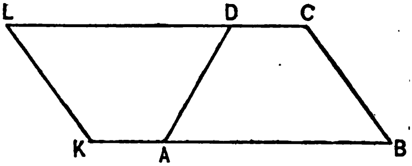


FIG. 6.

The trapezoid is $\frac{1}{2}$ of *KBCL*.

∴ The area of the trapezoid *ABCD* equals $\frac{1}{2}$ of (*DC* + *AB*) × the height of the trapezoid.

Consequently, the *area of a trapezoid* equals one-half the sum of its parallel sides multiplied by the distance between them.

248. If one of the bases, *e.g.* DC, of a trapezoid were to become smaller and smaller, we should ultimately have a triangle. Hence, the *area of a triangle* equals one-half of its base multiplied by its height.

BOARD MEASURE

249. Lumber is measured by the board foot. A **board foot** is a rectangular solid 12 in. by 12 in. and 1 in. in height.

If a board is 1 in. thick, the number of board feet in it is measured by the number of square feet in its upper or lower surface. Thus, a board 8 ft. long, 8 in. wide, and 1 in. thick contains $8 \times \frac{2}{3}$ board feet. If a board is more than 1 in. thick, the number of board feet in it is measured by the area, in square feet, of its upper or lower surface multiplied by the number denoting the thickness in inches. Thus, a board 9 ft. by 14 in. and $2\frac{1}{2}$ in. thick contains $9 \times \frac{14}{12} \times 2\frac{1}{2}$ board feet.

The number of board feet in a board less than 1 in. in thickness is measured by the number of square feet in its upper or lower surface. Thus: A board 10 ft. by 15 in. and $\frac{2}{3}$ in. thick contains $10 \times \frac{15}{12}$ board feet.

EXERCISE 137 (Oral)

1. How many board feet are in a board 8 ft. by $1\frac{1}{2}$ ft. and 1 in. thick?

2. How many board feet are in a board 9 ft. by 16 in. and $\frac{3}{4}$ in. thick? Find the cost of the board at $3\frac{1}{2}$¢ per board foot.

3. How many board feet are in a plank 24 ft. by 15 in. and 2 in. thick? In a board 20 ft. by 12 in. and 3 in. thick?

4. How many board feet are in a railroad tie 24 ft. by 8 in. and 6 in. thick?

5. Find the number of board feet in each of the following pieces of lumber:

(*a*) 18 ft. by 16 in. and 1½ in. thick.
(*b*) 12 ft. by 8 in. and 6 in. thick.
(*c*) 24 ft. by 9 in. and 3 in. thick.
(*d*) 16 ft. by 8 in. and 4 in. thick.
(*e*) 18 ft. by 12 in. and 3 in. thick.
(*f*) 21 ft. by 16 in. and 3 in. thick.
(*g*) 30 ft. by 14 in. and 2½ in. thick.
(*h*) 28 ft. by 15 in. and 3½ in. thick.

EXERCISE 138 (Written)

1. Find the cost of 480 boards, each 1½ in. by 10 in. and 16 ft. long, @ $27.50 per M. ("Per M" means "by the 1000" board feet.)

2. Find the cost of 840 boards, each 1¾ in. by 10 in. and 12 ft. long each, @ $25 per M.

3. How many board feet are in a board 24 ft. long, 12 in. wide at one end and 16 in. wide at the other end, and 2¾ in. thick?

4. How many board feet are in a board 18 in. wide at one end, 12 in. wide at the other end, 2½ in. thick, and 28 ft. long?

5. How many board feet are in a cubical block of wood, each of whose dimensions is 2 ft.?

6. How many board feet are in a beam 9 in. by 9 in. and 54 ft. long?

7. How many cubic feet are equivalent to 120 board feet?

8. How many board feet are in a beam 11 in. by 12 in. and 36 ft. long?

9. A piece of lumber contains 2980 cu. in. Express this in board feet, understanding that the piece is more than 1 in. in thickness.

10. Find the number of board feet in a beam 12 in. by 6 in. at one end, 9 in. by 6 in. at the other end, and 27 ft. long?

MASONRY AND BRICKLAYING

250. **Stone work** is estimated by the perch, also by the cubic foot.

251. A perch of masonry is 24¾ cu. ft. It is a wall 1 rd. long, 1½ ft. wide, and 1 ft. high. Perch is, in Great Britain, another name for rod. Of the 24¾ cu. ft. in a perch, 22 cu. ft. are allowed for stone and 2¾ cu. ft. are allowed for mortar, *i.e.* eight-ninths for stone and one-ninth for mortar.

In estimating the number of perches of masonry in the walls of a building, the outside dimensions of the walls are taken. This method of reckoning counts the corners twice. In estimating the amount of material, the computation should be exact, inside and outside dimensions being reckoned.

252. Twenty-two common bricks, *i.e.* bricks 8 in. by 4 in. by 2 in., are reckoned as a cubic foot.

EXERCISE 139 (Written)

1. How many perches of masonry are in a wall 84 ft. long, 16 ft. high, and 1½ ft. thick? How many common bricks are required to build this wall?

2. A cellar is 36 ft. long, 18 ft. wide, and 8 ft. deep. How many cubic feet of masonry are in its walls if they are 2 ft. thick? What is the actual number of cubic feet in the walls?

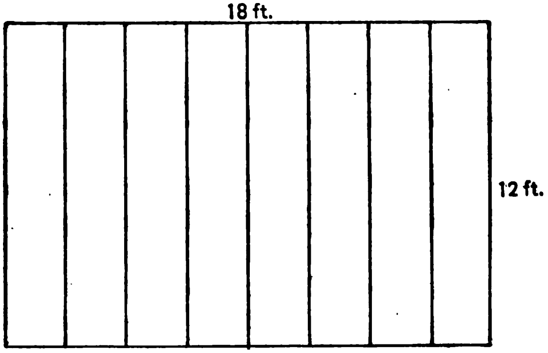
3. How many bricks are necessary to build a wall 25 ft. long, 12 ft. high, and 24 in. in thickness? What is the cost of the bricks at $9 per M?

4. How many cubic feet of mortar are required to build a wall 600 ft. long, 10 ft. high, and 18 in. in thickness? How many common bricks are required to build this wall?

5. At $9.50 per M, what is the cost of the bricks required to build a house 40 ft. by 36 ft. and 40 ft. high to the eaves, the highest point of the gable end being 52 ft. above the ground, and the walls 1½ ft. thick? Allow 270 cu. ft. for openings.

CARPETING

253. 1. How many yards of carpet are needed for a room 18 ft. by 12 ft., if the carpet is 27 in. wide?



SOLUTION. (1) Let the carpet be placed crosswise.

Number of strips = 18 ft. ÷ 27 in. = 18 ft. ÷ $2\frac{1}{4}$ ft. = 8.

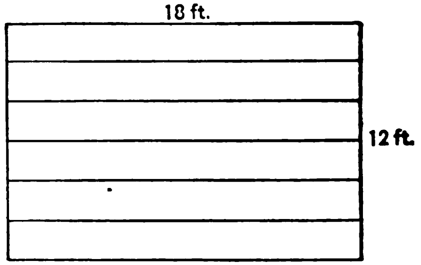
∴ Number of yards = 12 × 8 ÷ 3 = 32. *Ans.* 32 yd.

(2) Let the carpet be placed lengthwise.

Number of strips = 12 ft. ÷ $2\frac{1}{4}$ ft. = $5\frac{1}{3}$.

Here 6 strips are needed. The fraction of a strip may be either cut off or turned under.

∴ Number of yards = 6 yd. × 6 = 36 yd. *Ans.* 36 yd.



To find the number of yards of carpet required to carpet a room:

1. Draw a diagram of the room.
2. Find the number of strips.
3. Multiply the number of strips by the number of yards in one strip.

EXERCISE 140 (Written)

1. How many yards of carpet are required to cover a room 18 ft. by 16 ft. with carpet 30 in. wide, if the strips are laid crosswise?

2. How many yards of carpet $\frac{3}{4}$ of a yard wide are needed to carpet a room 24 ft. by 18 ft., if the carpet runs lengthwise?

3. How many yards of matting 36 in. wide are required to cover a room 16 ft. by 12 ft., the matting being laid crosswise?

4. How many yards of carpet 30 in. wide are needed to carpet a room 24 ft. long, 20 ft. wide, if the carpet is laid crosswise?

5. Find the cost of carpeting a room 16 ft. by 14 ft. with carpet 27 in. wide at 90¢ a yard, the strips being laid lengthwise.

FOREIGN MONEY

254. The unit of money in the British Isles is the **sovereign**, or **pound sterling.** Its value in United States money is $4.8665.

ENGLISH MONEY

4 farthings = 1 penny (*d.*)
12 pence = 1 shilling (*s.*)
20 shillings = 1 pound sterling (£)
5 shillings = 1 crown
21 shillings = 1 guinea
£ 1 = 20*s.* = 240*d.* = 960 farthings

The abbreviations £, *s.*, *d.*, are the initial letters of the names of the Roman coins, libra, solidus, denarius.

EXERCISE 141 (Oral)

1. How many pence are in 2*s.*? in 4*s.*? in 8*s.*? in 10*s.*?

2. How many shillings are in $\frac{1}{2}$ sovereign? in £ 2? in £ 2 10*s.*? in £ 5 10*s.*?

3. Reduce to shillings 24*d.*; 120*d.*; 144*d.*

4. Reduce $\frac{1}{8}$ of a sovereign to lower denominations.

5. What is the value in United States money of $\frac{1}{2}$ of a sovereign?

6. What part of £ 2 is 5*s.*? What part of £ 1 is 2*s.* 6*d.*? What part of £ 1 is 6*s.* 8*d.*?

7. What decimal of £ 1 is 3*s.* 4*d.*? 8*s.*? 12*s.*?

8. To how many shillings is $\frac{3}{4}$ of a sovereign equivalent?

255. *Example* **1.** Reduce 17*s.* $6\frac{1}{2}$*d.* to the decimal of a pound sterling.

SOLUTION. 17*s.* 6½*d.*
12
210½*d.*
4
842 farthings.

842 farthings = $\frac{842}{960}$ of £1 = $\frac{84.2}{96}$ of £1 = £.8771.

$$\begin{array}{r|l} 12 & 84.2 \\ \hline 8 & 7.01666 \\ \hline & .8770833 \end{array}$$

EXERCISE 142 (Written)

1. Reduce 13*s.* 6*d.* to the decimal of £1.
2. Reduce 12*s.* 6*d.* to the decimal of £1.
3. Reduce 17*s.* 8¾*d.* to the decimal of £1.
4. Reduce 18*s.* 7½*d.* to the decimal of £1.
5. Convert £5 12*s.* 6*d.* into United States currency.
6. Convert £8 11*s.* 8*d.* into United States currency.
7. Convert £9 0*s.* 6d. into United States currency.
8. Change $114.85 into English currency.
9. Change $175.15 into English currency.
10. Express $1 as a decimal of £1.
11. Express $1 in shillings and the decimal of a shilling.
12. Express 1*s.* as a decimal of $1.
13. Find in United States currency the difference between 25¢ and 1*s.*
14. A man earns 3*s.* 9*d.* a day. Find the least number of days in which he can earn an exact number of half-sovereigns.
15. When tea sells for 2*s.* 8*d.* per lb., what is the least number of pounds corresponding in value to an exact number of sovereigns?
16. From £20 take £14 14*s.* 10½*d.*
17. Divide £125 10*s.* 8*d.* by 8.
18. When yearlings sell for £2 17*s.* 6*d.*, how many can be bought for £71 17*s.* 6*d.*?

FOREIGN COINS AND THEIR VALUES

Country	Standard	Monetary Unit	Value	Other Coins
Argentine Republic	double	1 peso = 100 centavos	gold, $.96476 silver, $.9352	argentine, $4.8238
Austria	gold	1 krone = 100 heller	gold, $.20265 silver, $.1735	ducat, $2.2878
Belgium	double	1 franc = 100 centimes	gold, $.19295 silver, $.1735	
Brazil	gold	1 milreis = 1000 reis	gold, $.54636 silver, $.486	
Central American States, except Costa Rica	silver	1 peso = 100 centavos	silver, $.384	doubloon, $15.7389
Costa Rica	gold	1 colon = 100 centimes	gold, 2 colons = $.9307	
Chile	gold	1 peso = 100 centavos	gold, $.365	condor, $7.2998 doubloon, $3.649 escudo, $1.824
Cuba	double	1 peseta = 100 centimes	gold, $.19295 silver, $.1735	peso, $.926
Denmark	gold	1 krone = 100 ore	gold, $.26799 silver, $.249	
Egypt	gold	1 pound = 100 piastres	gold, $4.943	
France	double	1 franc = 100 centimes	gold, $.19295 silver, $.1735	
German Empire	gold	1 mark = 100 pfennigs	gold, $.23821 silver, $.2078	krone, $2.3821
Great Britain	gold	1 pound sterling	gold, $4.8665	
Greece	double	drachma	gold, $.193	
Italy	double	1 lira = 100 centesime	gold, $.193	
Japan	gold	yen	gold, $.4984	
Mexico	double	1 dollar or peso = 100 centavos	gold, $.9839	

Country	Standard	Monetary Unit	Value	Other Coins
Netherlands .	double	1 florin = 100 cents	gold, $.40195 silver, $.3928	
Norway . . .	gold	1 krone = 100 ore	gold, $.26799	
Peru	gold	1 sol = 100 centavos	gold, $.48665	
Portugal . . .	gold	1 milreis = 1000 reis	gold, $1.0804	
Russia . . .	gold	1 ruble = 100 kopecks	gold, $.5146 silver, $.748	
Spain	double	1 peseta = 100 centimos	gold, $.19295 silver, $.1735	
Sweden . . .	gold	1 krona = 100 öre	gold, $.26799 silver, $.2494	
Switzerland . .	double	1 franc = 100 centimes	gold, $.19295 silver, $.1735	
Venezuela . .	double	1 Bolivar	gold, $.19295 silver, $.1735	

Silver coins fluctuate in value. The value of a foreign silver coin depends upon the market value of silver and the amount of silver the coin contains.

EXERCISE 143 (Written)

1. Express the value of £1 in French currency; in German currency; in Austrian currency.

2. Change to United States currency (*a*) 450 francs, (*b*) 550 pesetas, (*c*) 280 Norwegian krones, (*d*) 900 marks, (*e*) 200 colons, (*f*) 720 Austrian krones, (*g*) 450 Danish krones.

3. Find in United States currency the difference in value between 1000 francs and $193.

4. Find the value of $1000 (*a*) in the money of Costa Rica; (*b*) in Austrian currency; (*c*) in Cuban currency; (*d*) in Swedish currency.

5. What is the equivalent in United States money of 200,000 bolivars (Venezuela)?

6. What is the equivalent in United States money of 10,000 Mexican gold dollars?

7. Express $1 in Peruvian monetary units.

8. Express $1 in Norwegian monetary units.

9. Find in United States currency the difference between the value of 10,000 Japanese yen and $5000.

10. What is the value of 10,000 Portuguese milreis?

SPECIFIC GRAVITY

256. The ratio of the weight of any given volume of a substance to the weight of an equal volume of another substance taken as a standard is called the **specific gravity** of that substance.

The **standard** taken for solids and liquids is water. Specific gravity, then, simply means how many times as heavy as water a substance is. Thus, cast iron is 7.21 times as heavy as water, and hence its specific gravity is 7.21. Cork is about one-fourth as heavy as water, and hence its specific gravity is $\frac{1}{4}$.

257. TABLE OF SPECIFIC GRAVITIES

Ash . . .	.84	Ebony .	1.33	Steel . .	7.83	Clay . .	1.2
Beech . .	.85	Glass . .	2.89	Copper .	8.95	Mercury .	13.57
Brass . .	8.40	Gold . .	19.26	Silver . .	10.47	Bar iron .	7.79
Butter . .	.94	Granite .	2.78	Lead . .	11.38	Platina .	21.5
		Ice . . .	.92				

EXERCISE 144 (Written)

1. A cubic foot of water weighs 1000 oz. Find the weight of 1 cu. ft. of steel.

2. Find the weight of 1 cu. in. of water. Find the weight of 1 gal. of water.

3. Find the weight of 1 bushel measure filled with water.

4. A cubic foot of marble weighs 2700 oz. Find the specific gravity of marble.

5. A cubic foot of sea water weighs $64\frac{3}{4}$ lb. Find the specific gravity of sea water.

6. A cubic foot of goat's milk weighs 65 lb. Find how many times as heavy as water goat's milk is.

7. The mercury in the barometer exactly counterbalances the pressure of the atmosphere. If the barometer is 30 in. high, find the pressure of atmosphere upon every square inch of surface.

8. A swimming pool of fresh water is 25 ft. by 16 ft. and 5 ft. deep. Find the weight of the water it contains.

9. A block of granite is 6 ft. by 4 ft. and 2 ft. thick. Find its weight in tons.

10. Find the weight of 1 cu. in. of gold; of 1 cu. in. of silver; of 1 cu. in. of platina; of 1 cu. in. of lead.

11. A block of ice 3 ft. by 2 ft. and 1 ft. thick weighs how many pounds?

12. What is the weight in tons of 1 cu. yd. of clay?

13. Find the weight of the butter required to fill a box 16 in. by 9 in. by 8 in.

14. A cellar is 18 ft. by 12 ft. and 8 ft. deep. Find the weight of the water required to fill it. The weight of 1 cu. ft. of air is $\frac{1}{773}$ of the weight of 1 cu. ft. of water. Find the weight of the air required to fill the cellar.

15. Find the weight of the air in a hall 27 ft. by 24 ft. and 15.46 ft. high.

16. A cubic foot of coal weighs $81\frac{1}{4}$ lb. Find the specific gravity of coal.

17. How many cubic inches of copper weigh just as much as 1 cu. in. of platina?

18. Find the weight of a block of ebony 4 ft. long, 9 in. wide, and 8 in. thick.

19. Find the weight of a block of ash 12 in. long, 8 in. wide, and 6 in. thick.

20. How many times as heavy as glass is mercury?

21. What is the weight of a bar of iron 2 in. by 2 in. and 8 ft. long?

22. Find the weight of a beam of beech wood 8 in. by 6 in. and 12 ft. long.

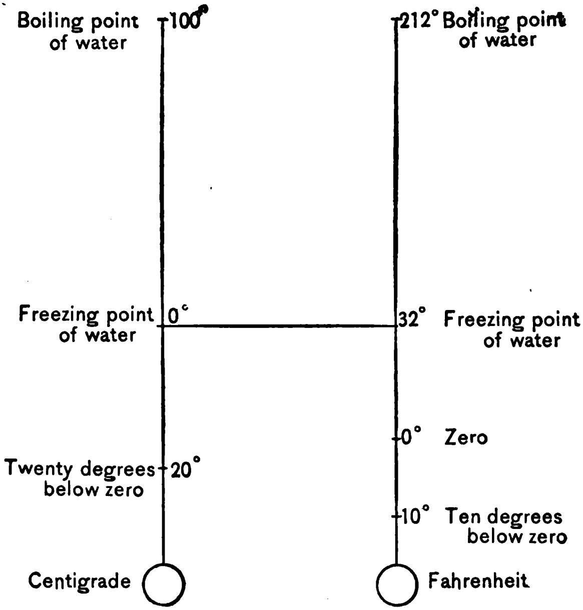
23. Which is the heavier, 3 cu. ft. of steel or 8 cu. ft. of glass?

MEASURE OF TEMPERATURE

258. A **thermometer** is an instrument for measuring heat. It consists of a glass tube ending in a bulb. The tube is carefully graduated, and is partially filled with mercury or some other liquid. The mercury thermometers are the only ones in common use. The principle of the thermometer is that substances expand with heat, according to a natural law.

259. There are two different styles of thermometer in general use, — the Centigrade and the Fahrenheit. The Centigrade thermometer marks the melting point of ice 0°, and the boiling point of water 100°. The interval between these points is divided into 100 parts or degrees, so that the change in the volume of the mercury between any two consecutive marks is $\frac{1}{100}$ of the change from 0° to 100°. The Centigrade thermometer was first proposed by Celsius, a Swedish astronomer, who lived in the first half of the eighteenth century. It is the only thermometer used in science.

To the ordinary individual, the best known thermometer is Fahrenheit's. This thermometer divides the interval from the melting point of ice to the boiling point of water into 180°. It marks the melting point of ice 32°, and the boiling point of water 212°.


Boiling point of water
100°
212° Boiling point of water
Freezing point of water
0°
32° Freezing point of water
0° Zero
Twenty degrees below zero
20°
10° Ten degrees below zero
Centigrade
Fahrenheit

Fahrenheit, the inventor of the thermometer which bears his name, took for the zero of his scale the lowest temperature observed by him at Dantzic in the winter of 1709, which resulted from mixing snow and sal ammoniac.

260. Notation. 92° C. means 92 degrees on the Centigrade thermometer.

45° Fahr. means 45 degrees on Fahrenheit's thermometer.

+ 10° means 10 degrees above zero.

– 10° means 10 degrees below zero.

261. *Example* 1. Express 92° Fahr. on the Centigrade scale.

SOLUTION. 92° Fahr. is (92° – 32°) Fahr. above the melting point of ice. Hence, we have to express the value of this difference, namely, 60° Fahr. on the Centigrade scale.

$$180° \text{ Fahr.} = 100° \text{ C.}$$

$$\therefore 1° \text{ Fahr.} = \tfrac{100}{180}° \text{ C.} = \tfrac{5}{9}° \text{ C.}$$

$$\therefore 60° \text{ Fahr.} = \tfrac{300}{9}° \text{ C.} = 33\tfrac{1}{3}° \text{ C.}$$

$$\therefore 92° \text{ Fahr.} = 33\tfrac{1}{3}° \text{ C.}$$

Example 2. Express –8° Fahr. on the Centigrade scale.

SOLUTION. –8° Fahr. is 40° Fahr. below freezing.

$$1° \text{ Fahr.} = \tfrac{5}{9}° \text{ C.}$$

$$\therefore 40° \text{ Fahr.} = \tfrac{200}{9} \text{ C.} = 22\tfrac{2}{9}° \text{ C.}$$

$$\therefore 40° \text{ Fahr. below freezing is } -22\tfrac{2}{9}° \text{ C.}$$

$$\therefore -8° \text{ Fahr.} = -22\tfrac{2}{9}° \text{ C.}$$

EXERCISE 145 (Written)

1. Express 86° Fahr. on the Centigrade scale.
2. Express 77° Fahr. on the Centigrade scale.
3. Express 203° Fahr. on the Centigrade scale.
4. Express 248° Fahr. on the Centigrade scale.
5. Express 68° Fahr. on the Centigrade scale.
6. Express 54° Fahr. on the Centigrade scale.
7. Express 38° Fahr. on the Centigrade scale.
8. Express 23° Fahr. on the Centigrade scale.
9. Express 14° Fahr. on the Centigrade scale.
10. Express –13° Fahr. on the Centigrade scale.
11. Express –40° Fahr. on the Centigrade scale.
12. Express –90° Fahr. on the Centigrade scale.

262. *Example* **1.** Express 48° C. on the Fahrenheit scale.

SOLUTION. $100° \text{ C.} = 180° \text{ Fahr.}$

$$\therefore 1° \text{ C.} = \tfrac{9}{5}° \text{ Fahr.}$$

$$\therefore 48° \text{ C.} = \tfrac{432}{5}° \text{ Fahr.} = 86\tfrac{2}{5}° \text{ Fahr.}$$

Hence, the result is $86\frac{2}{5}°$ Fahr. above freezing. $\therefore 48° \text{ C.} = (86\frac{2}{5}° + 32°) \text{ Fahr.} = 118\frac{2}{5}° \text{ Fahr.}$

Example **2.** Express $-20°$ C. on the Fahrenheit scale.

SOLUTION. $1° \text{ C.} = \tfrac{9}{5}° \text{ Fahr.}$

$$\therefore 20° \text{ C.} = \tfrac{180}{5}° \text{ Fahr.} = 36° \text{ Fahr.}$$

Hence, the result is 36° Fahr. below freezing. But 32° Fahr. below freezing is 0° Fahr.

$$\therefore 36° \text{ Fahr. below freezing is } -4° \text{ Fahr.}$$

$$\therefore -20° \text{ C.} = -4° \text{ Fahr.}$$

EXERCISE 146 (Written)

1. Express 35° C. on the Fahrenheit scale.
2. Express 55° C. on the Fahrenheit scale.
3. Express 25° C. on the Fahrenheit scale.
4. Express 20° C. on the Fahrenheit scale.
5. Express 18° C. on the Fahrenheit scale.
6. Express 8° C. on the Fahrenheit scale.
7. Express $-10°$ C. on the Fahrenheit scale.
8. Express $-20°$ C. on the Fahrenheit scale.
9. Express $-14°$ C. on the Fahrenheit scale.
10. Express $-18°$ C. on the Fahrenheit scale.
11. Express $-24°$ C. on the Fahrenheit scale.
12. Express $-273°$ C. on the Fahrenheit scale.

TABLE OF MELTING POINTS

Mercury	−40° C.	Lead	326° C.	Gold	1035° C.
Sulphur	113° C.	Zinc	415° C.	Cast iron	1100° to 1200° C.

13. Give the above table in the Fahrenheit scale.

14. There is a temperature below zero that is indicated by the same number of degrees on the Centigrade and the Fahrenheit scale. Find it.

15. Water attains its maximum density at 4° C. Express this temperature on Fahrenheit's scale.

REVIEW QUESTIONS

1. Define triangle, quadrilateral, parallelogram, trapezoid, parallel lines.

2. How do you find the area of a parallelogram? of a trapezoid? of a triangle?

3. What is a board foot?

4. How is the number of board feet in a piece of lumber obtained?

5. What is a perch of masonry?

6. What is the monetary unit in Great Britain? Give its equivalent in United States money.

7. How do you find the value of British money in United States currency?

8. What is specific gravity?

9. What is the standard used in estimating the specific gravity of solids and liquids?

10. What is a thermometer?

11. What are the two thermometers in common use?

12. How do they differ?

13. How do you express Fahrenheit degrees on the Centigrade scale?

14. How do you express Centigrade degrees on the Fahrenheit scale?

15. How many degrees are there between the boiling and freezing points on the Fahrenheit scale?

16. Why is the Centigrade thermometer so called?

17. Why is the Fahrenheit thermometer so called?

18. Who was the inventor of the Centigrade thermometer?

CHAPTER XV

RATIO, PROPORTIONAL PARTS, WORK, MOTION, ETC.

NOTE. We shall give in this chapter some problems which have been current for centuries. We shall, so far as we go, classify these problems and show how to solve them. We shall also give examples which are not so old. The difficult examples in this chapter are not intended for grammar grade pupils; they are placed here to gratify the desires of those who take interest in problem solving.

263. The first number in a ratio is called the **antecedent**, and the second number is called the **consequent.**

Since a ratio is a quotient, we may multiply antecedent and consequent by the same number without affecting the value of the ratio. It follows also that the terms of a ratio must be both abstract, or both concrete.

264. Two numbers vary directly if an increase in one of them produces a corresponding increase in the other. Thus, the distance a train travels in any given time varies directly with the number of miles it goes in one hour. An increase in the rate increases the distance.

The price of any number of bushels of corn varies directly with the price of one bushel of corn; the greater the price of one bushel, the greater the price of a number of bushels.

265. Two numbers vary inversely if an increase in one produces a corresponding decrease in the other number. Thus, the number of hours a train takes to run a certain distance varies inversely with the number of miles it runs in one hour. The greater the number of hours, the smaller the rate of running.

Take a rectangle, let its area be fixed; then the width of the rectangle varies inversely as its length; the greater the length, the smaller is the width.

266. *Example* 1. Divide 343 in the ratio 4:3.

SOLUTION. The first part $= \frac{4}{3}$ of the second part $= 1\frac{1}{3}$ times second part.

$\therefore$ First part + second part $= 2\frac{1}{3}$ times second part.

$\therefore$ $2\frac{1}{3}$ times second part $= 343$.

$\therefore$ Second part $= 343 \div 2\frac{1}{3} = 147$.

$\therefore$ First part $= 147 \times \frac{4}{3} = 196$.

ANOTHER SOLUTION. The first part contains 4 as many times as the second part contains 3. Hence, out of every 7 in 343, first part contains 4 and second part 3.

$\therefore$ First part $= \frac{4}{7}$ of 343.

$\therefore$ Second part $= \frac{3}{7}$ of 343.

Example 2. Divide 136 into parts proportional to 2, 3, 4.

SOLUTION. $2 + 3 + 4 = 9$.

$\therefore$ First part $= \frac{2}{9}$ of 136.

$\therefore$ Second part $= \frac{3}{9}$ of 136.

$\therefore$ Third part $= \frac{4}{9}$ of 136.

Example 3. A rows downstream at the rate of $8\frac{2}{5}$ miles per hour, and back at the rate of $3\frac{1}{2}$ miles per hour. How far may he row so as to be gone $8\frac{1}{2}$ hours in all?

SOLUTION. We shall first determine the time of rowing down, and the time of rowing back.

The time of rowing 1 mi. down $= (1 \div 8\frac{2}{5})$ hr. $= \frac{5}{42}$ hr.

The time of rowing 1 mi. back $= (1 \div 3\frac{1}{2})$ hr. $= \frac{2}{7}$ hr.

But, time down and time up vary directly with the time of going 1 mi.

∴ We divide $8\frac{1}{2}$ in the ratio $\frac{5}{42}:\frac{2}{7}$, or (multiplying the terms of the ratio by 42), we divide $8\frac{1}{2}$ in the ratio 5:12.

∴ Time down $= \frac{5}{17}$ of $8\frac{1}{2}$ hr. $= 2\frac{1}{2}$ hr.
∴ Time back $= \frac{12}{17}$ of $8\frac{1}{2}$ hr. $= 6$ hr.
∴ Distance down $= 2\frac{1}{2} \times 8\frac{2}{5}$ mi. $= 21$ mi.

EXERCISE 147 (Written)

1. Divide $130 between two persons in the ratio 8:5.
2. Divide $985 in the ratio 3:2.
3. Divide $3240 in the ratio 4:5.
4. Divide $7128 proportional to 1, 2, 3.
5. Divide $7225 proportional to 4, 6, 7.
6. Divide 2916 proportional to 7, 9, 11.
7. Divide 4745 proportional to 5, 6, 7, 8.
8. Divide 6728 proportional to 13, 14, 15, 16.
9. Divide 5040 proportional to 21, 22, 23, 24.
10. Divide 2592 proportional to 15, 17, 19, 21.
11. Divide 95 in the ratio $\frac{1}{2}:\frac{1}{3}$.
12. Divide 1736 in the ratio $\frac{1}{3}:\frac{1}{4}$.
13. Divide 365 in the ratio $2\frac{1}{3}:3\frac{3}{4}$.
14. Divide 1521 in the ratio $7\frac{1}{2}:4\frac{1}{5}$.
15. Divide 4225 proportional to $1\frac{1}{2}$, $1\frac{2}{3}$, $2\frac{1}{4}$.
16. Divide 2189 proportional to $1\frac{1}{5}$, $2\frac{1}{3}$, $3\frac{1}{10}$.

17. A, B, and C enter into partnership, A contributing $2450, B, $3500, and C, $4000. They gain $1990. Find each person's share of the profits, supposing each person's gain is proportional to his investment.

18. Divide $2700 among A, B, and C, so that A shall receive $4 as often as B receives $5 and C receives $6.

19. The silver coins of Great Britain are coined out of standard silver which contains 37 parts of pure silver to 3 parts of copper. If a Troy pound of standard silver makes 66 shillings, find the number of grains of pure silver in a crown. (1 crown = 5*s.*)

20. Divide $1620 among A, B, and C, so that A shall receive as much as B and C together, and B shall receive $\frac{2}{7}$ of the shares of A and C together.

21. A father bequeathes his property, valued at $13,000, to his two sons and three daughters, so that each daughter shall receive an equal portion, and each son two-thirds as much as a daughter. Find the share of each daughter.

22. A, B, and C purchase a farm. A pays $\frac{7}{10}$ of the total cost, B, $\frac{1}{6}$ of the total cost, and C, the remainder, amounting to $4000. Find the cost of the farm.

23. A and B own an oil well valued at $30,000. A's interest is $\frac{3}{5}$ and B's, $\frac{2}{5}$. They agree to take C as a partner, on the condition that each shall, in future, own $\frac{1}{3}$ of the well. If C pays them $10,000, how shall they divide the money?

24. Two men, A and B, rent a tract of land for grazing purposes, paying therefor $750. A puts in 200 head of cattle and B, 400 sheep. How much rent should each pay, if 2 head of cattle are counted as an equivalent of 11 sheep?

25. Common salt is composed of 71 parts of chlorine to 46 parts of sodium. Find the weight of each element in 585 lb. of common salt.

26. If 3 men can earn as much as 5 boys, and 7 men and 10 boys working together earn $520, find the share of a man and of a boy.

27. Six turkeys and 11 ducks cost \$16. Find the cost of a turkey and of a duck, if 2 turkeys cost as much as 7 ducks.

28. How far may a person ride in a carriage at the rate of 9 mi. an hour so that he may return in $4\frac{1}{2}$ hr., traveling back at the rate of $4\frac{1}{2}$ mi. an hour?

29. A man rows down a stream at the rate of 8 mi. an hour, and up the stream at the rate of 3 mi. an hour. How far may he row so as to return to the starting point in 11 hr.?

267. *Example* 1. How many men are required to mow 160 A. in 16 da., if 27 men mow 90 A. in 7 da.?

SOLUTION. 27 men mow 90 A. in 7 da.

$\therefore$ 27 men mow $\frac{90\text{ A.}}{7}$ in 1 da.

$\therefore$ 1 man mows $\frac{90\text{ A.}}{7 \times 27}$, *i.e.* $\frac{10\text{ A.}}{21}$ in 1 da.

$\therefore$ 1 man mows $\frac{10\text{ A.}}{21} \times 16$ in 16 da.

$\therefore$ 1 man mows $\frac{160\text{ A.}}{21}$ in 16 da.

$\therefore$ number of men required to mow 160 A. in 16 da. = $160 \div \frac{160}{21} = 21$. *Ans.* 21 men.

Example 2. If 28 horses can plow 87 A. in 11 da., how much can 21 horses plow in $5\frac{1}{2}$ da.?

ANALYTICAL SOLUTION. 28 horses plow 87 A. in 11 da.

$\therefore$ 28 horses plow $\frac{87\text{ A.}}{11}$ in 1 da.

$\therefore$ 1 horse plows $\frac{87\text{ A.}}{11 \times 28}$ in 1 da.

$$\therefore\ 21\text{ horses plow }\frac{87\text{ A.}\times 21}{11\times 28}\text{ in 1 da.}$$

$$\therefore\ 21\text{ horses plow }\frac{87\text{ A.}\times 21}{11\times 28}\times\frac{11}{2}\text{ in }5\tfrac{1}{2}\text{ da.}$$

$$\frac{87\text{ A.}\times \overset{3}{\cancel{21}}}{\cancel{11}\times \underset{4}{\cancel{28}}}\times\frac{\cancel{11}}{2}=32\tfrac{5}{8}\text{ A., }\textit{Ans.}$$

EXERCISE 148 (Written)

Solve analytically:

1. If 19 bu. of wheat cost $14.25, find the price of 11 bu. of wheat.

2. Seven men can dig a trench in 16 da. How long will it take 10 men to do the same work?

3. If 11 sheep cost $71.50, find the cost of 17 sheep at the same rate.

4. If 14 men can do a piece of work in 11 da., how long will it take 21 men to do the same work?

5. A garrison of 2500 men has provisions for 6 wk. How long will the provisions last if the garrison is reënforced by 500 men?

6. If 24 men do a piece of work in 16 da., how long will it take 36 men to do twice as much work?

7. A contractor finds that it will take 20 masons 18 da. to build a wall. How many additional men must he employ in order to complete the wall in 15 da.?

8. If 6 men or 8 boys do a piece of work in 14 da., how long will it take 4 boys and 4 men working together to do the same work?

9. If 5 cows cost as much as 2 horses, and 5 horses cost $875, find the cost of 7 cows.

10. Sixteen men can do a piece of work in 9 da.; 15 men and 15 boys can do the same work in 6 da. How long will it take 30 boys to do the work?

11. If 24 men can build $\frac{3}{8}$ of a wall in 15 da., how many men can build $\frac{1}{2}$ of the remaining portion in 15 da.?

12. If $\frac{4}{11}$ cwt. cost $1.92, find the cost of $\frac{5}{12}$ T. of the same commodity.

13. If 2 cwt. 75 lb. of sugar cost $13.75, how much sugar could be bought for $23.85?

14. If 42 lb. of wheat cost $56\frac{1}{4}$¢, find the cost of 19 cwt. 74 lb.

15. The hotel bill, for board and rooms, of 4 men for 9 da. is $108; at this rate, find the bill for 7 men for 7 da.

16. If 5 T. of rock are carried 120 mi. for $4.50, find the cost of carrying 110 T. of rock 180 mi.

17. If 5 men in 8 da. earn $60, at the same rate how much will 9 men earn in 6 da.?

18. Twelve men build a wall in 10 da. How many men can build twice as large a wall in $\frac{3}{5}$ of the time?

19. If 6 men in 4 da. of 8 hr. each, earn $76.80, at this rate how much will 11 men earn in 5 da. of 9 hr. each?

20. Five men can pave a street 30 ft. long and 24 ft. wide in 2 da. How many men will it take to pave a street 200 ft. long 36 ft. wide, in $12\frac{1}{2}$ da.?

21. If the freight on 5 T. 18 cwt. for 84 mi. is $28, find how far 1 T. 2 cwt. may be carried for $11.

22. If 1 T. 5 cwt. is carried 147 mi. for $49, how far should 10 T. be carried for $368?

23. If 85 bu. of oats last 120 horses 17 da., how long would 75 bu. last 90 horses?

MIXTURES

268. *Example* 1. A grocer mixes 5 lb. of tea worth 60¢ per pound, 4 lb. of tea worth 72¢ per pound, and 3 lb. of tea worth 80¢ per pound. Find the value of 1 lb. of the mixture.

SOLUTION. 5 lb. @ 60¢ is worth $3.00.
4 lb. @ 72¢ is worth $2.88.
3 lb. @ 80¢ is worth $2.40.
∴ 12 lb. of the mixture is worth $8.28.
∴ 1 lb. of the mixture is worth $.69.

Example 2. In what ratio must tea worth 60¢ be mixed with tea worth 80¢, that the mixture may be worth 74¢ per pound?

SOLUTION. By selling tea worth 60¢ per pound at 74¢ per pound, there is a gain of 14¢ per pound.

By selling tea worth 80¢ per pound at 74¢ per pound, there is a loss of 6¢ per pound.

The loss and the gain must balance each other.

By taking 6 lb. of the tea worth 60¢, the gain would be 84¢.

By taking 14 lb. of the tea worth 80¢, the loss would be 84¢.

∴ the required ratio is 6 : 14 or 3 : 7.

Example 3. In what proportion must sugar worth $4\frac{1}{2}$¢, $5\frac{1}{4}$¢, and 6¢ per pound, respectively, be mixed that the mixture may be worth 5¢ per pound?

SOLUTION. By selling sugar worth $4\frac{1}{2}$¢ per pound at 5¢ per pound, there is a gain of $\frac{1}{2}$¢ per pound.

By selling sugar worth $5\frac{1}{4}$¢ per pound at 5¢ per pound, there is a loss of $\frac{1}{4}$¢ per pound.

Hence, the gain on the first kind of sugar is twice the loss on the second kind of sugar.

∴ If we take 1 lb. of the first kind of sugar and 2 lb. of the second kind of sugar, the loss and the gain are balanced.

Again. By selling sugar worth 6¢ per pound at 5¢ per pound, there is a loss of 1¢ per pound. We can balance this loss by taking more sugar at 4½¢ per pound.

The gain on 1 lb. at 4½¢ per pound is ½¢ per pound.

The loss on 1 lb. at 6¢ per pound is 1¢ per pound.

Hence, if we take 2 lb. of the sugar worth 4½¢ per pound and 1 lb. of the sugar worth 6¢ per pound, the loss and the gain balance each other.

We have, therefore,

1 lb. @ 4½¢ to balance 2 lb. @ 5¼¢.

2 lb. @ 4½¢ to balance 1 lb. @ 6¢.

Hence, 3 lb. @ 4½¢, 2 lb. @ 5¼¢, and 1 lb. @ 6¢ will make the required mixture.

∴ The proportion is 3 lb., 2 lb., 1 lb.

We can add any number of pounds at 4½¢, provided we add the corresponding number of pounds at 5¼¢. By doing so we get an entirely different proportion. Thus, if we take 4 lb. @ 4½¢ and 8 lb. @ 5¼¢, and add to the mixture we have already obtained, we have 7 lb., 10 lb., 1 lb. Similarly, we can take any number of pounds at 4½¢ and the corresponding number of pounds at 6¢ per pound and get another entirely different proportion. Thus, if we take 10 lb. @ 4½¢ and 5 lb. @ 6¢, and add to the last mixture obtained, we have 17 lb., 10 lb., 6 lb.

We see, therefore, that problems of this character have any number of solutions.

EXERCISE 149 (Written)

1. A grocer mixes 27 lb. of tea @ 72¢ with 9 lb. of tea @ 84¢. At what price should he sell the mixture?

2. A dealer mixes 30 lb. of sugar at $4\frac{1}{2}$¢ per pound with 120 lb. of sugar at $5\frac{3}{4}$¢ per pound. At what price should he sell the mixture so as to make a profit of $\frac{1}{2}$¢ per pound?

3. If 3 gal. of spirits at $3.40 per gallon are mixed with $4\frac{1}{2}$ gal. at $3.90 per gallon, at what price per gallon should the mixture be sold so as to make a profit of $\frac{1}{10}$ of cost?

4. A dealer mixes, in the ratio of 5 : 8, two kinds of feed, which cost respectively 42¢ and 55¢ per bushel. At what price per bushel should he sell the mixture so as to make a profit of $\frac{1}{5}$ of his outlay?

5. In what ratio must tea worth 44¢ per pound be mixed with tea worth 60¢ per pound that the mixture may be worth 55¢ per pound? How many pounds of each kind are in 288 lb. of the mixture?

6. What quantity of tea at 54¢ per pound must be mixed with 128 lb. of tea at 72¢ per pound so that the mixture may be worth 66¢ per pound?

7. A dealer mixes tea which cost him 60¢ per pound with 120 lb. of tea which cost him 80¢ per pound, and sells the mixture at 80¢ per pound, thereby gaining $\frac{1}{5}$ of his outlay. How many pounds of tea at 60¢ per pound did he put in the mixture?

8. By mixing 30 lb. of coffee with 40 lb. of coffee at 40¢ per pound, and selling the mixture for $27, a grocer makes a profit of $2.30. Find the cost per pound of the first kind of coffee.

9. By mixing 12 gal. of wine at $3.70 per gallon with 8 gal. of wine of an inferior quality, and selling the mixture at $4.24 per gallon, a profit of $\frac{1}{8}$ of the cost is realized. Find the price per gallon of the inferior wine.

10. In a cask there are 12 gal. of wine and 18 gal. of water; in a second cask there are 9 gal. of wine and 3 gal. of water. How many gallons must be drawn from each cask to produce a mixture half wine and half water?

Hint. Twelve-thirtieths of first cask is wine, $\frac{9}{12}$ of second cask is wine, required mixture is $\frac{1}{2}$ wine.

WORK AND TIME

269. *Example* 1. A can do a piece of work in 6 da., and B can do the same piece of work in 8 da. In what time can A and B do the work together?

ANALYTICAL SOLUTION. A does $\frac{1}{6}$ of the work in 1 da.
B does $\frac{1}{8}$ of the work in 1 da.
$\therefore$ A and B together do $(\frac{1}{6} + \frac{1}{8})$ of the work in 1 da.
$\therefore$ A and B do $\frac{7}{24}$ of the work in 1 da.
$\therefore$ A and B do $\frac{1}{24}$ of the work in $\frac{1}{7}$ of 1 da.
$\therefore$ A and B do $\frac{24}{24}$ of the work in $\frac{24}{7}$ of 1 da.
$\therefore$ A and B do the work in $3\frac{3}{7}$ da.

Example 2. A cistern has three pipes. The first pipe fills the cistern in 12 hr., the second, in 15 hr., and the third empties it in 10 hr. In what time will the cistern be filled, if all three pipes run together, and the cistern is empty when the pipes start running?

ANALYTICAL SOLUTION. The first pipe fills $\frac{1}{12}$ of the cistern in 1 hr.
The second pipe fills $\frac{1}{15}$ of the cistern in 1 hr.
The third pipe empties $\frac{1}{10}$ of the cistern in 1 hr.

∴ The three pipes fill $(\frac{1}{12}+\frac{1}{15}-\frac{1}{10})$ of the cistern in 1 hr.

∴ The three pipes fill $\frac{1}{20}$ of the cistern in 1 hr.

∴ The three pipes fill $\frac{20}{20}$ of the cistern in 20 hr.

∴ The cistern is filled in 20 hr.

Example 3. A and B do a piece of work in $3\frac{11}{15}$ hr.; A alone, in 7 hr. In what time does B do the work?

SOLUTION. A and B together do $\dfrac{1}{3\frac{11}{15}}$ of the work in 1 hr.; *i.e.* A and B together do $\frac{15}{56}$ of the work in 1 hr.

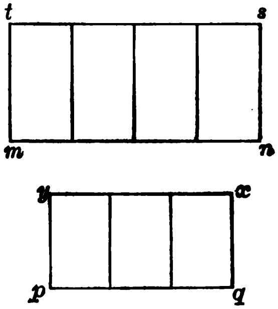
But A alone does $\frac{1}{7}$ of the work in 1 hr.

∴ B alone does $(\frac{15}{56}-\frac{1}{7})$ of the work in 1 hr.

∴ B alone does $\frac{1}{8}$ of the work in 1 hr.

∴ B does the work in 8 hr.

Example 4. A and B can mow a field in 21 hr. A can do $\frac{3}{4}$ as much work as B. Find the time in which each does the work.



SOLUTION. Represent B's work by the rectangle *mnst*.

Represent A's work by the rectangle *pqxy*.

A will do in $\frac{7}{3}$ hr. the work A and B do together in 1 hr.

∴ A will do in $\frac{7}{3}$ hr. × 21 the work A and B do together in 21 hr.

∴ A will do in 49 hr. the work A and B do together in 21 hr.

B will do in $\frac{7}{4}$ hr. the work A and B do together in 1 hr.

∴ B will do in $\frac{7}{4}$ hr. × 21 the work A and B do together in 21 hr.

∴ B will do in $36\frac{3}{4}$ hr. the work A and B do together in 21 hr.

A's time, 49 hr.; B's time, $36\frac{3}{4}$ hr.

EXERCISE 150 (Written)

1. If a person can do a piece of work in 7 da., what is his day's work? If he can do the work in $4\frac{1}{2}$ da., what is his day's work?

2. If B can copy a manuscript in $5\frac{3}{4}$ hr., how much can he copy in 1 hr.?

3. John travels $\frac{5}{16}$ of the distance between two cities in 1 hr. How many hours will it take him to travel the remainder of the distance?

4. A can do a piece of work in 3 hr., B in 4 hr., and C in 5 hr. How long will it take the three working together to do the work?

5. A can do a piece of work in 4 hr., B in 5 hr., and A, B, and C together in $1\frac{1}{2}$ hr. How long would it take C alone to do the work?

6. A, B, and C can do a piece of work in 6, 8, and 10 da. respectively. If they begin the work together, what part of the work remains to be done at the end of the second day?

7. A, B, and C can build a fence in 10, 15, and 20 hr. respectively. They work together for 4 hr., when B quits. In what time can A and C finish the work?

8. A cistern has two pipes. One can fill it in 20 min., and the other can empty it in 30 min. If the cistern is empty, in what time can it be filled, if both pipes begin to flow at the same instant?

9. A cistern has three pipes, two inflowing and one for discharge. The inflowing pipes would fill it in 3 hr. and 4 hr. respectively, while the discharge pipe would empty it in $1\frac{1}{2}$ hr. After the first two pipes have been running for $1\frac{1}{2}$ hr. the discharge pipe is opened. How soon will the cistern be emptied?

10. A man working alone plants $\frac{2}{5}$ of a number of trees in 10 da., and with the aid of his son plants the remainder of the trees in 12 da. more. In how many days can the son, working alone, plant the trees? In how many days can both together plant the trees?

11. A can do a piece of work in 3 hr., and B can do it in 4 hr. B works unaided for 1 hr. and 40 min., when A comes to help him. In what time can both finish the work?

12. A and B can do a piece of work in 8 da., A and C in 10 da., and B and C in 12 da. How long will it take all three together to do the work? How long will it take each alone to do the work?

13. A and B reap a field in 15 hr. B does $\frac{2}{3}$ as much work as A. Find the time in which each can do the work.

14. A and B working together can copy a book in 18 da. Find the time in which each can copy the book, supposing A's work is to B's work in the ratio 3 : 5.

MOTION IN THE SAME DIRECTION, OR IN OPPOSITE DIRECTIONS

270. *Example* 1. A starts to overtake B, who is 100 yd. ahead of him. A travels 11 yd. to B's 9 yd. How far must A travel in order to overtake B?

SOLUTION. A gains on B 2 yd. in every 11 yd. he goes.

∴ A gains on B 1 yd. in every 5½ yd. he goes.

∴ A gains on B 100 yd. in every 550 yd. he goes.

Ans. 550 yd.

Example 2. A freight train moving at the rate of 18 mi. an hour is 78 mi. ahead of a passenger train moving in the same direction at the rate of 30 mi. an hour. Find the distance the passenger train must run to overtake the freight train.

SOLUTION. In 1 hr. the passenger train gains on the freight (30 – 18) mi., *i.e.* 12 mi.

∴ In (78 ÷ 12) hr. the passenger train will overtake the freight train.

∴ The passenger train will overtake the freight train in 6½ hr.

In 6½ hr. the passenger train goes 30 mi. × 6½ = 195 mi.

Example 3. A hare is 50 leaps ahead of a greyhound, and takes 4 leaps to the greyhound's 3; but 2 of the greyhound's leaps equal 3 of the hare's. How many leaps must the greyhound take to catch the hare?

NOTE. This hare and greyhound have been running for several generations, and still attract attention, doubtless, by the leaps they take.

SOLUTION:

Step 1. In this problem, as in others involving motion, find first the relation between the rates of motion of the objects moving.

The hare makes 1⅓ leaps to the greyhound's 1 leap. But 1 leap of the greyhound = 1½ leaps of the hare.

∴ While the hare makes 1⅓ of her leaps, the greyhound goes a distance = 1½ of the hare's leaps.

∴ The ratio of their rates of motion is $1\frac{1}{3} : 1\frac{1}{2}$, or (multiplying by 6) 8 : 9.

Hence, in 9 yd., the greyhound gains on the hare 1 yd.

Hence, in 9 rd., the greyhound gains on the hare 1 rd.

Hence, in a distance measured by 9 leaps of the hare, the greyhound gains on the hare 1 leap of the hare.

∴ The distance the greyhound runs to overtake the hare is 50 leaps of the hare $\times 9 = 450$ leaps of the hare.

But 3 of these leaps $= 2$ of the greyhound's.

∴ The greyhound takes $(450 \times \frac{2}{3})$leaps $= 300$ leaps.

EXERCISE 151 (Written)

1. Two ships leave New York for Glasgow, one on Monday morning at 9 o'clock, and the other on the following morning at 9 o'clock. The rate of the first ship is 15 mi. an hour, and the rate of the second is 21 mi. an hour. How far will the ships be from New York City when the second ship overtakes the first?

2. Dallas and Galveston are 315 mi. apart. A train leaves Dallas for Galveston at 8 o'clock A.M. at the rate of 30 mi. an hour. At the same time, a train leaves Galveston for Dallas at the rate of 33 miles an hour. When will the trains meet? How far will the trains be from Dallas when they meet?

3. Paris, Texas, is 584 mi. from St. Louis. A passenger train leaves Paris for St. Louis at 6.50 P.M. Three hours later a freight train leaves St. Louis for Paris. When and where will they meet, the rates being respectively 24 mi. and 16 mi. per hour?

4. A man walking at the rate of 4 mi. an hour is overtaken by a train 88 yd. long, and is passed in 10 sec. Find the rate of the train.

5. A train going at the rate of 40 mi. an hour passes a man walking in the same direction at the rate of 4 mi. an hour, in 6 sec. What is the length of the train?

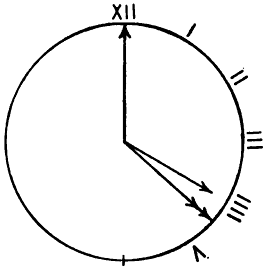
6. Two trains start from the same station and travel in the same direction. The first train leaves at 7 A.M., and the second train at 9 A.M. How many miles from the station will they be together, if the rate of the first train is 30 mi. per hour and the rate of the second train is 45 mi. per hour?

7. A train 132 yd. long, and moving at a uniform speed, overtakes a man walking on the railway at 4 mi. per hour, and passes him in $7\frac{1}{2}$ sec. Two minutes later the train overtakes a second man, and passes him in 7.2 sec. When one man overtakes the other man, how many miles ahead of the men will the train be?

8. A hare is 84 of her own leaps ahead of a greyhound, and makes 7 leaps to the greyhound's 6; but 5 of the greyhound's leaps are equivalent to 7 of the hare's. How many leaps must the hound make to catch the hare?

CIRCULAR MOTION: CLOCKS

271. *Example* 1. At what time between 4 and 5 o'clock are the hands of a clock together?



SOLUTION. At 4 o'clock the hour hand is 20 minute spaces ahead of the minute hand.

In 1 hr. the minute hand goes 60 minute spaces.

In 1 hr. the hour hand goes 5 minute spaces.

∴ Ratio of rates of motion of minute hand and of hour hand is 60 : 5, or 12 : 1.

∴ In 12 min. the minute hand gains on the hour hand 11 minute spaces.

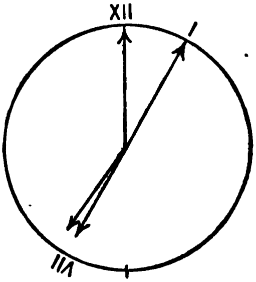
∴ In $1\frac{1}{11}$ min. the minute hand gains on the hour hand 1 minute space.

∴ In $1\frac{1}{11}$ min. × 20 the minute hand gains on the hour hand 20 minute spaces.

But $1\frac{1}{11}$ min. $\times 20 = 21\frac{9}{11}$ min.

∴ The hands are together $21\frac{9}{11}$ min. after 4 o'clock.

Example 2. At what time after 7 o'clock do the hour and minute hands first point in opposite directions?



SOLUTION. They will point in opposite directions whenever they are 30 minute spaces apart.

At 7 o'clock the hour hand is 35 minute spaces ahead of the minute hand.

∴ As soon as the minute hand gains on the hour hand 5 minute spaces, they will point in opposite directions.

But the minute hand gains on the hour hand 5 minute spaces in $1\frac{1}{11}$ min. × 5, *i.e.* in $5\frac{5}{11}$ min. (See Example 1.) *Ans.* $5\frac{5}{11}$ min. past 7 o'clock.

Example 3. When, between 5 and 6 o'clock, will the hour and minute hands be at right angles?

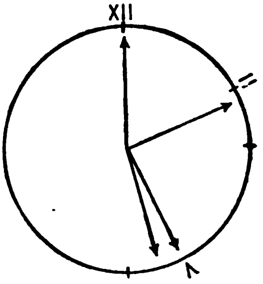


FIG. 1.

SOLUTION. The hour and minute hands will be at right angles when they are 15 minute spaces apart.

∴ They will be at right angles between 5 and 6 o'clock when the minute hand gains on the hour hand (25 − 15) minute spaces, *i.e.* 10 minute spaces.

$1\frac{1}{11}$ min. × 10 = $10\frac{10}{11}$ min. *Ans.* $10\frac{10}{11}$ minutes past 5 o'clock.

They will also be at right angles when the minute hand gains on the hour hand (25 + 15) minute spaces, *i.e.* 40 minute spaces. (See Fig. 2.) $1\frac{1}{11}$ min. × 40 = $43\frac{7}{11}$ min.

∴ The hands will be at right angles to each other at $10\frac{10}{11}$ min. past 5 o'clock and at $43\frac{7}{11}$ min. past 5 o'clock.

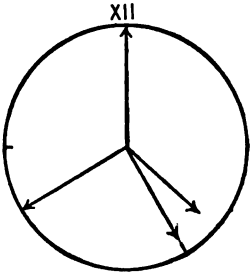


FIG. 2.

To the young learner the following suggestions may be of use:

To solve a question in circular motion: first, draw a diagram showing the things that move; second, find the ratios of the rates of motion of the things moving; third, having found the relative rates of motion of the objects, proceed as in a simple exercise involving motion in a straight line.

EXERCISE 152 (Written)

1. What angle do the hour and minute hands of a clock make with each other at 1 o'clock? at 2 o'clock? at 3 o'clock? at 4 o'clock? at 5 o'clock? at 6 o'clock? at 12 o'clock?

2. What angle do the hour and minute hands of a clock make when they point to positions 8 minute spaces apart? 12 minute spaces apart? 19 minute spaces apart? 23 minute spaces apart? 27 minute spaces apart? 50 minute spaces apart?

3. How many minute spaces apart do the hands of a watch indicate when they make an angle of 36°? 66°? 84°? 114°? 126°? 144°? 162°? 174°?

4. Find at what time between the hours of 2 and 3 o'clock the hands of a clock are together; between 3 and 4 o'clock; between 5 and 6 o'clock; between 7 and 8 o'clock; 9 and 10 o'clock; 10 and 11 o'clock; 11 and 12 o'clock.

5. At what time do the hands of a watch point in opposite directions between

(*a*) 1 and 2 o'clock? (*d*) 9 and 10 o'clock?
(*b*) 3 and 4 o'clock? (*e*) 11 and 12 oclock?
(*c*) 8 and 9 o'clock? (*f*) 12 and 1 o'clock?

6. At what time after 3 o'clock do the hands of a watch first point in opposite directions? When after 6 o'clock do they first point in opposite directions? When after 10 o'clock?

7. At what time or times are the hands of a watch at right angles between (*a*) 2 and 3 o'clock? (*b*) 3 and 4 o'clock? (*c*) 4 and 5 o'clock? (*d*) 6 and 7 o'clock? (*e*) 8 and 9 o'clock? (*f*) 9 and 10 o'clock? (*g*) 11 and 12 o'clock? (*h*) 12 and 1 o'clock?

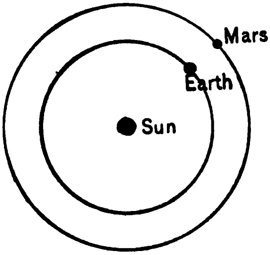
8. At what time after 4 o'clock do the hands of a watch first point to positions 8 minute spaces apart? 35 minute spaces apart? 24 minute spaces apart? 50 minute spaces apart?

9. Find the time, between 5 and 6 o'clock, when the minute hand is $\frac{1}{6}$ of the circumference of the dial in advance of the hour hand. The time between 9 and 10 o'clock when the minute hand is $\frac{1}{4}$ of the circumference in advance of the hour hand.

PLANETARY MOTION

272. The planets revolve around the sun as the center, in orbits nearly circular, whose planes almost coincide.

Example. The earth revolves around the sun in 365 da., nearly. Mars revolves around the sun in 687 da., nearly. Suppose they are in a straight line on the same side of the sun, in how many days will they be again on the same side of the sun in a straight line?



DEDUCTIVE SOLUTION:

The daily motion of the earth is $\frac{360}{365}^\circ$.

The daily motion of Mars is $\frac{360}{687}^\circ$.

$\therefore$ In 1 da. the earth passes Mars $\frac{360}{365}^\circ - \frac{360}{687}^\circ$.

When the earth passes Mars 360°, they will be again in a straight line.

$\therefore$ $(\frac{360}{365}^\circ - \frac{360}{687}^\circ) \times$ the required number $= 360^\circ$.

$\therefore$ The required number $= 360^\circ \div (\frac{360}{365}^\circ - \frac{360}{687}^\circ)$.

Dividing dividend and divisor by 360°,

$\therefore$ The required number $= 1 \div (\frac{1}{365} - \frac{1}{687})$.

The **period of revolution** of each of the planets is given in the following table:

Mercury	88 days	Jupiter	4,333 days
Venus	225 days	Saturn	10,759 days
The earth	365 days	Uranus	30,687 days
Mars	687 days	Neptune	60,181 days

EXERCISE 153 (Written)

1. How many days elapse between the time the earth and Mercury are on the same side of the sun in a straight line until they are again on the same side of the sun in a straight line?

2. (*a*) Work a like problem for the earth and Venus.
   (*b*) Work a like problem for the earth and Jupiter.
   (*c*) Work a like problem for the earth and Saturn.
   (*d*) Work a like problem for the earth and Uranus.
   (*e*) Work a like problem for the earth and Neptune.

3. (*a*) Work a like problem for Venus and Mars.
   (*b*) Work a like problem for Uranus and Neptune.

CURRENTS

273. It is a principle in physical science that if **two forces act on a body free to move,** their combined effect is equal to the sum of the forces if they act in the same direction, and equal to their difference if they act in opposite directions.

274. *Example* 1. A boat is rowed 18 mi. downstream in 3 hours. In what time can the boat return, if the rate of the current is $1\frac{1}{2}$ mi. per hour?

SOLUTION. The rate of rowing per hour down the stream is 18 mi. ÷ 3, *i.e.* 6 mi.

The rate of the current is $1\frac{1}{2}$ mi. per hour.

∴ The rate of rowing in still water is 6 mi. $-$ $1\frac{1}{2}$ mi. $=$ $4\frac{1}{2}$ mi.

∴ The rate of rowing against the current is $4\frac{1}{2}$ mi. $-$ $1\frac{1}{2}$ mi., *i.e.* 3 mi.

∴ The time of rowing against the current is 18 mi. ÷ 3 mi. = 6.

∴ It takes 6 hr. to return.

Example 2. A man rows 15 mi. and back in $7\frac{1}{2}$ hr. He rows 3 mi. with the current in the same time he rows 2 mi. against the current. How long does it take him to row down? What is the rate of the current?

SOLUTION. His rate of rowing down the stream is $1\frac{1}{2}$ times as fast as his rate of rowing up the stream.

∴ His time of rowing down the stream is $(1 \div 1\frac{1}{2})$ of the time of rowing up the stream.

∴ His time of rowing down the stream is $\frac{2}{3}$ of the time of rowing up the stream.

∴ The time of rowing down the stream is to the time of rowing up the stream in the ratio $2:3$.

Divide $7\frac{1}{2}$ hr. in the ratio $2:3$.

∴ The time of rowing down the stream is $\frac{2}{5}$ of $7\frac{1}{2}$ hr. = 3 hr.

∴ The time of rowing up the stream is $\frac{3}{5}$ of $7\frac{1}{2}$ hr. = 5 hr.

∴ The rate of rowing downstream is 15 mi. ÷ 3 = 5 mi.

∴ The rate of rowing upstream is 15 mi. ÷ 5 = 3 mi.

In rowing down we add the rate of the current to the rate in still water, and in rowing up we subtract the rate of the current from the rate in still water.

∴ The difference of the rates down and up stream is twice the rate of the current.

∴ Rate of the current is 1 mi. per hour.

EXERCISE 154 (Written)

1. In still water a boat's crew rowed 6 mi. and back in 2 hr. In what time could they row 6 mi. and back again, if there was a current at the rate of 3 mi. an hour?

2. A steamboat's rate down a river is 14 mi. per hour, and it goes in $2\frac{1}{2}$ hr. down the river as far as it goes in $3\frac{1}{2}$ hr. up the river. At what rate does the river flow?

3. A crew can row 20 mi. and back in $6\frac{1}{2}$ hr. They can row $1\frac{1}{4}$ mi. against the current in the same time they can row 2 mi. with the current. Find the time of rowing up, and also the rate of the current.

4. A boat is rowed in still water 4 mi. and back in 1 hr. 46 min. 40 sec. In what time can the boat be rowed 4 mi. and back, if there is a current of $1\frac{1}{2}$ mi. per hour?

MISCELLANEOUS PROBLEMS

275. *Example* 1. A can beat B 5 yd. in a race of 100 yd.; B can beat C 10 yd. in 200 yd. How many yards can A beat C in 400 yd.?

SOLUTION:

B's speed is $\frac{95}{100}$ of A's speed.
C's speed is $\frac{190}{200}$ of B's speed.
∴ C's speed is $\frac{190}{200}$ of ($\frac{95}{100}$ of A's speed).
∴ C's speed is $\frac{190}{200}$ of $\frac{95}{100}$ of A's speed.
∴ C's speed is $\frac{361}{400}$ of A's speed.
∴ C's speed and A's speed are in the ratio 361 : 400.
∴ In 400 yd. A beats C 400 yd. – 361 yd., or 39 yd.

NOTE. The following problem has been current in English and American text-books for at least a generation. We give two solutions to illustrate how a so-called difficult problem may become simple by a proper mode of attack.

Example 2. A train is traveling from one station to another. After traveling an hour it breaks down and is delayed for an hour. It then proceeds at $\frac{3}{5}$ of its former speed, and arrives 3 hr. late. Had it gone 50 mi. farther before the breakdown, it would have arrived 1 hr. 20 min. sooner. Find the rate of traveling per hour and the length of the line.

DEDUCTIVE SOLUTION. From the second condition of the problem it follows that:

I. The train loses 1 hr. 20 min. in going 50 mi.

The rate after the breakdown and the rate before the breakdown are in the ratio 3 : 5.

∴ The time taken to go 50 mi. after the breakdown is to the time taken to go 50 mi. before the breakdown in the ratio 5 : 3.

∴ A distance requiring 3 hr. at the original speed requires 5 hr. at the speed after the breakdown. . . . (*a*).

∴ The loss in time is $\frac{2}{3}$ of the original time.

Two-thirds of the original time in going 50 mi. = 1 hr. 20 min. = $\frac{4}{3}$ hr.

∴ The original time in going 50 mi. = $\frac{4}{3}$ hr. ÷ $\frac{2}{3}$ = 2 hr.

∴ The original speed = 50 mi. ÷ 2 = 25 mi.

II. The train is 3 hr. late.

∴ The train lost, after the breakdown, 2 hr.

∴ The original time in going from the breakdown to the destination was 3 hr. (See (*a*) above.)

∴ The original time for the entire distance was 4 hr.

∴ The length of the line is 25 mi. × 4 = 100 mi.

ANALYTICAL SOLUTION:

1 hr. 20 min. is lost in going 50 mi.
∴ 80 min. is lost in going 50 mi.
∴ 1 min. is lost in going $\frac{5}{8}$ mi.
∴ 120 min. is lost in going 75 mi.
∴ 2 hr. is lost in going 75 mi.

But 2 hr. is the entire time lost.

∴ The distance traveled after the breakdown is 75 mi.

Again, since the train at its original speed goes as far in 3 hr. as it went in 5 hr. at its speed after the breakdown,

∴ In 3 hr. at the original speed it goes 75 mi.

∴ In 1 hr. at the original speed it goes 25 mi.

∴ The length of the line is 75 mi. + 25 mi. = 100 mi.

EXERCISE 155 (Written)

1. A can beat B 50 yd. in a 1000-yd. race, and B can beat C 50 yd. in a 1000-yd. race. How far can A beat C in a 1000-yd. race?

2. The first of five clerks can copy 3 lines as quickly as the second can copy 4, the second can copy 5 as quickly as the third can copy 3, the third can copy 7 as quickly as the fourth can copy 8, the fourth can copy 14 as quickly as the fifth can copy 17. How many lines can the fifth copy while the first is copying 490?

3. If 9 yd. of calico are worth 4 yd. of linen, and 6 yd. of linen are worth 5 yd. of flannel, and 16 yd. of flannel are worth 13 yd. of cambric, how many yards of cambric are worth 15 yd. of calico?

4. A train travels for 1 hr., when it meets with an accident and is delayed 30 min. It then proceeds at $\frac{3}{4}$ of its original speed and arrives 1 hr. 30 min. late. Had it gone 36 mi. farther before the accident, it would have arrived 1 hr. late. Find the rate of traveling and the entire distance traveled.

5. A's rate of working is to B's in the ratio 4 : 3, and B's is to C's in the ratio 2 : 1. How long will it take C to do what A would do in 6 days?

6. A sum of money is to be divided among 11 men and 18 boys, and 5 men are to receive as much as 9 boys. When 3 men and 3 boys have received their shares, what fraction of the whole sum remains?

REVIEW QUESTIONS

1. Define the terms *antecedent*, *consequent*.

2. Why must the terms of a ratio be both abstract or both concrete?

3. When do numbers vary directly? When do numbers vary inversely?

4. If the area of a triangle is given, do the altitude and the base vary directly or inversely?

5. What effect on the value of a ratio has multiplying its terms by the same number?

6. How do you divide a number in any given ratio?

7. What is the first objective point in the solution of questions involving motion?

8. What are the relative rates of motion of the hands of a clock?

9. How long does it take the minute hand to gain on the hour hand 11 minute spaces?

10. If the hands of a watch or clock are together, in how many minutes will the minute hand be 120° ahead of the hour hand?

11. If you knew the rate of rowing in still water, and the rate of a current, how would you find the rate of rowing down the stream? Upon what principle in physical science does the answer to this question depend?

CHAPTER XVI

PERCENTAGE

276. *Example* 1. A man invests $750 and gains 4¢ on every dollar invested. Find his total gain, and the gain on $100.

SOLUTION. $1 gains 4¢.

∴ $750 gains 4¢ × 750 = $30.

∴ $100 gains 4¢ × 100 = $4.

The gain is $30.

The gain on $100 is $4.

In this example the rate of gain on $1 is 4¢.

The rate of gain on $100 is $4.

The number indicating the gain on $100 is called the **gain per cent.**

The ratio of the gain to the investment, or the ratio of the gain on $1 to $1 is called the **rate.** In the above example the gain on $1 is 4¢. The ratio of 4¢ to $1 is the rate.

The result, *i.e.* the whole gain, is often called the **percentage.**

277. Per cent means "by the hundred," or "on every hundred." The symbol % stands for the words *per cent.*

278. Rate per cent means so many on a hundred. Thus, 5% means 5 on every hundred. A rate of 5 on every 100 is the same as a rate of 1 in 20, or .05 in 1.

The number denoting the rate on one or unity is called the **rate.**

279. The result of taking a rate per cent of a quantity is called **percentage.**

The term *percentage* is also used to designate computation on the basis of 100. This chapter is headed "Percentage." Percentage is also used occasionally in the same sense as per cent.

280. *Example* 1. A man gains 5% on an investment of $850. Find his gain.

SOLUTION. To gain 5% of a sum of money means to gain $5 on every $100 of the sum of money.

On $100 there is a gain of $5.

∴ On $1 there is a gain of $$\frac{5}{100}$.

∴ On $850 there is a gain of $$\frac{5}{100} \times 850 =$ $42.50.

Another Point of View

To gain 5% is to gain 5 times 1%.

1% means 1 in every 100.

∴ 1% of $850 = $$\frac{850}{100}$.

∴ 5% of $850 = $$\frac{850}{100} \times 5 =$ $42.50.

In this example the rate per cent of gain is 5.

The rate of gain is $\frac{5}{100}$, $\frac{1}{20}$, or .05.

We may say a train has a speed of 60 mi. an hour, a speed of 1 mi. a minute, or a speed of 88 ft. a second. The three statements imply one and the same rate of motion. In the first statement we estimate the speed on the basis of 1 hr. of time. In the second statement we estimate the speed on the basis of 1 min. of time, and in the third statement we estimate the speed on the basis of 1 sec. of time.

Similarly, a business man may estimate his gains and losses on the basis of any particular sum of money. If he makes 100 the basis for estimating his gains or his losses, then his rate of gain or loss is called rate per cent. If he makes $1 the basis for estimating his gains or his losses, then his rate of gain or loss is simply a rate on $1 and is called rate. This is not rate per cent. When we use the phrase "rate per cent," the basis of computation is 100. When we use the word *rate* alone, the basis of computation is unity.

Example **2.** What is the rate, when the rate per cent is 7?

The rate on 100 is 7.
∴ The rate on 1 is $\frac{7}{100}$ or .07.
∴ The rate is $\frac{7}{100}$ or .07.

Example **3.** When the rate is $\frac{1}{8}$, what is the rate per cent?

Rate on 1 is $\frac{1}{8}$.
∴ Rate on 100 is $\frac{100}{8} = 12\frac{1}{2}$.
∴ Rate per cent is $12\frac{1}{2}$.

EXERCISE 156 (Oral)

Express each of the following rates per cent as a rate:

1. 60.
2. 80.
3. 75.
4. 90.
5. 40.
6. 45.
7. 55.
8. 30.
9. 35.
10. 25.
11. 20.
12. 10.
13. 9.
14. 8.
15. 5.
16. 4.
17. 15.
18. $33\frac{1}{3}$.
19. $16\frac{2}{3}$.
20. $14\frac{2}{7}$.
21. $12\frac{1}{2}$.
22. $11\frac{1}{9}$.
23. $9\frac{1}{11}$.
24. $8\frac{1}{3}$.
25. $6\frac{2}{3}$.
26. $5\frac{5}{9}$.
27. $6\frac{1}{6}$.
28. $3\frac{1}{3}$.
29. $13\frac{1}{3}$.
30. $23\frac{1}{3}$.
31. $36\frac{2}{3}$.
32. $43\frac{1}{3}$.
33. $53\frac{1}{3}$.
34. $66\frac{2}{3}$.
35. $28\frac{4}{7}$.
36. $42\frac{6}{7}$.
37. $57\frac{1}{7}$.
38. $71\frac{3}{7}$.
39. $85\frac{5}{7}$.
40. $22\frac{2}{9}$.
41. $36\frac{4}{11}$.
42. $55\frac{5}{11}$.
43. $41\frac{2}{3}$.
44. $58\frac{1}{3}$.
45. $91\frac{2}{3}$.
46. $13\frac{1}{3}$.
47. $46\frac{2}{3}$.
48. $73\frac{1}{3}$.
49. $86\frac{2}{3}$.
50. $27\frac{7}{9}$.
51. $38\frac{8}{9}$.
52. $61\frac{1}{9}$.
53. $18\frac{3}{4}$.
54. $31\frac{1}{4}$.
55. $43\frac{3}{4}$.
56. $37\frac{1}{2}$.
57. $62\frac{1}{2}$.
58. $87\frac{1}{2}$.
59. $68\frac{3}{4}$.
60. $88\frac{8}{9}$.

281. *Example* 1. Express the rate $.01\frac{1}{4}$ as a rate per cent.

The rate on 1 is $.01\frac{1}{4}$.

∴ The rate on 100 is $.01\frac{1}{4} \times 100 = 1\frac{1}{4}$.

∴ The required rate per cent is $1\frac{1}{4}$.

Example 2. Express the rate $.001\frac{3}{4}$ as a rate per cent.

The rate on 1 is $.001\frac{3}{4}$.

∴ The rate on 100 is $.001\frac{3}{4} \times 100 = .1\frac{3}{4} = \frac{1\frac{3}{4}}{10} = \frac{7}{40}$.

∴ The rate per cent is $\frac{7}{40}$.

(*a*) Bear in mind, we multiply or divide by 100 by moving, respectively, the decimal point two places to the right or two places to the left.

(*b*) Remember in expressions such as $.004\frac{3}{4}$, the $\frac{3}{4}$ means here, as elsewhere, $\frac{3}{4}$ of the unit to its immediate left; in this instance $\frac{3}{4}$ of 1 thousandth.

EXERCISE 157 (Oral)

Express each of the following rates as a rate per cent:

1. .04.
2. .05.
3. .08.
4. $.12\frac{1}{2}$.
5. .25.
6. $.06\frac{2}{3}$.
7. $\frac{5}{16}$.
8. $\frac{7}{11}$.
9. $\frac{2}{11}$.
10. $\frac{18}{25}$.
11. $\frac{17}{25}$.
12. $\frac{17}{30}$.
13. $\frac{19}{30}$.
14. $\frac{25}{30}$.
15. $\frac{15}{16}$.
16. $\frac{4}{15}$.
17. $\frac{7}{15}$.
18. $\frac{11}{15}$.
19. $\frac{13}{15}$.
20. $\frac{5}{8}$.
21. $\frac{7}{8}$.
22. $\frac{10}{11}$.
23. $\frac{7}{16}$.
24. $\frac{3}{4}$.
25. $\frac{3}{400}$.
26. $\frac{1}{200}$.
27. $.003\frac{1}{3}$.
28. .005.
29. $.006\frac{2}{3}$.
30. $.016\frac{2}{3}$.
31. $.033\frac{1}{3}$.
32. $.01\frac{3}{7}$.
33. $.007\frac{1}{7}$.
34. .002.
35. .006.
36. $.002\frac{1}{2}$.
37. $.005\frac{5}{7}$.
38. $.01\frac{1}{9}$.
39. $.11\frac{1}{9}$.
40. $.04\frac{6}{11}$.
41. $.008\frac{2}{3}$.
42. .0075.
43. $.0008\frac{1}{3}$.
44. $.0016\frac{2}{3}$.
45. .0024.

EXERCISE 158 (Written)

Find:

1. 9% of $84.
2. 8% of $425.
3. 6% of $800.
4. 5% of $2000.
5. 8% of $3250.
6. 7% of $4500.
7. 10% of $2250.
8. 11% of $4000.
9. 12% of $7250.
10. 4% of $3600.
11. 5% of $983.
12. 8% of $750.
13. 6% of $850.
14. 7% of $1250.
15. 25% of $4840.
16. 30% of $3290.
17. 40% of $4500.
18. 50% of $3250.
19. 75% of $4000.
20. 70% of $3500.
21. 80% of $2450.
22. 100% of $7800.
23. 16% of $3200.
24. 18% of $9200.
25. 125% of $4000.
26. 225% of $5400.

282. It is well to fix firmly in mind **the following relations**:

100% of a number = the number
110% of a number = $1\frac{1}{10}$ of the number.
$16\frac{2}{3}\%$ of a number = $\frac{1}{6}$ of the number.
20% of a number = $\frac{1}{5}$ of the number.
25% of a number = $\frac{1}{4}$ of the number.
$33\frac{1}{3}\%$ of a number = $\frac{1}{3}$ of the number.
50% of a number = $\frac{1}{2}$ of the number.
$66\frac{2}{3}\%$ of a number = $\frac{2}{3}$ of the number.
75% of a number = $\frac{3}{4}$ of the number.
$12\frac{1}{2}\%$ of a number = $\frac{1}{8}$ of the number.
$37\frac{1}{2}\%$ of a number = $\frac{3}{8}$ of the number.
$62\frac{1}{2}\%$ of a number = $\frac{5}{8}$ of the number.
$87\frac{1}{2}\%$ of a number = $\frac{7}{8}$ of the number.

EXERCISE 159 (Written)

Find:

1. $33\frac{1}{3}\%$ of $9600.

2. $66\frac{2}{3}\%$ of $3240.

3. 25% of $4920.

4. 20% of $4500.

5. $16\frac{2}{3}\%$ of $636.

6. $83\frac{1}{3}\%$ of $792.

7. $14\frac{2}{7}\%$ of $147.

8. $28\frac{4}{7}\%$ of $287.

9. $42\frac{6}{7}\%$ of $910.

10. $57\frac{1}{7}\%$ of $1260.

11. $12\frac{1}{2}\%$ of $3280.

12. $37\frac{1}{2}\%$ of $4640.

13. $62\frac{1}{2}\%$ of $5720.

14. $11\frac{1}{9}\%$ of $9981.

15. $88\frac{8}{9}\%$ of $9360.

16. $77\frac{7}{9}\%$ of $8480.

17. $8\frac{1}{3}\%$ of $5640.

18. $41\frac{1}{3}\%$ of $9120.

19. $58\frac{1}{3}\%$ of $7560.

20. $6\frac{2}{3}\%$ of $4515.

21. $13\frac{1}{3}\%$ of $4845.

22. $26\frac{2}{3}\%$ of $3900.

23. $46\frac{2}{3}\%$ of $2400.

24. $5\frac{5}{9}\%$ of $3600.

25. $27\frac{7}{9}\%$ of $5418.

26. $116\frac{2}{3}\%$ of $672.

27. The composition of a piece of coal taken from the Texas and Pacific Coal Company's mine is given as follows: moisture, 5.46%; combustible matter, 35.66%; fixed carbon, 49.17%; ash, 9.71%. Find the amount of each constituent in 2000 lb. of coal. Check your answer.

28. The analysis of a specimen of lignite is given as follows:

Moisture, 29.07%; combustible matter, 28.96%; fixed carbon, 24.47%; ash, 17.60%. Find the amount of each constituent in a ton of lignite. Check.

29. Distilled water is composed of two gases, $11\frac{1}{9}\%$ by weight being hydrogen, and $88\frac{8}{9}\%$ by weight being oxygen. Find the weight of each gas that can be obtained from 10 lb. of water.

30. A bookkeeper receives a salary of $1800 per annum. If he spends 62½% of his salary, and saves the remainder, how much does he spend? How much does he save?

31. An auriferous ore contains .5% of gold. How many pounds Avoirdupois of gold would 2240 lb. of this ore yield?

32. A copper ore contains 5½% of copper. Find the number of pounds of copper in 500 lb. of this ore.

33. A owns 16⅔% of a boat valued at $12,300. What is the value of A's share of the boat?

34. The estimated value of the exports of the United States for 1899 was $1,275,000,000. The percentages of exports from United States' ports for that year are given as follows: New York, 37.4%; Boston, 10.43%; Philadelphia, 5.05%; Baltimore, 8.9%; New Orleans, 6.4%· Galveston, 7.17%. Find the value of the exports from these cities.

35. The value of the crude rubber exported from Brazil to the United States in 1898 was $13,300,000. In the year 1899, this increased 38¼%. Find the increase, and also find the value of the export in 1899.

36. The value of the raw silk imported by the United States from Japan in 1898 was $12,500,000. In 1899, the imports of raw silk increased 67.2%. Find the increase, also find the value of the raw silk imported from Japan in 1899.

37. The value of the total exports of the United States for the year 1902 is estimated at $1,355,481,680. Of this value, the agricultural products constitute 62.81%, the mining products, 2.88%, and the manufactures, 29.8%. Find the value of the exports of the agricultural products, of the mining products, and of the manufactures.

38. The composition of cast iron, steel, and wrought iron is given as follows:

	CAST IRON	STEEL	WROUGHT IRON
Carbon	3.81%	.65%	.10%
Silicon	1.68%	.07%	.05%
Phosphorus. . . .	.70%	.03%	.15%
Sulphur	.60%	.02%	.05%
Manganese	.41%	.40%	.07%
Iron	92.80%	98.83%	99.58%

Find the weight of each constituent in 2 T. of cast iron. In 2 T. of steel. In 2 T. of wrought iron.

39. Dead Sea water contains 22.86% of solid matter. Find the weight of solid matter obtained from evaporating 1 T. of Dead Sea water.

283. *Example.* If 15.5% of a number equals 22.785, what is the number?

SOLUTION:

$$15.5\% \text{ of the number} = 22.785.$$

$$\therefore \quad 1\% \text{ of the number} = \frac{22.785}{15.5}.$$

$$\therefore 100\% \text{ of the number} = \frac{22.785}{15.5} \times 100 = 147.$$

$\therefore$ The number is 147.

EXERCISE 160 (Written)

1. If 5% of a number equals 185, what is the number?

2. If 12½% of a man's salary is $156, what is the man's salary?

3. If 37½% of a man's property is valued at $324, what is the value of his property?

4. The number 360 is equal to 5% of what number? 6% of what number? 8% of what number? 9% of what number?

5. The number 120 is equal to 3% of what number? 4% of what number? 5% of what number? 6% of what number? 8% of what number? 12% of what number?

6. $750 is 6% of what sum of money? $6\frac{2}{3}$% of what sum of money? $7\frac{1}{2}$% of what sum of money? $12\frac{1}{2}$% of what sum of money?

7. $108 is 30% of what sum? 25% of what sum? $33\frac{1}{3}$% of what sum? $44\frac{4}{9}$% of what sum?

8. $450 is $6\frac{1}{4}$% of what sum? $6\frac{2}{3}$% of what sum? $8\frac{1}{3}$% of what sum? $12\frac{1}{2}$% of what sum? $16\frac{2}{3}$% of what sum?

9. $420 is $37\frac{1}{2}$% of what sum? $62\frac{1}{2}$% of what sum? $87\frac{1}{2}$% of what sum?

10. In 1900 the commercial value of silver was 3% of the value of gold. Find the number of ounces of silver equivalent in value to 126 ounces of gold.

11. Of the population of the United States in 1900, 27,849,760 were married. This number was 36.5% of the population. Find the population in 1900.

12. The census of 1900 gives the number of married men in the United States as 14,003,798. This number was 35.9% of the number of males. Find the male population.

13. The census of the same year gives the number of married women in the United States as 13,845,963. This number was 37.2% of the entire number of females. Find the female population.

14. Of the number of illiterates above 10 years of age in the United States 15.5%, according to the census of 1900, can neither read nor write. This number is 955,840. Find the number of illiterates above 10 years of age in the United States in 1900.

15. Of the average value of the raw cotton exported from the United States in 5 years 50.4% went to England. If the export of raw cotton to England amounts in value to $107,500,000, find the average value of the raw cotton exported to England.

16. Of the sheep exported from the United States 86% are shipped to England. If the value of the sheep shipped to England in a certain year was $1,685,800, find the total value of the export of sheep for that year.

17. Of the pupils attending school in a certain city 6% are in the senior class of the high school. If the senior class numbers 21, find the number of pupils attending school in that city.

18. In a certain city the number of pupils promoted at the end of the scholastic year was 2765. This number was 79% of the number of pupils in school. Find the number of pupils attending school in that city.

19. The foreign-born population of a city is 10,668. This number is $3\frac{1}{2}$% of the population of the city. Find the population of the city.

20. Of the water of the Dead Sea 22.857% is saline material. If a quantity of Dead Sea water is evaporated, and the saline material left behind weighs 914.28 lb., what is the weight of the water before evaporation?

284. *Example* 1. A man invests \$8400 and makes a clear profit of \$1400. Find the rate per cent of profit.

SOLUTION:

On \$8400 the gain is \$1400.

$\therefore$ On \$1 the gain is $\$\frac{1400}{8400}$.

$\therefore$ On \$100 the gain is $\$\frac{1400}{8400} \times 100 = \$16\frac{2}{3}$.

$\therefore$ The profit is $16\frac{2}{3}\%$.

Example 2. The population of Fargo, N. D., in 1890 was 5664, and in 1900 was 9589. Find the rate per cent of increase in population in 10 years.

SOLUTION. Population in 1890 = 5664.
Population in 1900 = 9589.
$\therefore$ The increase = 3925.

In 5664 there is an increase of 3925.

$\therefore$ In 1 there is an increase of $\frac{3925}{5664}$.

$\therefore$ In 100 there is an increase of $\frac{3925}{5664} \times 100 = 69.3$, nearly.

$\therefore$ The rate per cent of increase is 69.3, or the increase is 69.3%.

Example 3. What per cent of 12 is 5?

SOLUTION:

$12 = 100\%$ of 12.

$\therefore 1 = \frac{1}{12}$ of 100% of 12.

$\therefore 5 = \frac{5}{12}$ of 100% of $12 = 41\frac{2}{3}\%$ of 12; or proceed thus:

$5 = \frac{5}{12}$ of 12. The rate $\frac{5}{12} = 41\frac{2}{3}\%$.

EXERCISE 161 (Written)

1. What per cent of 15 is 12?
2. What per cent of 20 is 4? is 7? is 11? is 13?
3. What per cent of 40 is 2? is 8? is 12? is 17?

4. What per cent of 90 is 9? is 12? is 27? is 11¼?

5. What per cent of 120 is 15? is 18? is 45? is 80?

6. What per cent of 480 is 28.8? is 33.6? is 40? is 90? is 500?

7. What per cent of 1728 is 345.6? is 155.52? is 288? is 108.5? is 3456?

8. What per cent of 231 is 21? is 77? is 34.65?

9. What per cent of 5280 is 88? is 440? is 330? is 110?

10. What per cent of a bushel is a quart?

11. What per cent of a mile is 8 rd.?

12. What per cent of an acre is a square rod?

13. What per cent of a chain is a yard?

14. What per cent of 1 sq. ch. is 1 sq. rd.?

15. What per cent of 1 gal. is 1 pt.?

16. What per cent of 1 sq. rd. is 7 sq. yd. 5 sq. ft. 9 sq. in?

17. What per cent of a rod is 1 yd. 2 ft. 6 in.?

18. What per cent of 1 mi. is 1 knot?

19. What per cent of 1 mi. is an arc of 1′ measured on the 40th parallel of latitude? (See table, page 261.)

20. What per cent of 1 mi. is 1 ch.? is 8 ch.?

21. What per cent of 1 sq. mi. is the S. W. ¼ of N. E. ¼ of a section of land?

22. What per cent of a common year is 73 da.? is 219 da.? is 292 da.?

23. A meter is 39.37 in. What per cent of a meter is 1 yd.? What per cent of 1 yd. is 1 meter?

24. A kilometer is 1000 meters. What per cent of 1 mi. is 1 km.?

25. What per cent of 1 lb. Avoirdupois is 1 lb. Troy?

26. What per cent of 1 oz. Avoirdupois is 1 oz. Troy?

27. What per cent of the area of each of the following states consists of irrigated land?

	ACRES IRRIGATED	AREA IN SQUARE MILES
(*a*) California	1,446,114	158,360
(*b*) Colorado	1,611,270	103,925
(*c*) Louisiana	201,685	48,720
(*d*) Montana	951,054	146,080
(*e*) Nevada	501,168	110,700
(*f*) Oregon	388,110	96,030
(*g*) Utah	629,290	84,970
(*h*) Washington	135,470	69,180
(*i*) Wyoming	605,230	97,890

28. Find the increase per cent in the population of each of the following cities for the ten years from 1890 to 1900:

	POPULATION IN 1890	POPULATION IN 1900
Mobile	31,076	38,469
Little Rock	25,874	38,307
Los Angeles	50,395	102,479
Denver	106,713	133,859
Pensacola	11,750	17,747
Savannah	43,189	54,244
Springfield	24,963	34,159
Evansville	50,756	59,007
Dubuque	30,311	36,297
Kansas City, Kan.	38,316	51,418
Lexington	21,567	26,369
Kansas City, Mo.	132,716	163,752
Minneapolis	164,738	202,718

29. The total production of butter in the United States in 1899 is estimated at 1,430,000,000 lb. Of this quantity 16,002,000 lb. was exported. Find the per cent of the total production exported.

30. For the year 1899 the total production of cheese in the United States is estimated at 300,000,000 lb. Of this quantity 27,203,200 lb. was exported. Find the per cent of cheese exported.

31. The number of farms in the United States June 1, 1900, was 5,739,657. The number of farms operated by owners was 3,713,371. The number of farms operated by tenants was 752,920. The number of farms operated by share tenants was 1,273,366. Find the per cent operated by owners, by tenants, and by share tenants respectively.

32. The world's production of wool in 1901 was 2,700,000,000 lb. nearly. Of this quantity the United States produced 303,000,000 lb. What per cent of the world's production was produced in the United States?

PROFIT AND LOSS

285. *Example* **1.** A dealer sold two pianos for $240 each. On the first piano he made a profit of 20%, and on the second piano he lost 20%. Did he gain or lose by the transaction, and how much?

SOLUTION:

$$100\% \text{ of the cost of first piano} = \text{cost of first piano.}$$

$$20\% \text{ of the cost of first piano} = \text{profit on first piano.}$$

$$\therefore 120\% \text{ of the cost of first piano} = \$240.$$

$$\therefore 1\% \text{ of the cost of first piano} = \frac{\$240}{120}.$$

$$\therefore 100\% \text{ of the cost of first piano} = \frac{\$240}{120} \times 100 = \$200.$$

Again:

100% of the cost of second piano = cost of second piano.

20% of the cost of second piano = loss on second piano.

$\therefore$ 80% of the cost of second piano = \$240.

$\therefore$ 1% of the cost of second piano $= \dfrac{\$240}{80}$.

$\therefore$ 100% of the cost of second piano $= \dfrac{\$240}{80} \times 100 = \$300$.

$\therefore$ The two pianos cost \$200 + \$300 = \$500.

The two pianos sold for \$240 × 2 = \$480.

$\therefore$ The dealer lost \$20.

Example 2. A man sold a horse for \$126, thereby losing 20%. What should have been the selling price of the horse to have a profit of 15%?

SOLUTION. 100% of cost = cost.

20% of cost = loss.

$\therefore$ 80% of cost = \$126 (selling price).

$\therefore$ 1% of cost $= \dfrac{\$126}{80}$.

$\therefore$ 115% of cost $= \dfrac{\$126}{80} \times 115 = \$181.125$.

But 115% of cost = cost + 15% of cost.

$\therefore$ Required selling price = \$181.12.

Example 3. How should goods be marked so that a dealer may give a discount of 20% off the marked price and still make a profit of 15% on the cost price of the goods?

SOLUTION. Here we have to find the relation of the marked price to the cost price.

Every \$1 of cost price produced a profit of 15% of cost price, *i.e.* every \$1 of cost becomes \$1.15.

Again :

100% of the marked price = the marked price.

20% of the marked price = discount.

∴ 80% of the marked price = selling price = 1.15 × cost.

$$\therefore\quad 1\% \text{ of the marked price} = \frac{1.15 \times \text{cost}}{80}.$$

$$\therefore\ 100\% \text{ of the marked price} = \frac{1.15 \times \text{cost}}{80} \times 100$$

$$= 1.43\tfrac{3}{4} \text{ cost.}$$

∴ What originally cost \$1 must be marked \$1.43¾.

∴ What originally cost \$100 must be marked \$143¾.

∴ The goods must be marked 43¾% above cost.

Note. Per cent of gain and per cent of loss are always estimated on the cost price.

EXERCISE 162 (Written)

1. A man sold a house and lot for \$3835 and gained thereby 18% of his outlay. What was his outlay?

2. By selling a piano for \$270, a dealer loses 10%. Find the cost of the piano.

3. By gaining 25% of his capital, a merchant increases his capital to \$4550. What was his original capital?

4. If goods are bought for \$20 and sold for \$22.50, what is the gain per cent?

5. If the cost is \$48 and the selling price is \$52, what is the gain per cent?

6. If the cost is \$190 and the selling price is \$152, what is the loss per cent?

7. If the cost is \$145 and the selling price is \$159.50, what is the gain per cent?

8. If the cost is \$118 and the selling price is \$128.62, what is the gain per cent?

9. If the selling price is $106.70 and the gain is 10%, what is the cost?

10. If the selling price is $75.25 and the loss 12½%, what is the cost?

11. If the selling price of a rug is $90 and the gain is 20%, what is the cost of the rug?

12. If the selling price is $89.25 and the loss is 15%, what is the cost?

13. If the selling price is $95 and the gain is 18¾%, what is the cost? What is the profit?

14. By selling a horse for $168, a man gains 40% of the cost. What is the cost of the horse?

15. By selling velvet at $4.55 a yard, a clothier makes a profit of 8⅓%. Find the cost per yard of the velvet.

16. At an auction a dealer buys goods at 20% below the market price. If he sells these goods at the market price, what is his gain per cent?

17. A lawyer collects a debt. He charges 3% for collection, and remits to his client, after deducting his fee, $952.54. Find the amount of the debt.

18. If property was sold for $11,778 at a loss of 2½%, what was the value of the property?

19. Two horses were sold for $200 each, one at a gain of 20%, and the other at a loss of 20%. Did the seller gain or lose by the transaction, and how much?

20. A sells goods to B at a profit of 10%, and B sells them to C at a profit of 15%. If C paid $253 for the goods, find A's cost price.

21. A merchant increases his capital 18¾%, and at the end of a second year he also increases his capital 18¾%. If his capital is then $14,440, what was it at first?

22. A merchant begins business with a certain capital. The first year he adds 25% to his capital, the next year he adds 25% to the capital of the previous year, the third year he loses 40% of his capital, and is then worth $10,800. How much capital did he begin with?

23. If it takes $81,415 for the running expenses of the schools of a city, and if 95% of the tax levied for school purposes goes to the support of the schools, what should be the amount levied for school purposes so that the expenses may be just met?

24. By selling tea at 65¢ per pound a merchant gained $118.75. If his gain was $62\frac{1}{2}$%, how many pounds of tea did he sell?

25. By selling turkeys at $2.50 each a dealer makes a profit of 60%. What would have been his gain per cent had he sold the turkeys for $1.75 each?

26. By selling wine at $2.10 a gallon, a merchant makes a profit of 20%. What would be his gain per cent should he sell the wine for $2 a gallon?

27. A merchant mixes tea which cost him 60¢ per pound with tea which cost him 70¢ per pound in the proportion of 5 lb. of the former tea to 6 lb. of the latter. If he sells the mixture at 80¢ per pound, find his gain per cent.

28. A horse is sold for $145.50 at a loss of 3%. At what price should the horse have been sold so as to make a profit of 10%?

29. If oranges are bought at 5 for 3¢ and sold at 3 for 5¢, what is the gain per cent?

30. If 325 lb. of sugar are bought for $13, at what price per pound must it be sold so as to make a profit of 25%?

31. If 15 yd. of silk are bought for $26.25, at what price per yard should it be sold to make a profit of 20%?

32. By selling cloth at the rate of 17½¢ a yard a clothier loses 12½%. What should the selling price per yard be so as to make a profit of 20%?

33. A merchant buys butter at 18¢ per pound and sells 120 lb. for $25.20. What is his gain per cent?

34. A grocer buys cheese at 9¢ per pound and sells it at the rate of 8 lb. for $1. What is his gain per cent?

35. A merchant mixes two kinds of tea in the ratio 5 : 2. If the teas are worth respectively 68¢ and 75¢ per pound, what should be the selling price per pound so as to make a profit of 20%?

36. If I buy a horse for $80 and sell him for $71, what is the loss per cent?

37. By selling a horse for $83.30 I lost 15%. What did the horse cost?

38. Two men, A and B, buy two horses, each paying the same price. A sells his horse for $90 at a loss of $14\frac{2}{7}$%. B sells his horse at a loss of 6%. Find the selling price of B's horse.

39. A merchant buys 70 yd. of cloth at 50¢ a yd. and sells it at a gain of 15%. He buys also 70 yd. of silk at 90¢ a yard. On both he gains 10%. At what price per yard does he sell the silk?

40. I buy a quantity of barley and intrust it to an agent to sell. The agent sells it at an advance of 25% on the cost, and after deducting a commission of 2% he remits the balance, $539. How much did the barley cost me?

41. A fruit dealer buys a crate of oranges for $2.50 He sells them at 2¢ each, making a profit of 60%. How many oranges are in the crate?

42. If a merchant buys goods at 25% below the retail price and sells them at the retail price, what is his gain per cent?

43. A coal dealer buys 120 T. of coal at $4 a ton. He sells $\frac{1}{2}$ of the coal at an advance of 25%, $\frac{1}{3}$ of it at an advance of 50%, and the remainder at an advance of 10%. Find his entire profit and his gain per cent on the coal.

44. A man buys 120 acres of land at $15 an acre. An agent sells the land and remits $2850, after deducting 5% of the selling price. At what per cent above the original cost did the agent sell the land?

45. (*a*) How should goods be marked so as to make a profit of 12% after deducting 20% from the marked price?

(*b*) How should they be marked so as to make a profit of 17% after deducting 10% from the marked price?

(*c*) How should they be marked so as to make a profit of 20% after deducting 25% from the marked price?

(*d*) How should they be marked so as to make a profit of 10% after deducting 10% from the marked price?

(*e*) By taking 30% off the marked price a merchant neither gains nor loses. How were the goods marked?

46. A discount of 20% in price of tea at 75¢ per pound enables me to buy 4 lb. more for my money than I could formerly have bought for $7.50. How much money have I?

47. A merchant marks his goods at 15% above cost. If he gives, in a bill amounting to $57.50, a discount of $1.50, what per cent of profit does he make?

48. If 32 lb. of sugar are sold for what 36 lb. of sugar cost, what is the gain per cent?

REVIEW QUESTIONS

1. What is meant by the words *per cent?* What is the meaning of *rate per cent?* of *rate?*

2. Express each of the following fractions as a rate per cent: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$, $\frac{1}{12}$.

3. What fraction of a number is $16\frac{2}{3}\%$ of the number? $33\frac{1}{3}\%$ of the number? $66\frac{2}{3}\%$ of the number? 25% of the number? 75% of the number? 80% of the number?

4. What fraction of a number is $3\frac{1}{3}\%$ of the number? 5% of the number? $6\frac{2}{3}\%$ of the number?

5. What fraction of a number is 125% of the number? 150% of the number? 225% of the number? 450% of the number? 380% of the number? $166\frac{2}{3}\%$ of the number? $133\frac{1}{3}\%$ of the number? $116\frac{2}{3}\%$ of the number? $119\frac{1}{9}\%$ of the number?

6. Is there a greatest possible gain per cent?

7. Is there a greatest possible loss per cent? If so, what is it?

8. If you found an ax on a road and sold it immediately for $1, what would be the gain per cent?

9. If you had a 25¢ piece and lost it, what would be the loss per cent?

10. If the cost price and the rate per cent of gain were given you, how would you find the gain?

11. If the cost price and the loss per cent are given, how do you find the actual loss?

12. If I should give you the loss and the loss per cent, how would you find the cost price?

13. If you are given the gain and the gain per cent, how do you find the cost?

14. If I give you 84% of a number, how do you find 95% of the number?

15. If I give you 120% of a number, how do you find the number?

16. If you are given 75% of a number, how do you find the number?

17. If I give you the cost price of an article, and the gain on the article, how do you find the gain per cent?

18. If I give you the cost price and the loss, how do you find the loss per cent?

19. If the selling price of an article is given and also the gain per cent, how do you find the cost of the article?

20. If the selling price of an article and the loss per cent are given, how do you find the cost price of the article?

21. How would you find the selling price, if the cost price and the gain per cent were given?

22. How would you find the selling price, if the cost price and the loss per cent were given?

23. If I give you the rate per cent, how do you find the rate per thousand?

24. In a mortuary report, if the rate of deaths per thousand is 30, what is the rate per cent?

25. The school tax in a certain city is 50¢ on the $100. Express this rate of taxation as a rate per cent. What is this rate on the basis of $1?

26. If the rate of insurance on the life of a person is $31 per $1000, what is the rate per cent of insurance?

27. If the rate per cent charged for insuring property is 1¼, what is the rate per $1000?

28. On what is the rate per cent of profit or of loss always estimated?

29. What is meant by profit? What is meant by loss?

COMMERCIAL DISCOUNTS

286. **Commercial** or **trade discount** is an allowance made upon the list price of goods, or on the amount of a bill.

Discounts are reckoned upon the basis of 100; in other words, as so many per cent.

In some cases several discounts are allowed. If there are two or more discounts, the first is reckoned on the list or catalogue price; the next is reckoned on the remainder after deducting the first discount; the third is reckoned on the second remainder; and so on.

Example 1. If watches are listed at $35 each and a discount of 10% is allowed, what is the cost of one of the watches?

SOLUTION. 10% of $35 = $3.50 = the discount.
$35 − $3.50 = $31.50.
∴ The cost is $31.50.

Example 2. What is the cost of goods to a purchaser, if the list price is $750 and discounts of 20%, 15%, and 10% are allowed?

SOLUTION. First discount = 20% of $750 = $\frac{1}{5}$ of $750 = $150.

∴ The first remainder = $750 − $150 = $600.
The second discount = 15% of $600 = $90.
$600 − $90 = $510.
∴ The second remainder = $510.
The third discount = 10% of $510 = $51.
$510 − $51 = $459.
∴ The cost to the purchaser is $459.

A little consideration will enable us to solve the above examples in another way.

A discount of 10% is equivalent to a discount of 10¢ on the dollar. If 10¢ is allowed on every dollar, then 90¢ is paid on every dollar. In other words, .9 of the list price is paid.

∴ In Example 1 the cost price of the watch = .9 of \$35 = \$31.50.

In Example 2 the first remainder = .8 of \$750.

The second remainder = .85 of first remainder.

∴ The second remainder = .85 of .8 of \$750.

Similarly, the third remainder = .9 of .85 of .8 of \$750.

∴ The third remainder = $.9 \times .85 \times .8 \times \$750 = \$459$.

Since the order in which we multiply factors is immaterial, therefore the order in which we reckon commercial discounts is also immaterial.

Example 3. If goods are listed at \$1200 and discounts of 20%, 10%, 5% are allowed, find the cost price and the single discount equivalent to the three discounts.

SOLUTION. The cost of the goods = $1200 \times .8 \times .9 \times .95 = \$820.80$.

∴ The total discount = $\$1200 - \$820.80 = \$379.20$.

On \$1200 the discount = \$379.20.

∴ On \$1 the discount $= \dfrac{\$379.20}{1200}$.

∴ On \$100 the discount $= \dfrac{\$379.20}{1200} \times 100 = \$31.60$.

∴ The single discount is 31.6%.

Another Method

On every dollar of list price there is paid $.8 \times .9 \times .95$ of \$1 = \$.684.

∴ On every dollar of list price there is a discount of \$1 – \$.684 = \$.316.

∴ On every \$100 of list price there is a discount of \$31.60.

∴ The single rate of discount = 31.6%.

EXERCISE 163 (Oral)

1. What is paid for goods if the list price is \$40 and a discount of 20% is given?

2. Find the cost, when the list price is \$85 and the discount is 25%.

3. Find the cost, when the list price is \$120 and the discount is 15%.

4. Find the cost, if the list price is \$150 and the discount is 10%.

5. Find the cost, if the list price is \$270 and the discount is $33\frac{1}{3}$%.

6. Find the cost, if the list price is \$140 and the discount is 25%.

7. Find the cost, if the list price is \$350 and the discount is 20%.

8. Find the cost, if the list price is \$1000 and the discount is 18%.

EXERCISE 164 (Written)

1. A suit of clothes is marked \$70, and is sold with discounts of 25% and 10% for cash. Find the selling price.

2. On a bill of \$900 two discounts of 20% and 15% are allowed. What is the net amount of the bill? What single discount would be equivalent to the two discounts?

3. If goods are marked \$175 and sold for \$122.50, what is the discount?

4. On a bill of $1500 discounts of 25%, 15%, and 6% are allowed. Find the net cash amount of the bill.

5. A piano is listed at $450, with discounts of 20%, 12½%, and 10%. Find the cost price to the purchaser.

6. Find the cash value of a bill of $320 with discounts of 15%, 10%, and 5%.

7. Suppose you were offered a single discount of 45%, or two discounts of 30% and 20%, which would you take? What would be the difference in a bill of $1000?

8. A dealer buys a quantity of goods marked $550, with a discount of 20%. If he sells the goods at 6% above the marked price, what is his gain per cent?

9. If I buy goods listed at $150, with a discount of 10%, and sell them at 12% above the marked price, find my gain per cent.

10. A bookseller buys 100 books marked $1.50 each, at a discount of 20% and sells them at the marked price. What is his gain per cent?

11. A bookseller bought 100 books at $1.25 each. He make a profit of 20% after giving a discount of ¼. At what price did he mark each book, and what was his profit?

12. Find the cost price in each case, if the list price and the rates of discount are as follows:

	LIST PRICE	DISCOUNTS
(*a*)	$480	20%, 30%, 10%
(*b*)	$1000	40%, 5%, 4%
(*c*)	$775	25%, ⅕
(*d*)	$880	12½%, 4%
(*e*)	$720	⅛, 16%
(*f*)	$960	12½%, 12%
(*g*)	$1200	10%, 8%, 5%
(*h*)	$1760	25%, 12½%, 16⅔%

13. Find the per cent of profit made by a merchant who buys goods at discounts of 20 %, 10 %, and sells them at the marked price.

14. If the discounts are 20% and 12½% and a man sells goods marked $1000 at 2% above the marked price, what are his profit and rate per cent of gain?

287. Given cost price and the discounts to find the list price.

Example 1. If the cost price is $840 and the discount is 20%, what is the list price?

SOLUTION:

List price − 20% of the list price = 80% of the list price.

$$\therefore\ 80\%\text{ of the list price} = \$840.$$

$$\therefore\ 1\%\text{ of the list price} = \frac{\$840}{80}.$$

$$\therefore\ 100\%\text{ of the list price} = \frac{\$840}{80} \times 100 = \$1050.$$

Or,

$$.80 \times \text{list price} = \$840.$$

$$\therefore\ \text{List price} = \frac{\$840}{.8} = \$1050.$$

Example 2. If the cost is $1323 and the discounts are 25%, 12½%, 10%, find the list price.

SOLUTION:

$$.75\text{ of }.87\tfrac{1}{2}\text{ of }.90\text{ of list price} = \$1323.$$

$$\therefore\ .75 \times .87\tfrac{1}{2} \times .9 \times \text{list price} = \$1323.$$

$$\text{Now }.75 \times .87\tfrac{1}{2} \times .9 = \text{some number.}$$

∴ We have some known number multiplied by the list price, and we know the product.

∴ List price $= \$1323 \div (.75 \times 87\tfrac{1}{2} \times .9) = \$1323 \div (\tfrac{3}{4} \times \tfrac{7}{8} \times \tfrac{9}{10}) = \$1323 \div \tfrac{189}{320} = \$1323 \times \tfrac{320}{189} = \$2240.$

15. Find the list price in each case, if the cost prices and rates of discount are as follows:

	Cost	Discounts
(*a*)	$441	20%, 12½%, 10%
(*b*)	$3528	10%, 20%, 30%
(*c*)	$2244	15%, 12%
(*d*)	$4845	¼%, 33⅓%
(*e*)	$2565	43%, 10%
(*f*)	$1344	30%, 20%, 20%
(*g*)	$2736	20%, 10%, 5%
(*h*)	$1782	46%, 45%

COMMISSION AND BROKERAGE

288. A very large proportion of the buying and selling of the produce of the country is done through commission merchants and brokers. A commission merchant is usually intrusted with the goods bought or sold. A broker merely buys and sells. Brokers usually deal in stocks, bills of exchange, and other negotiable paper.

The fee which a commission merchant charges for his services, or which an agent who buys and sells land, or collects rents, charges, is called a **commission.** The fee which a broker charges is called **brokerage.**

Commission and brokerage are sometimes reckoned as a rate per cent of the buying price when goods or other commodities are bought, or of the selling price when they are sold.

As a general rule a specified sum is charged for a transaction. Thus, if a bale of cotton or a bushel of wheat is sold, the fee is more likely to be a fixed sum than a rate per cent of the selling price.

The person who employs another person to buy, to sell, or to collect money is called a **principal.** The person who transacts business for a principal is called an **agent.** Commission merchants and brokers act in the capacity of agents.

289. *Example* 1. A commission merchant sold 400 boxes of oranges at \$2.75 per box, charging 5% commission. What was his commission, and how much did he remit?

SOLUTION:

400 boxes of oranges @ \$2.75 = \$1100.

5% of \$1100 = \$55, the commission.

\$1100 – \$55 = \$1045, amount remitted.

Example 2. A commission merchant sells 4500 bu. of wheat, charging $2\frac{1}{2}\%$ commission. If his commission is \$94.50, what is the price of wheat per bushel, and how much does the principal receive?

SOLUTION:

$2\frac{1}{2}$ of the selling price = \$94.50.

$$\therefore \quad 1\% \text{ of the selling price} = \frac{\$94.50}{2\frac{1}{2}}.$$

$$\therefore\ 100\% \text{ of the selling price} = \frac{\$94.50}{2\frac{1}{2}} \times 100 = \$3780.$$

The selling price of 4500 bu. = \$3780.

$$\therefore \text{ The selling price of 1 bu.} = \frac{\$3780}{4500} = \$.84.$$

The amount remitted = \$3780 – \$94.50 = \$3685.50.

Or proceed thus:

$$2\tfrac{1}{2}\% = \tfrac{1}{40}.$$

$\frac{1}{40}$ of the selling price = \$94.50.

∴ The selling price = \$3780.

Example **3.** A real estate agent remits to his principal \$9788.75, being the amount of the sales of four city lots after deducting a commission of $4\frac{1}{2}\%$. Find the selling price of the four lots and the agent's commission.

SOLUTION. The selling price $-$ $4\frac{1}{2}\%$ of the selling price $=$ $95\frac{1}{2}\%$ of the selling price.

$$95\tfrac{1}{2}\% \text{ of the selling price} = \$9788.75.$$

$$\therefore\ 1\% \text{ of the selling price} = \frac{\$9788.75}{95\frac{1}{2}}.$$

$$\therefore\ 100\% \text{ of the selling price} = \frac{\$9788.75}{95\frac{1}{2}} \times 100 = \$10{,}250.$$

$$\therefore\ \$10{,}250 - \$9788.75 = \$461.25, \text{ the commission.}$$

EXERCISE 165 (Written)

1. What is the commission for selling \$200 A. of land at \$40 an acre, the rate of commission being 4%?

2. An agent charges 6% for selling real estate. If he sells 350 A. of land at \$50 an acre, find his commission and the sum he remits to his principal. What per cent of the selling price does the principal receive?

3. If 200 boxes of oranges are sold at \$3.75 a box by a commission merchant who charges 6%, what is the commission? How much is remitted to the principal?

4. If 800 bu. of wheat are sold on a commission of $\frac{1}{2}$¢ per bushel, what is the commission?

5. How many bushels of barley at 45¢ per bushel must a commission merchant sell at $2\frac{1}{2}\%$ commission to make an annual salary of \$1080?

6. If 480 bbl. of flour at \$3.90 a barrel are sold on commission at a rate of 3%, find the commission. What is the net amount realized from the sale?

7. A broker sells 2500 lb. of beef at 12¢ a pound. What is his brokerage at 1½%?

8. Thirty-six hundred gallons of oil are sold by a broker at 45¢ per gallon. If he charges 3½% brokerage, find his brokerage and the amount remitted.

9. A commission merchant sells 1500 T. of hay at $15 per ton, and charges 5% commission. Find his commission. How much does he remit? What per cent of the selling price does he remit?

10. A coal dealer receives a commission of 90¢ a ton for selling anthracite coal, and 80¢ a ton for selling bituminous coal. If he sells an equal quantity of each kind of coal, and if his entire commission is $1530, how many tons of each kind does he sell?

11. A broker charges 35¢ for selling a bale of cotton. How many bales must he sell to realize for himself $295.40? If a bale contains 500 lb., and the brokerage is equivalent to ⅞%, find the price of cotton per pound.

12. A commission merchant sells 1200 doz. eggs at 18¢ per dozen. Find his commission at 8%.

13. If a commission of $70.98 is paid for buying 40 A. of land at $54.60 an acre, what is the rate per cent of commission?

14. If $284 is received for selling grain on a commission of ⅛¢ per bushel, how many bushels of grain were sold?

15. If an agent charges 3% for collecting debts, and in one month his commission from this source is $126, what is the amount collected?

16. A real estate agent's commission at 4½% is $351. Find the amount of his sales and the amount he remits to his principal.

17. A broker buys 10,000 bu. of wheat at 79½¢, and sells the wheat the next day at 79⅞¢, charging ⅛¢ a bushel for buying and ⅛¢ for selling. Find his commission.

18. An agent remits to his principal $9184.50 as the net proceeds from the sale of 12,000 bu. of wheat. Find the agent's commission if he charges at the rate of 2½%. Find also the selling price of wheat per bushel.

19. After deducting a commission of 3%, an agent remits $1813.90 from the sale of oats at 46¾¢ per bushel. How many bushels of oats were sold?

20. When the market price of pork is 13.6¢ per lb., how many pounds can be bought for $520.20, if 2% is charged for brokerage?

21. How many acres of land can be bought for $4635 at $30 an acre, by a real estate agent who charges a commission of 3%?

REVIEW QUESTIONS

1. What is commission? What is brokerage?

2. What is the meaning of the terms, *agent*, *broker*, *principal?*

3. When an agent sells property at a rate per cent, how does he calculate his commission?

4. If an agent buys goods at a rate per cent, how does he find his commission?

5. If a broker sells grain and charges a fraction of a cent a bushel for selling, how is his commission calculated?

6. If a broker buys grain and charges so much a bushel, how do you find the number of bushels that can be bought for a sum of money?

7. If you were given the commission and the rate per cent of commission, how would you find the amount of sales? How would you find the total remittance for these sales?

8. If you were given the remittance for a sale and the rate per cent of commission, how would you determine the amount of the sale?

9. If you shipped to a commission merchant a number of barrels of apples, and a month later he sold them and sent you the proceeds after deducting his commission of 4%, how would you determine from these data the price your apples brought in the market?

10. Which usually receives the greater commission, a commission merchant or a broker? Give a reason for your answer.

CHAPTER XVII

INTEREST

290. **Interest** is the price paid for the use of money.

The sum loaned is called the **principal.**

Interest is always reckoned as a rate per cent of the principal. The rate per cent is for 1 year, unless something to the contrary is expressly stated.

The **amount** is the sum of the principal and the interest.

In the commercial world, a month is reckoned as 30 da.

Bankers and others who make a business of lending money find the interest by means of tables. These tables are generally made out on the basis of 30 days to a month.

In reckoning interest for periods less than 1 year, the exact number of days between the dates of lending and paying is now generally made use of. Those who do not make use of interest tables reckon the time between dates in the manner explained in § 196.

291. *Example.* Find the interest on $780 for 1 yr. 2 mo. 10 da. at 7%.

SOLUTION. 1 yr. 2 mo. 10 da. $= 1\frac{70}{360}$ yr. $= 1\frac{7}{36}$ yr.

$\$780 \times .07 =$ int. for 1 yr.

$\$780 \times .07 \times \frac{43}{36} =$ int. for $1\frac{7}{36}$ yr.

$$\frac{\$780 \times .07 \times 43}{36} = \$65.22. \quad \textit{Ans.}$$

292. A concrete quantity which is contained an exact number of times in another concrete quantity is called an **aliquot part** of that quantity.

Thus, $2\frac{1}{2}$ yd. is an aliquot part of 10 yd.; $2.75 is an aliquot part of $11.

Example. Find the interest on $780 for 1 yr. 2 mo. 10 da. at 7%.

SOLUTION BY ALIQUOT PARTS:

$$\begin{array}{lrl} & \$780 & \\ & .07 & \\ \cline{2-2} & \$54.60 & = \text{int. for } 1 \text{ yr.} \\ 2 \text{ mo.} = \frac{1}{6} \text{ of } 1 \text{ yr.} & 9.10 & = \text{int. for } 2 \text{ mo.} \\ 10 \text{ da.} = \frac{1}{6} \text{ of } 2 \text{ mo.} & 1.52 & = \text{int. for } 10 \text{ da.} \\ \cline{2-2} & \$65.22 & = \text{int. for } 1 \text{ yr. } 2 \text{ mo. } 10 \text{ da.} \end{array}$$

Both the above methods are natural methods of solution. There are various artificial methods of solving interest problems. These, as a general rule, possess no great advantage over the two methods illustrated. Later we shall refer to special methods of solution. For the present, practice in the methods shown is most essential.

EXERCISE 166 (Written)

Find the interest on:

1. $600 for 1 yr. at 4%; for 1 yr. at 5%; for 1 yr. at 6%; for 1 yr. at 8%.

2. $850 for 1 yr. at 7%; for 1 yr. at 8%; for 1 yr. at 9%.

3. $950 for 1 yr. at 3%; for 1 yr. at 4%; for 1 yr. at 8%.

4. $982 for 2 yr. at 4%; for 3 yr. at 5%; for 1 yr. 6 mo. at 6%.

5. $1000 for 1 yr. 3 mo. at 6%; for 1 yr. 5 mo. at 6%.
6. $1200 for 1 yr. 4 mo. at 8%; for 1 yr. 5 mo. at 7%.
7. $1500 for 1 yr. 7 mo. at 9%; for 1 yr. 8 mo. at 8%.
8. $1600 for 2 yr. 3 mo. at 5%; for 1 yr. 7 mo. at 6%.
9. $2000 for 1 yr. 6 mo. at 4%; for 1 yr. 9 mo. at 5%.
10. $3500 for 10 mo. at 7%; for 7 mo. at 8%.
11. $156.40 for 1 yr. at 4½%.
12. $185.50 for 1 yr. at 4½%.
13. $375.60 for 5 mo. at 6%.
14. $928.40 for 2 mo. at 8%.
15. $735.60 for 3 mo. at 7%.
16. $1200 for 2 mo. at 8%.
17. $1350.50 for 5 mo. at 5%.
18. $393.80 for 7 mo. at 4%.
19. $385.40 for 8 mo. at 5%.
20. $934.54 for 9 mo. at 8%.
21. $2713.64 for 10 mo. at 9%.
22. $3800 for 11 mo. at 7%.
23. $2825 for 10 mo. at 7%.
24. $2700 for 1 yr. 1 mo. at 6%.
25. $3280 for 1 yr. 3 mo. at 5%.
26. $4500 for 8 mo. 15 da. at 6%.
27. $329.50 for 7 mo. 10 da. at 8%.
28. $982 for 9 mo. 18 da. at 6%.
29. $545 for 10 mo. 25 da. at 8%.
30. $775.24 for 11 mo. 24 da. at 7%.

293. *Example.* Find the interest on \$384.42 from Jan. 11, 1903, to April 30, 1903, at 7%.

SOLUTION. *Step* 1. Find the number of days.

In January there are 20 da. after Jan. 11.
In February there are 28 da.
In March there are 31 da.
In April there are 30 da.
Total 109 da. 109 da. = $\frac{109}{360}$ of 1 yr.

Step 2. Multiply the principal by the rate, and that product by the number denoting the time expressed in years.

$$\$\overset{192.21}{\cancel{384.42}} \times .07 \times \frac{109}{\underset{180}{\cancel{360}}} = \frac{\$19.221 \times .07 \times 109}{18} = \$8.15. \textit{ Ans}$$

$$\begin{array}{r|r}
 & \$19.221 \\
 & 109 \\
\cline{2-2}
 & 172989 \\
 & 19221 \\
\cline{2-2}
 & \$2095.089 \\
 & .07 \\
\cline{2-2}
6 & \$146.65623 \\
\cline{2-2}
3 & 24.44 \\
\cline{2-2}
\multicolumn{1}{r}{} & 8.15
\end{array}$$

SOLUTION BY ALIQUOT PARTS. 109 da. = 3 mo. 19 da.

$$\begin{array}{r}
\$384.42 \\
.07 \\
\hline
\end{array}$$

	\$26.9094	= int. for 1 yr.
3 mo. = $\frac{1}{4}$ of 1 yr.	6.727	= int. for 3 mo.
15 da. = $\frac{1}{6}$ of 3 mo.	1.121	= int. for 15 da.
3 da. = $\frac{1}{5}$ of 15 da.	.224	= int. for 3 da.
1 da. = $\frac{1}{3}$ of 3 da.	.075	= int. for 1 da.
	\$8.147	= int. for 3 mo. 19 da.

Ans. \$8.15.

EXERCISE 167 (Written)

Find the interest on:

1. $450 from Jan. 12 to April 18 at 6%.
2. $783 from Feb. 14 to April 24 at 8%.
3. $2385 from Mar. 6 to Nov. 11 at 6%.
4. $3950 from July 4 to Dec. 6 at 6%.
5. $4280 from Aug. 31 to Nov. 18 at 8%.
6. $7335 from July 5 to Sept. 14 at 7%.
7. $3280 from Jan. 8 to July 5 at 4%.
8. $4592.40 from Jan. 10 to May 4 at 3%.
9. $384.75 from Mar. 10 to Aug. 14 at 10%.
10. $327.50 from April 6 to July 3 at 9%.

EXERCISE 168 (Written)

1. Find the amount of $800 for 1 yr. 2 mo. 15 da. at 4%.

   HINT. Find the interest and add it to the principal.

2. Find the amount of $580 for 3 yr. 3 mo. at 6%.
3. Find the amount of $750 for 10 mo. 15 da. at 8%.
4. Find the amount of $327.60 for 11 mo. 12 da. at 9%.
5. Find the amount of $326.54 for 8 mo. 8 da. at 5%.
6. Find the amount of $739.90 for 5 mo. 11 da. at 4%.
7. Find the amount of $843.90 for 6 mo. 14 da. at 8%.
8. Find the amount of $325 for 9 mo. 12 da. at 4½%.
9. Find the amount of $982 for 8 mo. 16 da. at 5%.
10. Find the amount of $375 for 7 mo. 14 da. at 6%.
11. Find the amount of $1280 from Jan. 10 to July 14 at 8%.
12. Find the amount of $3580 from Jan. 14 to Aug. 18 at 8%.

13. Find the amount of $1500 from March 11 to July 19 at 6%.

14. Find the amount of $4350 from Aug. 14 to Dec. 17 at 7%.

15. Find the amount of $1200 from Jan. 1 to July 8 at 7%.

EXACT INTEREST

294. Interest reckoned on the basis of 365 days to the year is called **exact interest.** Exact interest is used by the United States Government and sometimes in business transactions.

Example. Find the exact interest on $2384.50 from Jan. 12 to July 5 at 5%.

SOLUTION. From Jan. 12 to July 5 there are $(19 + 28 + 31 + 30 + 31 + 30 + 5)$ days $=$ 174 days.

$$\$2384.50 \times .05 \times \tfrac{174}{365} = \text{exact interest.}$$

$$\frac{\$2384.50 \times .05 \times 174}{365} = \$56.84\text{, nearly.}$$

EXERCISE 169 (Written)

Find the exact interest on:

1. $913 from Jan. 4 to Feb. 4 at 5%.
2. $731.11 from Jan. 14 to Jan. 28 at 7%.
3. $52.50 from Jan. 1 to April 28 at 7%.
4. $2745 from Feb. 1 to April 6 at 5%.
5. $1095.80 from March 6 to June 7 at 5%.
6. $1911.17 from March 1 to May 11 at 7%.
7. $1464.98 from Jan. 4 to May 30 at 6%.
8. $10,565.65 from May 13 to June 25 at 4%.
9. $834 from Feb. 5 to July 12 at 11%.
10. $3561.50 for 81 da. at 5%.

PROMISSORY NOTES

295. A written promise by one person to pay another person on demand, or after a specified time, a sum of money is called a **promissory note.**

The following are promissory notes written in standard form:

Galveston, Texas, March 7, 1903.
$380.75
Sixty days after date I promise to pay to
the order of................James Moore................
..................Three hundred eighty......$\frac{75}{100}$ Dollars
at......the First National Bank....................
Value received, with interest at 5%.
John Mosley.
No. 20. May 6/9, 1903.

Dallas, Texas, March 4, 1903.
$474.25
On demand I promise to pay to the order of
.....................John Anderson.....................
............Four hundred seventy-four......$\frac{25}{100}$ Dollars
at.............the First National Bank...............
Value received, with interest at 7%.
James Rowe.
No. 34. Due.......

The first of the above promissory notes is called a **time note**; the second is called a **demand note.**

The person who promises to pay is called the **maker.** John Mosley is the maker of the first note above.

The person to whom the money is to be paid is called the **payee.** The person who has legal possession of a note is called its **holder.**

The sum specified in a note is called its **face.** A time note is legally due on the date indicated. In some states 3 days more than are indicated in the note are allowed before the note is legally due. These days are called **days of grace.** The day on which a note is legally due is called the **day of maturity.**

A note made payable to the order of a person, or a note made payable to the bearer is **negotiable,** *i.e.* it may be transferred from one person to another person.

A note made payable to the payee only is **non-negotiable.**

When a note payable to the order of the payee is transferred, every holder before parting with it must *indorse* it, *i.e.* write his name on the back of it. Every *indorser* thus becomes liable for the payment of the note, if the maker fails to pay it. The holder in whose possession the note is at maturity presents it to the maker for payment. If the maker refuses to pay it, the holder engages a Notary Public to give to the indorser, or indorsers, a written notice of its non-payment. This notice is called a **protest.** A protest must be sent on the date of maturity; otherwise the indorsers are not held responsible for the payment of the note.

An indorser who writes over his signature the words *without recourse* is not held responsible for the payment of the note.

A note made payable to *the bearer* is negotiable without indorsement. In some states a note must contain the words *value received* in order to be legal.

If the words *with interest* are not in a note, no interest is charged. If, however, the note is not paid on the date of maturity, interest at the legal rate may be charged. If a note contains the words *with interest*, and no rate is specified, it is then understood that the note bears the rate of interest usually charged in the state where it is made.

When the time of payment is indicated in months, calendar months are understood.

A note drawn March 6, and payable two months after date, matures on May 6 in states where days of grace are not allowed, and on May 9 in states where days of grace are allowed.

296. The following table gives the legal rates of interest and the maximum rates allowed by agreement in the states and territories. It indicates also where days of grace are allowed on time notes and time drafts.

The usage with respect to sight drafts is somewhat different.

States and Territories	Legal Rate	Rate allowed by Contract	Days of Grace
Alaska	6	10	3
Alabama	8	8	3
Arkansas	6	10	3
Arizona	7	any rate	3
California	7	any rate	
Colorado	8	any rate	
Connecticut	6	(*a*)	
Delaware	6	6	
District of Columbia	6	10	
Florida	8	10	
Georgia	7	8	3
Idaho	7	12	
Illinois	5	7	
Indiana	6	8	3

States and Territories	Legal Rate	Rate allowed by Contract	Days of Grace
Iowa	8	10	3
Indian Territory	6	10	3
Kansas	6	10	
Kentucky	6	6	3
Louisiana	5	8	
Maine	6	any rate	3
Maryland	6	6	
Massachusetts	6	any rate	
Michigan	5	7	3
Minnesota	6	10	3
Mississippi	6	10	3
Missouri	6	8	3
Montana	10	any rate	
Nebraska	7	10	3
Nevada	7	any rate	3
New Hampshire	6	6	3
New Jersey	6	6	
New Mexico	6	12	3
New York	6	6	
North Carolina	6	6	3
North Dakota	7	12	
Ohio	6	8	
Oklahoma	7	12	3
Oregon	6	10	
Pennsylvania	6	6	
Rhode Island	6	any rate	3
South Carolina	7	8	3
South Dakota	7	12	3
Tennessee	6	6	
Texas	6	10	3
Utah	8	any rate	
Vermont	6	6	
Virginia	6	6	
Washington	7	12	
West Virginia	6	6	
Wisconsin	6	10	
Wyoming	8	12	3

(*a*) Any rate, but only 6% can be collected by law.

297. Find the date of maturity, and the interest on the following note when it shall have matured.

New York, N. Y., Jan. 12, 1903.

$480.75

Sixty days after date I promise to pay to the order of........*Thomas Rutgen*....................

....................*Four hundred eighty*......$\frac{75}{100}$ *Dollars*

at......*the Bank of Commerce*........................

Value received, with interest.

Jacob Cooper.

No. 78. Due.......

Solution. Count forward 60 da. from Jan. 12. Doing so, we get (19 + 28) days for January and February, making 47 da. in all.

$$60 - 47 = 13.$$

The note matures March 13, 1903.

The legal rate of interest in New York is 6%.

The interest = $\$480.75 \times .06 \times \frac{1}{6} = \$4.81$, nearly.

EXERCISE 170 (Written)

Find the date of maturity, interest, and amount due at legal rate of interest on each of the following indicated notes:

	Face	Date	Where Payable	Time
1.	$480	Jan. 12, 1903	Texas	60 da.
2.	$650	Jan. 4, 1903	Ala.	30 da.
3.	$900	Jan. 30, 1903	Ga.	45 da.
4.	$1000	Feb. 11, 1903	Fla.	30 da.

	FACE	DATE	WHERE PAYABLE	TIME
5.	$540	Mar. 6, 1903	Tenn.	1 mo.
6.	$825	April 4, 1903	Ky.	3 mo.
7.	$920	May 10, 1903	Cal.	2 mo.
8.	$530	June 8, 1903	N.Y.	3 mo.
9.	$950	July 6, 1902	N.J.	90 da.
10.	$720	Aug. 14, 1902	Penn.	60 da.
11.	$840	Nov. 10, 1902	Ohio	60 da.
12.	$730	Dec. 15, 1902	Mo.	45 da.

13. The following note was paid March 30, 1903. Find the amount due.

> *Austin, Texas, January 12, 1903.*
>
> *$325*
>
> *On demand I promise to pay to the order of*
>
> ----------------------*E. J. Villavaso*---------------------
>
> --------------*Three hundred twenty-five*------*Dollars.*
>
> *Value received, with interest at 6%.*
>
> *D. Rice.*
>
> *No. 91.* *Due*-------

Find the amount due on each of the following indicated demand notes:

	DATE	FACE	WHERE PAYABLE	PAID AFTER DATE
14.	Aug. 10, 1902	$600	La.	9 mo. 15 da.
15.	Sept. 4, 1902	$800	Miss.	6 mo. 18 da.
16.	Oct. 15, 1902	$550	Utah	3 mo. 15 da.
17.	Nov. 12, 1902	$650	Oregon	5 mo. 12 da.
18.	Dec. 4, 1902	$850	Ill.	8 mo. 15 da.
19.	July 20, 1902	$980	Va.	7 mo.

PARTIAL PAYMENTS

298. When payments are made on a note, these payments are known as **partial payments.** These payments and their dates of payment are written on the back of the note.

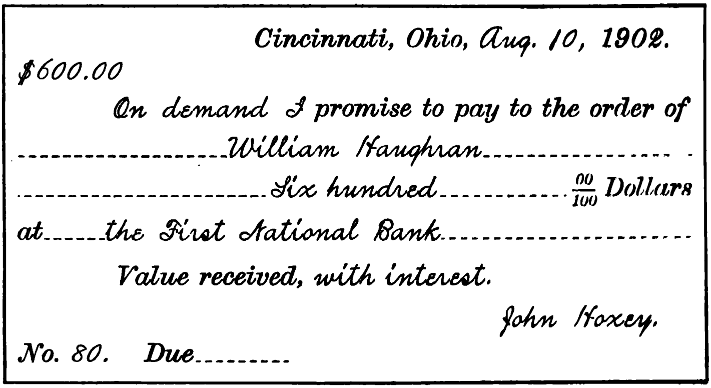
There are several methods of computing the amounts due on such notes. The best known and most widely used are the *United States Rule* and the *Merchants' Rule.*

UNITED STATES RULE

Find the amount of the principal until the time of the first payment, or until the sum of two or more payments equals or exceeds the interest.

Subtract the payment, or the sum of the two or more payments from the amount.

Proceed with the remainder as a new principal. Continue in this manner until the date of settlement.



Cincinnati, Ohio, Aug. 10, 1902.

$600.00

On demand I promise to pay to the order of

William Haughran

Six hundred $\frac{00}{100}$ *Dollars*

at the First National Bank

Value received, with interest.

John Hoxey.

No. 80. Due

This note was indorsed as follows:

1902, Dec. 15, $100.
1903, Feb, 12, $150.
1903, March 15, $150.

Find the amount due April 1, 1903.

SOLUTION:

DATES OF INDORSEMENT			TIME BETWEEN DATES			PAYMENTS
yr.	mo.	da.	yr.	mo.	da.	
1902	8	10		4	5	\$100
1902	12	15		1	27	\$150
1903	2	12		1	3	\$150
1903	3	15			16	
1903	4	1				

\$600 = first principal. $\$600 \times .06 \times \frac{4\frac{1}{6}}{12} = \$12.50.$
12.50 = int. for 4 mo. 5 da.
\$612.50 = amt. for 4 mo. 5 da.
\$100 = first payment.
\$512.50 = second principal. $\$512.50 \times .06 \times \frac{1.9}{12} = \$4.87.$
4.87 = int. for 1 mo. 27 da.
\$517.37 = amt. for 1 mo. 27 da.
150.00 = second payment.
\$367.37 = third principal. $\$367.37 \times .06 \times \frac{1.1}{12} = \$2.02.$
2.02 = int. for 1 mo. 3 da.
\$369.39 = amt. for 1 mo. 3 da.
150.00 = third payment.
\$219.39 = fourth principal. $\$219.39 \times .06 \times \frac{16}{360} = \$.59.$
.59 = int. for 16 da.
\$219.98 = amt. for 16 da. \$219.98. *Ans.*

EXERCISE 171 (Written)

1. A man borrows from a loan association \$3000 at 8% interest. If he pays \$100 at the end of every 3 mo. for 1 yr., find the amount due at the end of the year.

2. A man borrows from a building and loan company \$2000 at 8% interest. He pays in monthly installments of \$50 each. How much does he owe at the end of 6 mo.?

3. If you borrow \$1800 at 6% interest and pay annually \$600, how much do you owe at the end of 3 yr.?

4. If you borrow $1500 at 8% interest and pay semi-annually $300, how much do you owe at the end of 2 yr.?

5. An Indiana note bearing the maximum rate of interest, dated July 1, 1900, and drawn for $3000 had the following indorsements: Dec. 1, 1900, $400; July 1, 1901, $300; Nov. 1, 1901, $200; March 1, 1902, $500; Jan. 1, 1903, $500. Find the amount due April 1, 1903.

6. A Michigan note for $2000, dated June 15, 1900, and bearing the highest rate of interest allowed by contract, had the following indorsements: Nov. 15, 1900, $200; Jan. 5, 1901, $450; June 10, 1901, $200; Jan. 1, 1902, $200. Find the amount due July 1, 1902.

7. A Kentucky note for $1000, with interest, dated April 1, 1901, had the following indorsements: July 1, 1901, $400; Dec. 1, 1901, $200; Feb. 1, 1902, $300. Find the amount due April 1, 1902.

8. A Virginia note for $1500, with interest at 6%, and dated Jan. 2, 1902, had the following indorsements: March 3, 1902, $400; May 12, 1902, $300; July 11, 1902, $400. Find the amount due at date of settlement, Nov. 8, 1902.

NOTE. In finding interest in Partial Payment problems where settlement is made within a calendar year, use exact number of days between dates.

THE MERCHANTS' RULE

299. The Merchants' Rule for computing interest on partial payments is used when settlement is made within one year from the date of the note. The rule is as follows:

I. Compute the amount of the face of the note until the date of settlement.

II. Compute the amount of each indorsement from its date until the date of settlement, and add these amounts.

III. Subtract the sum of the amounts of the indorsements from the amount of the face of the note. The remainder will be the amount due on the date of settlement.

Example. A note for $500, with interest at 8%, dated Jan. 10, 1902, had the following indorsements: April 1, 1902, $100; May 10, 1902, $200; July 1, 1902, $100. Find the amount due Nov. 1, 1902.

SOLUTION. From Jan. 10 to Nov. 1 is 9 mo. 21 da.
From April 1 to Nov. 1 is 7 mo.
From May 10 to Nov. 1 is 5 mo. 21 da.
From July 1 to Nov. 1 is 4 mo.

$500 for 9 mo. 21 da. amounts to		$532.33
$100 for 7 mo. amounts to	$104.67	
$200 for 5 mo. 21 da. amounts to	207.60	
$100 for 4 mo. amounts to	102.67	
The payments amount to		414.94
Balance due Nov. 1,		$117.39

EXERCISE 172 (Written)

1. A note for $500, with interest at 6%, dated Jan. 2, 1902, had the following indorsements: March 2, 1902, $100; May 5, 1902, $150; July 10, 1902, $200. Find the amount due Nov. 1, 1902.

2. A note for $600, with interest at 8%, dated Feb. 1, 1902, was indorsed as follows: March 11, 1902, $200; May 15, 1902, $100; July 3, 1902, $100. Find the balance due Aug. 1, 1902.

3. A note for $800, with interest at 8%, dated Feb. 10, 1902, was indorsed as follows: March 15, 1902, $200; April 10, 1902, $100; June 3, 1902, $200. Find the amount due Nov. 1, 1902.

SAVINGS BANK DEPOSITS

300. Savings banks give interest on deposits at the rate of 3%, 3½%, or 4%. The interest is computed semi-annually and marked on the Dr. side of the bank book, for the depositor. The balance to the credit of the depositor is computed in the same way as the balance in problems in partial payments, by the Merchants' Rule.

Dr.							Cr.
1902				1902			
Jan. 8	Deposit,	$125	00	Feb. 10	Withdrawal,	$15	00
March 7	Deposit,	25	00	May 4	Withdrawal,	20	00
June 10	Deposit,	30	00		Balance,	145	00
		$180	00			$180	00
July 6	Balance,	$145	00				
" "	Int. to date,						

The above represents a page of the bank book of a depositor of a savings bank. On the left-hand side, or Dr. (debtor) side, are entered the deposits. On the right-hand, or Cr. (credit) side, are entered the withdrawals. The difference between the deposits and the withdrawals is marked **Balance**, and is written on the credit side, and then brought on the Dr. side. On July 6 the depositor has on the Dr. side the balance of $145 and the interest to date. How is the interest computed? The answer is:

(1) Find the interest on $125 from Jan. 8 to July 6.
Find the interest on $25 from March 7 to July 6.
Find the interest on $30 from June 10 to July 6.
Add the three results.

(2) Find the interest on $15 from Feb. 10 to July 6.
Find the interest on $20 from May 4 to July 6.
Add the two results.

(3) Subtract the latter sum from the former.

(A bank clerk would find these interests by means of interest tables.)

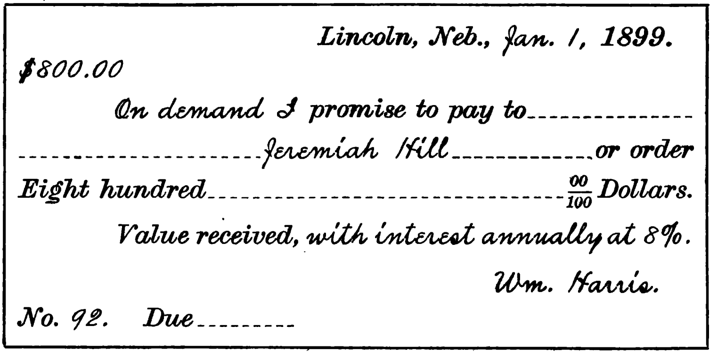
EXERCISE 173 (Written)

1. A depositor in a savings bank had on deposit July 8, 1902, $223.50. On Dec. 7, 1902, he deposited $12. On Sept. 13 he withdrew $5; on Oct. 14, $5; on Oct. 26, $5; and on Dec. 19, $10. Balance his account and find the interest to Jan. 8, 1903, rate of interest being $2\frac{1}{2}\%$.

2. A depositor in a savings bank had on deposit Jan. 1, 1902, $100. He made deposits as follows: Feb. 5, $30; March 5, $40; April 10, $15. He made withdrawals as follows: March 20, $20; May 10, $15. Balance his account and find the interest at 3% to July 1, 1902.

3. A student made the following deposits in a savings bank: Sept. 1, 1902, $250; Nov. 10, $100. He made the following withdrawals: Oct. 15, $90; Dec. 15, $100. Find the interest at 4% on his account Jan. 2, 1903.

ANNUAL INTEREST



Lincoln, Neb., Jan. 1, 1899.

$800.00

On demand I promise to pay to ---------------

--------------------- *Jeremiah Hill* ------------- *or order*

Eight hundred --------------------------------- $\frac{00}{100}$ *Dollars.*

Value received, with interest annually at 8%.

Wm. Harris.

No. 92. Due ---------

301. In some states if a note contains the words "with interest annually," and if the interest remains unpaid for a number of years, then the interest due at the end of each year bears interest until the date of settlement.

The *interest* by this method of reckoning is called **annual interest.** The interest we have up to this time considered is known as **simple interest.**

Example. How much is due on a note for \$800, dated Jan. 5, 1898, and bearing interest annually at 8%, if left unpaid, both in principal and interest, until March 10, 1903.

SOLUTION:

Interest on \$800 for 1 yr. at 8% = \$64.

Interest on \$800 for 5 yr. 2 mo. 5 da. at 8% = \$331.56.

yr.	mo.	da.
1903	3	10
1898	1	5
5	2	5

The first year's interest is due Jan. 5, 1899, and bears interest until March 10, 1903. The interest falling due at the end of the second year bears interest until March 10, 1903, and so on.

We have \$64 bearing interest for	4 yr.	2 mo.	5 da.
We have \$64 bearing interest for	3 yr.	2 mo.	5 da.
We have \$64 bearing interest for	2 yr.	2 mo.	5 da.
We have \$64 bearing interest for	1 yr.	2 mo.	5 da.
We have \$64 bearing interest for		2 mo.	5 da.
We have \$64 bearing interest for	10 yr.	10 mo.	25 da.

Interest on \$64 for 10 yr. 10 mo. 25 da. at 8% = \$56.73.
Interest on \$800 for 5 yr. 2 mo. 5 da. at 8% = \$331.56.
Amount due = \$800 + \$331.56 + \$56.73 = \$1188.29. *Ans.*

EXERCISE 174 (Written)

1. Find the amount due April 1, 1903, on a note for \$900, dated July 15, 1899, and bearing interest annually at 7%.

2. Find interest from March 6, 1898, to Jan. 1, 1903, on note for \$1400, interest payable annually at 8%.

3. Find amount due Jan. 1, 1903, on a note for \$1200, dated July 1, 1897, interest payable annually at 6%.

4. Find amount due Jan. 1, 1903 on note for \$1000, dated Jan. 1, 1898, interest payable semiannually at 5%.

5. Find the amount due at the end of 5 years on a coupon note, interest payable semiannually, if the face of the note is \$700, and the rate of interest is 7%.

302. *Example.* What principal will produce \$78.75 interest in 75 da. at $7\frac{1}{2}\%$?

SOLUTION:

Since principal × rate × time = interest,

$\therefore$ Principal $\times .075 \times \frac{75}{360} = \$78.75$.

$\therefore$ Principal $= \$78.75 \div (.075 \times \frac{75}{360}) = \$5040$.

ANALYTICAL SOLUTION:

\$1 produces $7\frac{1}{2}$¢ interest in 1 yr.

$\therefore$ $\$\frac{2}{15}$ produces 1¢ interest in 1 yr.

$\therefore$ $\$\frac{2}{15} \times 360$ produces 1¢ interest in 1 da.

$\therefore$ $\$\frac{2}{15} \times \frac{360}{75}$ produces 1¢ interest in 75 da.

$\therefore$ $(\$\frac{2}{15} \times \frac{360}{75}) \times 7875$ produces 7875¢ in 75 da.

$\$\frac{2}{15} \times \frac{360}{75} \times 7875 = \$5040$.

EXERCISE 175 (Written)

What principal will produce:

1. \$60 in $1\frac{1}{2}$ yr. at 8%?
2. \$120 in 2 yr. at 5%?
3. \$135 in 1 yr. 6 mo. at 9%?
4. \$36 in 3 yr. at 5%?
5. \$144 in 1 yr. 4 mo. at $4\frac{1}{2}\%$?
6. \$12 in 1 yr. at 4%?

7. \$21 in 1 yr. at $3\frac{1}{2}\%$?

8. \$84 in 3 yr. 6 mo. at 3%?

9. \$16.90 in 2 yr. 2 mo. at 4%?

10. \$42 in 2 yr. 4 mo. at 4%?

11. \$25.50 in 6 mo. at 5%?

12. \$5.40 in 4 mo. at 5%?

13. \$6.75 in 9 mo. at 3%?

14. \$50 in 8 mo. at 8%?

15. By investing money at 6% per annum, interest payable semiannually, I receive every 6 mo. \$339. How much have I invested?

303. *Example.* In what time will \$840 produce \$57.40, interest at 5%?

SOLUTION. Since principal × rate × time = interest,

$$\therefore\ \$840 \times .05 \times \text{time} = \$57.40.$$

$$\therefore\ \text{Time} = \$57.40 \div (\$840 \times .05) = \frac{\$5740}{\$840 \times 5} = 1\tfrac{11}{30}.$$

The time is $1\frac{11}{30}$ yr. Here we multiplied dividend and divisor by 100. $\frac{11}{30}$ of a year $= \frac{11}{30}$ of 12 mo. $= 4\frac{2}{5}$ mo.

Two-fifths of a month $= \frac{2}{5}$ of 30 da. = 12 da.

∴ The time is 1 yr. 4 mo. 12 da.

ANALYTICAL SOLUTION:

\$1 produces 5¢ interest in 1 yr.

∴ \$1 produces 1¢ interest in $\frac{1}{5}$ of 1 yr.

∴ \$1 produces 5740¢ interest in $\frac{5740}{5}$ of 1 yr.

∴ \$840 produces 5740¢ interest in $\frac{1}{840}$ of $\frac{5740}{5}$ of 1 yr.

$\frac{1}{840} \times \frac{5740}{5} = \frac{41}{30} = 1\frac{11}{30}$.

∴ The time is $1\frac{11}{30}$ yr. = 1 yr. 4 mo. 12 da.

NOTE. The actual computation by both methods is the same.

EXERCISE 176 (Written)

In what time will:

1. \$1088.75 produce \$87.10 interest at 8%?
2. \$144 produce \$21.60 interest at 5%?
3. \$215 produce \$6.45 interest at 5%?
4. \$1160 produce \$278.40 interest at 6%?
5. \$810 produce \$56.70 interest at 7%?
6. \$312.50 produce \$43.75 interest at 8%?
7. \$2220 produce \$216.45 interest at 3%?
8. \$1400 produce \$78.75 interest at 5%?
9. \$480 produce \$85.50 interest at 9½%?
10. \$3835 produce \$345.15 interest at 8%?
11. \$1380 produce \$88.55 interest at 3½%?
12. \$5400 produce \$267.75 interest at 7%?
13. \$7630 produce \$1335.25 interest at 6%?
14. \$2205 produce \$213.15 interest at 8%?
15. \$1250 produce \$45.94 interest at 7%?

304. *Example.* At what rate per cent will \$720 produce \$42.50 interest in 1 yr. 2 mo. 5 da.?

SOLUTION. Principal × time × rate = interest.

$\therefore \$720 \times (1 + \frac{1}{6} + \frac{5}{360}) \times \text{rate} = \$42.50.$

$\therefore \$720 \times 1\frac{13}{72} \times \text{rate} = \$42.50; \ \$720 \times 1\frac{13}{72} = \$850.$

$\therefore \$850 \times \text{rate} = \$42.50.$

$$\therefore \text{Rate} = \frac{\$42.50}{\$850} = .05.$$

∴ Rate per cent is 5, or rate is 5%.

ANALYTICAL SOLUTION. The interest on \$720 for 1 yr. 2 mo. 5 da. is \$42.50.

The interest on \$720 for $14\frac{1}{6}$ mo. is \$42.50.

The interest on \$720 for 1 mo. is $\frac{\$42.50}{14\frac{1}{6}}$.

The interest on \$720 for 12 mo. is $\frac{\$42.50 \times 12}{14\frac{1}{6}}$.

The interest on \$1 for 12 mo. is $\frac{\$42.50 \times 12}{14\frac{1}{6} \times 720}$.

The interest on \$100 for 12 mo. is $\frac{\$42.50 \times 12 \times 100}{14\frac{1}{6} \times 720} = \$5$.

The rate is 5%.

A *third method* may be given. Find the interest on the principal for the given time at 1%. Divide the given interest by the interest at 1%. This quotient is the required rate.

EXERCISE 177 (Written)

Find the rate per cent when the interest on —

1. \$750 for 1 yr. is \$45.
2. \$928 for 1 yr. is \$64.96.
3. \$880 for 1½ yr. is \$79.20.
4. \$945 for 6 mo. is \$37.80.
5. \$828 for 8 mo. is \$38.64.
6. \$1200 for 1 yr. 3 mo. is \$90.
7. \$1800 for 9 mo. is \$67.50.
8. \$2400 for 8 mo. is \$64.
9. \$2500 for 9 mo. 18 da. is \$100.
10. \$3000 for 7 mo. 12 da. is \$90.
11. \$3750 for 4 mo. 15 da. is \$112.50.
12. \$2754 for 2 mo. 20 da. is \$55.08.
13. \$4846 for 6 mo. 20 da. is \$121.15.
14. \$1440 for 7 mo. 10 da. is \$52.80.

Find the rate per cent when —

15. $1080 amounts to $1123.20 in 8 mo.

16. $1200 amounts to $1270 in 10 mo.

17. $1600 amounts to $1640 in 6 mo.

18. $2460 amounts to $2574.80 in 9 mo. 10 da.

19. $92 amounts to $102.12 in 2 yr.

20. $324 amounts to $333.72 in 8 mo.

305. *Example.* What principal will amount to $136.27 in 1 yr. 3 mo. 15 da. at 5%?

SOLUTION. We shall solve this question on the principle that the amount varies directly with the principal when the time and the rate are fixed. Thus, $2 at any given rate and for any given time will amount to twice as much as $1 at the same rate and for the same time.

The interest on $1 for 1 yr. is $.05.

∴ The interest on $1 for 3 mo. is $.0125.

∴ The interest on $1 for 15 da. is $\$.0020\frac{5}{6}$.

∴ The interest on $1 for 1 yr. 3 mo. 15 da. is $\$.0645\frac{5}{6}$.

∴ The amount of $1 for 1 yr. 3 mo. 15 da. is $\$1.0645\frac{5}{6}$.

$\$1.0645\frac{5}{6}$ × (*number* of dollars in the principal) = $136.27.

∴ (*Number* of dollars in the principal) = $136.27 ÷ $\$1.0645\frac{5}{6}$ = $128, nearly.

The process of dividing $136.27 by $1.0645\frac{5}{6}$ is performed by multiplying dividend and divisor by 6, and then proceeding in the usual manner.

EXERCISE 178 (Written)

What principal will amount to —

1. $840 in 1 yr. at 5%?

2. $749 in 1 yr. at 7%?

3. $645 in 1 yr. at $7\frac{1}{2}$%?

4. $903 in $1\frac{1}{2}$ yr. at 5%?

5. $414 in 3 yr. at 5%?

6. $255.30 in 2 yr. at $5\frac{1}{2}$%?

7. $12,540.45 in 2 yr. 3 mo. at 4%?
8. $168.35 in 7 mo. 6 da. at 6%?
9. $618.67 in 1 yr. 4 mo. at 5%?
10. $646.80 in 8 mo. at 4%?
11. $776.07 in 1 yr. 1 mo. at $4\frac{1}{2}$%?
12. $481.50 in 1 yr. at 7%?
13. $432.55 in 11 mo. at 8%?
14. $282.75 in 2 yr. 4 mo. 15 da. at $7\frac{1}{2}$%?
15. $2090.07 in 1 yr. 1 mo. 15 da. at 8%?
16. $2067.75 in 1 yr. 5 mo. at $10\frac{1}{2}$%?
17. $268.28 in 1 yr. 7 mo. at 6%?
18. $254.25 in 7 mo. 15 da. at $9\frac{1}{2}$%?
19. $25,346.25 in 1 yr. 3 mo. at 10%?
20. $843.70 in 8 mo. 3 da. at $7\frac{1}{2}$%?

SPECIAL METHODS FOR COMPUTING INTEREST

We shall illustrate two of these.

306. *Example.* Find the interest on $570 for 5 mo. 19 da. at 5%.

Six Per Cent Method

The interest on $1 for 1 yr. at 6% is $.06.
∴ The interest on $1 for 1 mo. at 6% is $.005.
∴ The interest on $1 for 1 da. at 6% is $$.000\frac{1}{6}$.
∴ The interest on $1 for 5 mo. at 6% is $.025.
∴ The interest on $1 for 19 da. at 6% is $$.003\frac{1}{6}$.
∴ The interest on $1 for 5 mo. 19 da. at 6% is $$.028\frac{1}{6}$.
∴ The interest on $570 for 5 mo. 19 da. at 6% is $$.028\frac{1}{6} \times 570 =$ $16.06.

To get the interest at 5%, subtract from the interest at 6% $\frac{1}{6}$ of the interest at 6%.

Interest at 6% = $16.06.
$\frac{1}{6}$ of interest at 6% = 2.68.
∴ Interest at 5% = 13.38.

In practice, **we multiply .005 by the number indicating the months; .000$\frac{1}{6}$ by the number indicating the days. Find their sum and multiply it by the number indicating the dollars in the principal.** This product gives the interest at 6%.

To find the interest at 7%, add $\frac{1}{6}$ of the interest at 6% to itself.

To find the interest at 7$\frac{1}{2}$%, add $\frac{1}{4}$ of the interest at 6% to itself.

To find the interest at 8%, add $\frac{1}{3}$ of the interest at 6% to itself.

To find the interest at 9%, add $\frac{1}{2}$ of the interest at 6% to itself.

To find the interest at 5%, subtract $\frac{1}{6}$ of the interest at 6% from itself.

To find the interest at 4$\frac{1}{2}$%, subtract $\frac{1}{4}$ of the interest at 6% from itself.

To find the interest at 4%, subtract $\frac{1}{3}$ of the interest at 6% from itself.

Twelve Per Cent Method

307. The interest on $1 for 1 yr. at 12% is $.12.
The interest on $1 for 1 mo. at 12% is $.01.
The interest on $1 for 1 da. at 12% is $.000$\frac{1}{3}$.

Example. Find the interest on $890 for 7 mo. 22 da. at 7%.

Find, first, the interest on $890 for 7 mo. 22 da. at 12%.
Interest on $1 for 7 mo. = $.07.
Interest on $1 for 22 da. = $.007$\frac{1}{3}$.

∴ Interest on $1 for 7 mo. 22 da. = $.077$\frac{1}{3}$.

∴ Interest on $890 for 7 mo. 22 da. = $.077$\frac{1}{3}$ × 890 = $68.83, nearly.

Interest at 5% = $\frac{5}{12}$ of the interest at 12%. $\frac{5}{12} = \frac{1}{4} + \frac{1}{6}$.

$\frac{1}{4}$ of $68.83 = $17.21, nearly.
$\frac{1}{6}$ of $68.83 = $11.47, nearly.
($\frac{1}{6} + \frac{1}{4}$) of $68.83 = $28.68, nearly.

Having found the interest at 12%, or 1% a month, we find the interest at —

3% by taking $\frac{1}{4}$ of this result.
4% by taking $\frac{1}{3}$ of this result.
6% by taking $\frac{1}{2}$ of this result.
5% by adding ($\frac{1}{4}$ and $\frac{1}{6}$) of this result.
7% by adding ($\frac{1}{4}$ and $\frac{1}{3}$) of this result.
8% by taking $\frac{2}{3}$ of this result.
9% by subtracting $\frac{1}{4}$ of this result from itself.

To compute interest at 12%:

Multiply the number indicating the months by .01, the number indicating the days by .000$\frac{1}{3}$; add, and multiply their sum by the number denoting the dollars in the principal.

NOTE. Taking into consideration the present usage in the commercial world, it seems that the 12% method is shorter and less involved than the 6% method.

EXERCISE 179 (Written)

REVIEW

1. Find the interest on $4000 for 1 yr. 1 mo. 2 da. at 9%.
2. Find the interest on $256.30 for 4 mo. 9 da. at 7%.
3. Find the interest on $30.85 for 11 mo. 6 da. at 5%.
4. Find the interest on $653 for 2 mo. 16 da. at 4%.

5. Find the interest on $2105.60 for 2 mo. 24 da. at 5%.

6. Find the amount of $805 for 10 mo. at 8%.

7. Find the amount of $507 for 1 yr. 12 da. at 8%.

8. What principal will produce $20.83 interest in 5 mo. at 5%?

9. What principal will produce $17.50 interest in 9 da. at 5%?

10. Find the rate of interest when $500 produces $2.92 interest in 1 mo.

11. Find the rate of interest when $250 produces $7.30 in 5 mo.

12. How much must I invest at 5% interest to have an annual income of $1200 from my investment?

13. A man buys a house and lot and rents it for $40 a month. Taxes and insurance cost him $120 a year. If his net receipts give him a profit of 6% on his investment, find the cost of the house and lot.

14. For how long a time must $3000 be loaned at 5% to produce $20 interest?

15. If I borrow $2400 at 7% interest and pay in principal and interest $2456, how long do I keep the money?

16. For how long a time must a sum of money be loaned at simple interest at 8% to produce in interest $\frac{1}{5}$ of itself?

17. For how long a time must a sum of money be loaned at simple interest at 6% to produce in interest $\frac{1}{10}$ of itself?

18. Find the exact interest of $1200 for 292 da. at 5%.

19. Find the exact interest of $7300 for 146 da. at 7%.

20. The exact interest of $10,800 at 5% is $324. Find the time.

21. In what time will $260 amount to $262.60 at 5%?

22. What sum must be deposited in a savings bank which pays 3½% interest to produce semiannually $8.75?

23. A man deposits his money in two banks. In one bank he has $572 which pays 3½%. The other bank gives 4% interest. If he receives as interest the same amount from both banks, how much money has he altogether?

24. If I invest half my money at 6% and the remainder at 4%, and derive an income of $650 annually, how much money have I invested?

REVIEW QUESTIONS

1. Define *principal*, *rate*, *per cent*, *interest*, *amount*.

2. How does exact interest differ from interest according to the common use of the term? What is the distinction between simple interest and annual interest?

3. How do you find the interest of a sum of money at a given rate and for a given time?

4. If you were given the interest, the rate, and the time, how would you find the principal?

5. Given the principal, interest, and time, how would you find the rate per cent? How would you find the rate?

6. Given the principal, the rate, and the interest, how would you find the time?

7. Given the principal, the amount, and the rate, how would you find the time?

8. Given the principal, the amount, and the time, how would you find the rate?

9. Given the amount, the rate, and the time, how would you find the principal?

10. If you knew the interest of a sum of money for a given time at 6%, how would you find the interest for the same sum for the same time at 5%? at 4%? at 8%?

11. If you knew the interest at 4%, how would you find the interest of the same sum at 7%? at 3%? at 5%? at 3½%?

12. If you were given the interest of a sum of money for a number of days, how would you determine from this the exact interest of the same sum for the same number of days?

13. For short periods of time, what is the ratio of the exact interest to the ordinary simple interest?

14. From the lender's point of view, which is preferable, annual interest or simple interest?

15. Write a promissory note for $200 dated Jan. 12, 1903, interest at 7%, and time 30 days.

16. A promissory note is drawn Jan. 15, 1903, payable 60 da. after date. Find its date of maturity in your state.

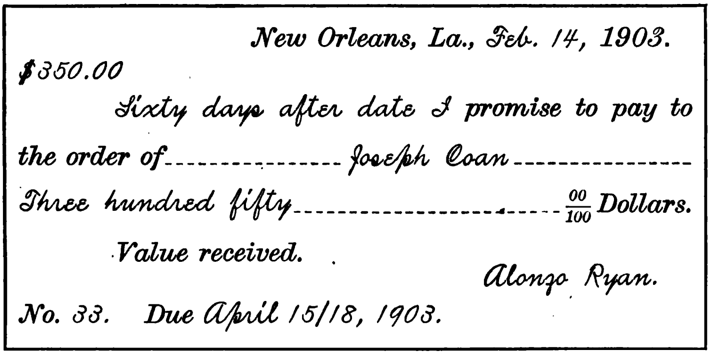
17. Explain the terms *maker*, *payee*, *holder*.

18. When is a promissory note negotiable? When is it non-negotiable?

19. State the Merchants' Rule for computing the interest on a note when partial payments are made. State the United States Rule for partial payments.

CHAPTER XVIII

BANK DISCOUNT, EXCHANGE



New Orleans, La., Feb. 14, 1903.

$350.00

Sixty days after date I promise to pay to the order of ---------------- Joseph Coan ----------------

Three hundred fifty --------------------------- $\frac{00}{100}$ Dollars.

Value received.

Alonzo Ryan.

No. 33. Due April 15/18, 1903.

308. The above time note is negotiable when indorsed. Supposing the payee, Joseph Coan, needs money, he can sell the note to a bank. The sum the bank gives him for the note is called the **proceeds** of the note. The difference between the proceeds and the face of the note is called the **bank discount.**

The **bank discount** is always a rate per cent of the value of the note on its day of maturity, reckoned from the date of the sale of the note to the day of maturity.

The **bank discount** is then the interest on the maturity value of the note computed from the date of discount to the date of maturity. This time is called the **term of discount.**

The maturity value of the note minus the **bank discount** is the **proceeds** of the note.

309. *Example.* Find the discount and the proceeds of the above note, if it was discounted at 8%, March 1, 1903.

SOLUTION. The bank charges discount from March 1 to April 18.

From March 1 to April 18 is 48 days.

Twelve Per Cent Method

The interest on \$1 for 1 da. at 12% = $\$.000\frac{1}{3}$.

The interest on \$1 for 48 da. at 12% = $\$.000\frac{1}{3} \times 48 = \$.016$.

The interest on \$350 for 48 da. at 12% = $\$.016 \times 350 = \$5.60$.

The interest at 12% = \$560.

The interest at 8% = \$3.73, nearly.

Bank discount = \$3.73.

Proceeds of the note = \$346.27.

EXERCISE 180 (Written)

Find the bank discount and the proceeds of the following indicated notes:

	DATE	WHERE DRAWN	TIME	FACE	DIS-C'TED	RATE OF DISC'T
1.	Jan. 12,	Atlanta, Ga.,	60 da.,	\$600,	Feb. 13,	8%.
2.	July 4,	Tuscaloosa, Ala.,	60 da.,	\$800,	Aug. 3,	8%.
3.	Mar. 3,	Nashville, Tenn.,	90 da.,	\$500,	Mar. 3,	6%.
4.	April 5,	Memphis, Tenn.,	60 da.,	\$700,	Apr. 5,	6%.
5.	May 7,	Key West, Fla.,	60 da.,	\$600,	May 10,	10%.
6.	May 9,	Cincinnati, O.,	20 da.,	\$900,	May 24,	8%.
7.	May 30,	Cleveland, O.,	90 da.,	\$750,	July 1,	6%.
8.	June 5,	Harrisburg, Pa.,	60 da.,	\$450,	July 5,	6%.
9.	Aug. 11,	Rochester, N. Y.,	30 da.,	\$800,	Aug. 11,	6%.
10.	Sept. 9,	Omaha, Neb.,	30 da.,	\$350,	Sept. 12,	10%.

	DATE	WHERE DRAWN	TIME	FACE	DIS-C'TED	RATE OF DISC'T
11.	Oct. 4,	San Antonio, Texas,	60 da.,	\$800,	Oct. 7,	10%.
12.	Oct. 14,	Guthrie, O. T.,	60 da.,	\$500,	Nov. 16,	12%.
13.	Nov. 10,	Augusta, Ga.,	90 da.,	\$600,	Nov. 25,	8%.
14.	Dec. 4,	Trenton, N. J.,	90 da.,	\$750,	Feb. 5,	6%.
15.	Jan. 10,	Springfield, Ill.,	45 da.,	\$650,	Feb. 9,	7%.
16.	Jan. 5,	Los Angeles, Cal.,	60 da.,	\$850,	Feb. 5,	9%.
17.	Jan. 30,	Portland, Oregon,	75 da.,	\$950,	Feb. 28,	9%.
18.	July 10,	Salt Lake City,	3 mo.,	\$380,	July 12,	9%.

Portland, Me., June 9, 1902.

\$250.00

Ninety days after date I promise to pay to the order of..............Rudolph Weber..............

Two hundred fifty..........................$\frac{00}{100}$ Dollars.

Value received, with interest at 6%.

Wm. Tucker.

No. 84. Due Sept. 7/10, 1902.

310. The above note was discounted July 8, 1903 at 8%. Find the bank discount and the proceeds.

Six Per Cent Method

SOLUTION. *Step* 1. Find the maturity value of the note.

The interest on \$1 for 1 mo. at 6% = \$.005.

The interest on \$1 for 1 da. at 6% = $\$.000\frac{1}{6}$.

The interest on \$1 for 3 mo. at 6% = \$.015.

The interest on \$1 for 3 da. at 6% = $\$.000\frac{1}{2}$.

The interest on \$1 for 3 mo. 3 da. = $\$.015\frac{1}{2}$.

The interest on $\$250$ for 3 mo. 3 da. $= \$.0155 \times 250 = \frac{\$.0155 \times 1000}{4} = \frac{\$15.50}{4} = \$3.87$.

Maturity value = $253.87.

Step 2. Find the term of discount.

The note was discounted July 8. The term of discount is the *exact* number of days from July 8 to Sept. 10, *i.e.* 64 da.

Step 3. Find the bank discount.

The bank discount is reckoned on the maturity value of the note. The rate is 8%. 64 da. = 2 mo. 4 da.

The interest of $\$1$ for 2 mo. at 6% $= \$.01$.

The interest of $\$1$ for 4 da. at 6% $= \$.000\frac{2}{3}$.

The interest of $\$1$ for 2 mo. 4 da. $= \$.010\frac{2}{3}$.

The interest of $\$253.87$ for 2 mo. 4 da. $= \$.010\frac{2}{3} \times 253.87$.

$\$.010\frac{2}{3} \times 253.87 = (\$.01 \times 253.87 = \$2.54, \text{nearly}) + (\$.000\frac{2}{3} \times 253.87 = \$.17, \text{nearly}) = \$2.71$.

The interest at 6% $= \$2.71$.

The interest at 8% $= \$2.71 + \frac{1}{3}$ of $\$2.71 = \$3.61$.

The *bank discount* $= \$3.61$.

The proceeds = maturity value − discount $= \$253.87 - \$3.61 = \$250.26$.

EXERCISE 181 (Written)

Find the discount and the proceeds of the following indicated notes:

	FACE	DATE	TIME	STATE	RATE OF INT.	RATE OF DISC'T	DISC'T
1.	$350,	Jan. 1,	45 da.,	Idaho,	10%,	12%,	Feb. 1.
2.	$395,	Jan. 10,	3 mo.,	Arkansas,	6%,	10%,	Jan. 20.
3.	$450,	Feb. 1,	30 da.,	Iowa,	6%,	8%,	Feb. 18.
4.	$600,	Mar. 1,	60 da.,	Colorado,	8%,	12%,	Mar. 31.

	Face	Date	Time	State	Rate of Int.	Rate of Disc't	Disc't
5.	$500,	Apr. 2,	60 da.,	Connecticut,	6%,	8%,	Apr. 17.
6.	$900,	Apr. 10,	30 da.,	Indiana,	6%,	8%,	Apr. 10.
7.	$1000,	Apr. 15,	60 da.,	Kansas,	8%,	10%,	Apr. 15.
8.	$750,	May 4,	60 da.,	Maryland,*	6%,	6%,	May 5.
9.	$800,	July 10,	30 da.,	Missouri,*	6%,	8%,	July 26.
10.	$400,	Aug. 15,	60 da.,	North Carolina,	6%,	6%,	Aug. 15.
11.	$850,	Nov. 11,	90 da.,	Washington	7%,	10%,	Nov. 12.
12.	$900,	Dec. 12,	45 da.,	Wisconsin,	6%,	6%,	Jan. 13.
13.	$1200,	Dec. 5,	3 mo.,	Texas,	6%,	10%,	Jan. 15.
14.	$1500,	Feb. 10,	90 da.,	West Virginia,	6%,	10%,	May 1.

311. *Example* **1.** For what sum must I draw a note payable in 48 da. so that when it is discounted I may realize $480, rate of discount being 8%?

SOLUTION:

Interest of $1 for 48 da. at 8% = $\$.010\frac{2}{3}$.

Proceeds of $1 for 48 da. at 8% = $\$.989\frac{1}{3}$.

$\$.989\frac{1}{3} \times$ (number denoting the dollars in the face of the note) = $480.

∴ The number denoting the dollars in the face of the note = $\$480 \div \$.989\frac{1}{3} = \$1440 \div \$2.968 = 485.18$, nearly.

Face = $485.18.

The note must be drawn for $485.18.

Example **2.** A note with interest at 7%, dated Houston, Texas, Jan. 4, 1903, and drawn for 60 da., is discounted Jan. 13, at 8%. If the proceeds of the note are $540, find its face.

SOLUTION. The note matures March 8, 1903.

The note is discounted Jan. 13. From Jan. 13 to March 8 = 54 da. = term of discount.

---

* In Maryland and Missouri banks count the day of discounting a note as well as the day of maturity.

Step 1. The interest of $1 for 2 mo. 3 da. at 7% = $.01225.

The maturity value of $1 of the face = $1.01225.

Step 2. The interest of $1.01225 for 54 da. at 8% = $.012147.

$1.01225 – $.012147 = $1.000103 = proceeds of $1 of face.

Step 3. Since the proceeds of $1 multiplied by the number of dollars gives the proceeds of that number of dollars, $1.000103 × (number of dollars in the face) = $540.

Number of dollars in the face = $540 ÷ $1.000103 = $539.94.

Face = $539.94.

EXERCISE 182 (Written)

Find (neglecting days of grace) the face of each of the following indicated notes:

PROCEEDS	DATE	TIME	INTEREST	DISCOUNT	DISCOUNTED
1. $735,	Mar. 3,	120 da.,	6%,	8%,	Apr. 4.
2. $825,	Mar. 10,	90 da.,	6%,	6%,	May 2.
3. $900,	July 9,	60 da.,	6%,	8%,	Aug. 30.
4. $583,	July 6,	90 da.,	7%,	10%,	Sept. 4.
5. $950,	Aug. 5,	60 da.,	8%,	10%,	Sept. 4.
6. $528,	Nov. 8,	4 mo.,	7%,	9%,	Dec. 8.
7. $995,	Dec. 5,	3 mo.,	6%,	8%,	Jan. 4.
8. $540,	Jan. 2,	60 da.,	6%,	9%,	Feb. 16.
9. $450,	Mar. 4,	90 da.,	8%,	8%,	May 3.
10. $750,	June 10,	120 da.,	6%,	7%,	Aug. 9.
11. $900,	Apr. 13,	45 da.,	7%,	8%,	Apr. 28.
12. $827,	May 14,	60 da.,	6%,	9%,	May 29.

13. If the value of a note at maturity is $550, and its proceeds at 8% is $539, find the term of discount.

14. If the value of a note at maturity is $850, and its proceeds when discounted at 6% is $845.75, find the term of discount.

15. If the maturity value of a note is $ 1200, and the proceeds of the note when discounted for 60 days is $ 1188, find the rate of discount.

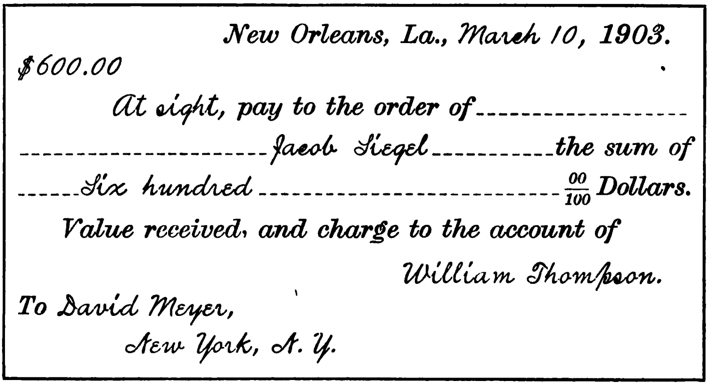
EXCHANGE

312. The method of paying debts to persons at a distance without the actual transmission of the money involved is called **exchange.**

Suppose William Thompson of New Orleans owes Jacob Siegel of New York $600. Suppose further, David Meyer of New York owes William Thompson of New Orleans $600. If William Thompson of New Orleans directs David Meyer of New York to pay Jacob Siegel of New York the amount named, the indebtedness of the parties concerned is thus cancelled without the risk of sending money from New Orleans to New York and from New York to New Orleans.

313. The written order of Thompson upon Meyer, requesting him to pay a sum of money is called a **draft.**

SIGHT DRAFT



New Orleans, La., March 10, 1903.

$600.00

At sight, pay to the order of

Jacob Siegel the sum of

Six hundred $\frac{00}{100}$ Dollars.

Value received, and charge to the account of

William Thompson.

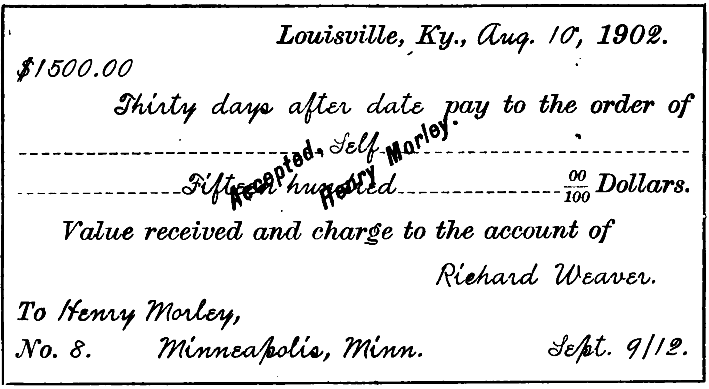
To David Meyer,
New York, N. Y.

The above draft is known as a **sight draft.** Thompson sends it by mail to Siegel, who on receiving it presents it to Meyer for payment. Thompson is called the **drawer.** Siegel is called the **payee.** Meyer is called the **drawee.**

If the draft were payable after date, or after sight, it would be called a **time draft.**

A **time draft,** on its receipt, is presented to the drawee for acceptance. To accept a draft is to write your name across the face of it. Acceptance is regarded as a promise to pay the sum specified in the draft.

TIME DRAFT



Louisville, Ky., Aug. 10, 1902.

$1500.00

Thirty days after date pay to the order of

Self

Fifteen hundred $\frac{00}{100}$ *Dollars.*

Value received and charge to the account of

Richard Weaver.

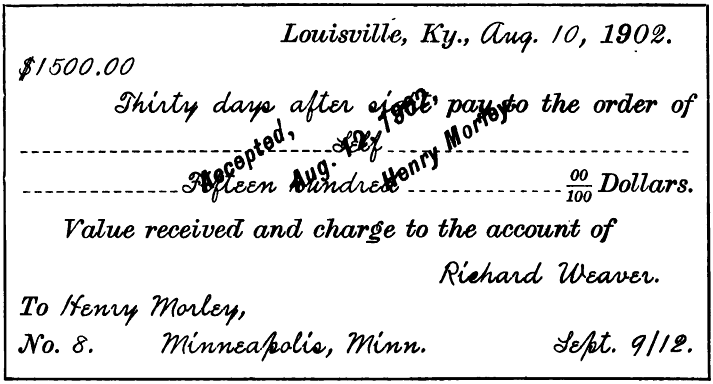
To Henry Morley,

No. 8. Minneapolis, Minn. *Sept. 9/12.*

Accepted, Henry Morley.

After acceptance the above draft is returned to Richard Weaver, who, if he desires, may have it discounted as a promissory note.

The date of maturity of a time draft payable after date is reckoned in the same manner as that of a promissory note.



Louisville, Ky., Aug. 10, 1902.

$1500.00

Thirty days after sight pay to the order of

Self

Fifteen Hundred $\frac{00}{100}$ *Dollars.*

Value received and charge to the account of

Richard Weaver.

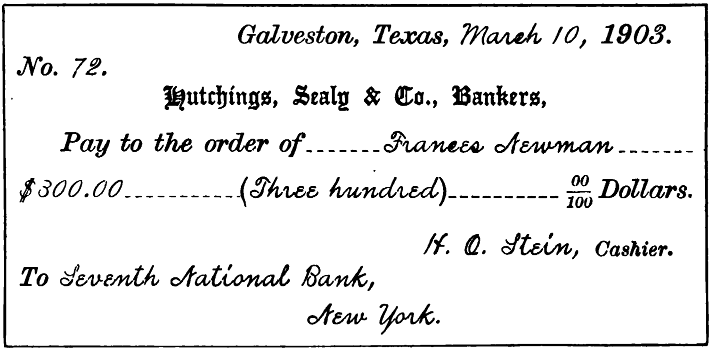
To Henry Morley,

No. 8. Minneapolis, Minn. Sept. 9/12.

Accepted, Aug. 12, 1902, Henry Morley

A time draft payable *after sight* matures as many days or months after date of acceptance as are indicated in face, without days of grace, or in three days more if days of grace are allowed in state where drawee resides.

314. Bank drafts form one of the most important mediums of exchange. If a merchant owes money in New York, Chicago, or in any other place, he can always purchase a draft from his home bank, and by transmitting this through the mails settle his indebtedness. The following is the form of a bank draft:



Galveston, Texas, March 10, 1903.

No. 72.

Hutchings, Sealy & Co., Bankers,

Pay to the order of Frances Newman

$300.00 (Three hundred) $\frac{00}{100}$ *Dollars.*

H. O. Stein, Cashier.

To Seventh National Bank,

New York.

The price of exchange is a matter of supply and demand. If, for instance, the Galveston banks freely offer New York or Chicago exchange, a purchaser will very probably get it for less than its face value. If exchange is scarce, its price is high. When exchange costs more than its face value, it is said to be at a *premium;* when it costs less than its face value, it is said to be at a *discount.* Exchange is *at par* when the cost of a draft is its face value. Exchange is always quoted as a rate per cent, or as so many dollars on a $1000. The exchange is always reckoned on the face value of the draft. Thus, $\frac{1}{8}\%$ premium, or $1.25 premium mean that the cost of a draft for $100 is 100\frac{1}{8}$, and the cost of a draft for $1000 is $1001.25. The rate of exchange is generally a fraction of 1%.

Express companies charge for money orders to any part of this country or Canada not over 30¢ for $100. It is reasonable to assume that a person will not pay much more than this rate for exchange.

EXERCISE 183 (Oral)

1. What is the cost of exchange on a sight draft for $400 at $\frac{1}{8}\%$ premium?

2. What is the cost of exchange on a sight draft for $300 at $\frac{1}{4}\%$ premium?

3. Find the cost of a draft on New York for $200 at $\frac{1}{8}\%$ premium.

4. Find the cost of a demand draft for $500 at $\frac{1}{4}\%$ premium.

5. Find the cost of a sight draft for $600 at $\frac{1}{4}\%$ premium.

6. Find how much must be paid for a sight draft for $1000 at $1.25 premium.

7. Find how much must be paid for a sight draft for $2000 at $2.50 premium.

8. Find the cost of remitting $800 from Dallas to Chicago by means of a bank draft when exchange is $\frac{1}{4}$% premium.

9. Find the cost of a sight draft on Chicago for $700, exchange being at $\frac{1}{4}$% discount.

10. Find the cost of a sight draft on New York for $400 when exchange is $\frac{1}{4}$% discount.

11. Find the cost of exchange on a draft for $1000, exchange being $\frac{3}{8}$% premium.

12. Find the cost of an express order for $1200 at the rate of 30¢ per $100.

13. If you had to remit $200 to Chicago when exchange is $\frac{1}{4}$% premium, which would you prefer, to buy exchange or to buy an express money order at the rate of 30¢ for $100?

14. What is the cost of an express money order for $1000?

15. What is the cost of a sight draft for $3000 at $1.50 premium?

16. What is the cost of a sight draft for $3000 at $2.50 premium?

315. *Example.* What is the cost of a draft on Buffalo for $800, payable in 30 da. at 6% interest, exchange being at the rate of $\frac{1}{4}$% premium?

SOLUTION:

The interest on $800 for 30 da. at 6% = $400.

Cost of exchange = $\frac{1}{4}$% of $800 = $2.00.

The cost of the draft = $800 + $2 − $4 = $798.

EXERCISE 184 (Written)

Find the cost of the following sight drafts :

	FACE	RATE OF EXCHANGE
1.	$2720,	$\frac{1}{4}$% premium.
2.	$3480,	$\frac{1}{2}$% premium.
3.	$5080,	$\frac{1}{8}$% premium.
4.	$6290,	$2.50 premium.
5.	$8290,	$1.25 discount.
6.	$9980,	$\frac{1}{4}$% discount.
7.	$5493,	$1.50 discount.
8.	$5280,	$1.50 premium.
9.	$6040,	$2.00 discount.
10.	$6090,	$1.25 premium.
11.	$5400,	$1.25 discount.
12.	$9870,	1% discount.
13.	$4500,	$\frac{1}{5}$% premium.
14.	$6920,	$\frac{1}{10}$% discount.
15.	$7780,	$\frac{1}{8}$% premium.
16.	$1234,	$\frac{1}{8}$% discount.

17. Find the cost of an express money order for $1400 at 30¢ for $100.

18. Find the cost of a draft for $10,000 at $\frac{1}{8}$% discount.

19. Find the cost of a draft for $600 payable in 30 da. without grace, the rate of interest being 6%, exchange being $\frac{1}{4}$% premium.

20. Find the cost of a draft for $1000 payable in 30 da., exchange being at par and the rate of interest being 8%.

21. Find the cost of a 45 da. draft for $900, exchange being at $\frac{1}{4}$% discount, and the rate of interest being 6%.

FOREIGN EXCHANGE

316. The settling of outstanding indebtedness between parties in different parts of this country by means of drafts is known as **domestic exchange.** The settling of debts between parties in this country and parties in foreign countries by means of drafts is called **foreign exchange.** Foreign drafts are known as **bills of exchange.** These are issued in duplicate, *i.e.* two are drawn exactly alike. The two bills are sent by different mails. As soon as one is paid, the other is void.

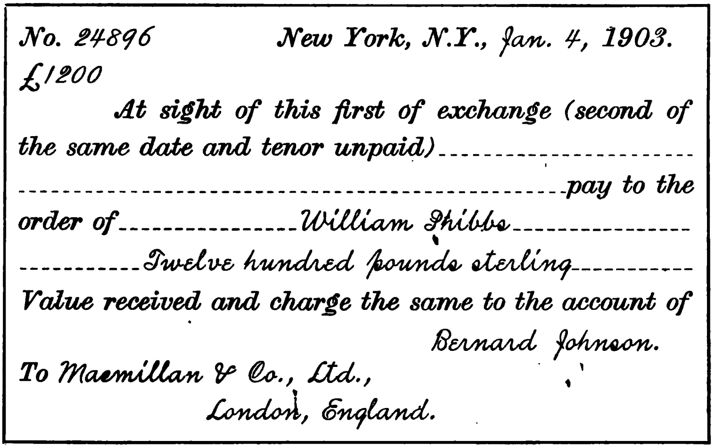
When we take into consideration the fact that the exports of this country for 1902 amounted, in round numbers, to $1,355,000,000, and that the imports for the same year amounted to upward of $900,000,000, we can readily appreciate the part bills of exchange play in the commerce of the world. While millions of dollars are owed by merchants in this country to merchants in foreign countries, and while merchants in foreign countries owe millions of dollars to merchants in this country, the actual settlement of their obligations involves annually the transfer of less than 5% of the amount of money.

The price paid for bills of exchange fluctuates. The intrinsic value of metallic coins, expressed in terms of the monetary unit of a country, is called the *par of exchange.* The value of foreign coins expressed in terms of the unit of United States currency is given in § 255.

Usually exchange is a little above or a little below the par of exchange.

Exchange on Great Britain is quoted at the number of dollars to £1; exchange on France is quoted at so many cents to the franc, or as so many francs to the dollar; exchange on Germany is quoted as so many cents to 4 marks, or as so many cents to a mark.

BILL OF EXCHANGE



No. 24896 *New York, N.Y., Jan. 4, 1903.*

£1200

At sight of this first of exchange (second of the same date and tenor unpaid) ---------- *pay to the order of* ---------- *William Shibbs* ---------- *Twelve hundred pounds sterling* ----------

Value received and charge the same to the account of

Bernard Johnson.

To Macmillan & Co., Ltd.,
London, England.

EXERCISE 185 (Written)

1. Find the cost of a draft on London for £1000, exchange being 4.872.

2. Find the cost of a bill of exchange for £550 15*s.*, exchange being 4.85.

3. Find the cost of a bill of exchange for 5750 francs, exchange being 5.15, *i.e.* 5.15 francs = \$1.

4. Find the cost of a draft on Manchester for £3500 6*s.* 8*d.* when exchange is quoted at 4.87½.

5. Find the cost of a draft on Berlin for 1480 marks when exchange is quoted at 95¼, *i.e.* 4 marks = 95¼¢.

6. Find the cost of a draft on Vienna for 8500 krones, rate of exchange being 1 krone = \$.202.

7. Find the cost of a draft for 1500 lira when exchange is quoted at 19.3, *i.e.* 19.3¢ = 1 lira.

8. Find the cost of each of the following drafts:

	FACE	WHERE PAYABLE	RATE OF EXCHANGE
(*a*)	£928 10*s.*	Liverpool	4.85
(*b*)	£850 6*s.* 8*d.*	Belfast	4.86¼
(*c*)	2500 marks	Berlin	24
(*d*)	9280 francs	Havre	5.16
(*e*)	8500 lira	Naples	19¼
(*f*)	£584 13*s.* 4*d.*	Dublin	4.85¾

317. *Example.* What is the face of a New York draft on Liverpool which costs $7297.50 when exchange is quoted at 4.86½?

SOLUTION. $\$4.865 = £1.$

$$\$1 = \frac{£1}{4.865}.$$

$$\$7297.50 = \frac{£7297.50}{4.865} = £1500.$$

EXERCISE 186 (Written)

Find the face of each of the following drafts:

	COST	WHERE PAYABLE	RATE OF EXCHANGE
1.	$5835	London	4.86¼
2.	$1218.75	Newcastle	4.87½
3.	$9063.05	Berlin	95.2
4.	$3724.90	Paris	19.3
5.	$718.24	Christiana	26.8
6.	$2366.82	Belfast	4.86½
7.	$1037	Paris	5.18½
8.	$5664.80	Hamburg	23.8

REVIEW QUESTIONS

1. What is bank discount?

2. What is meant by the proceeds of a note?

3. What is meant by the term of discount?

4. How is the bank discount on a note computed?

5. What is meant by the maturity value of a note?

6. What are days of grace?

7. How is the date of maturity found?

8. What is meant by exchange?

9. Name the parties to a sight draft?

10. What is a time draft?

11. What is meant by accepting a draft?

12. What is a bank draft?

13. What is meant by exchange being at par?

14. If drafts on another city are at par in your city, what may you infer?

15. How is the cost of exchange on a sight draft computed?

16. Why should the premium on exchange not exceed a fraction of 1%?

17. What do you mean by foreign exchange?

18. What do you mean by domestic exchange?

6. What must I pay for 5% stock so as to make a profit of 8% on my investment?

7. How much must be invested in Northwestern paying 6%, at 206, to derive an income of $900?

8. Which is the more profitable investment, 5% stock at 119⅞ or 8% stock at 219⅞?

9. How much must be paid for a $1000 bond at 87½?

10. City bonds bought at 89⅞ pay 5%. What rate of interest do they pay?

11. If I have $5000 stock in United States 4's at 102, what annual dividend do I receive? If I sold my stock what should it bring?

12. How much should be realized by selling $10,000 of each of the stocks quoted above at the highest prices paid March 11, 1903?

13. Which is the better investment, Illinois Central paying 6% dividend, at 137⅜, or United States 4's at 109⅞?

14. How many shares of stock must a broker sell to make $54.75 brokerage?

INSURANCE

323. **Insurance** is an indemnity against loss to property, and also a payment on the death of a person, or after the expiration of a specified time.

There are two kinds of insurance, **property insurance** and **life insurance.**

The price paid for insurance is called a **premium.** The written contract between the party insured and the insurance company is called **a policy.**

The premium on a life insurance policy is an annual payment of so many dollars on the $1000.

In the following table, the first column gives the age, and the second column gives the annual premium on $1000 insurance for a life insurance policy taken out at the age specified. The figures are taken from the book of rates of a reputable life insurance company.

AGE	ANNUAL PREMIUM	AGE	ANNUAL PREMIUM	AGE	ANNUAL PREMIUM
21	$18.65	38	$29.80	50	$47.00
24	$19.92	40	$31.50	52	$51.20
27	$21.50	42	$33.80	54	$56.40
30	$23.30	44	$36.40	56	$62.40
33	$25.30	46	$39.60	58	$69.00
35	$27.10	48	$43.00	60	$76.40

EXERCISE 190 (Written)

Use the rates given above.

1. Find the premium on a $5500 policy taken out at the age of 27 yr.

2. What is the premium on a life insurance policy for $4500 taken at 33 yr. of age?

3. If you insure your life at 24 yr. of age, what premium do you pay on a policy of $6500?

4. Find the premium on a life insurance policy of $10,500 taken at 38 yr. of age.

5. A man 52 yr. old insures his life for $8500; find his annual premium.

6. A man 56 yr. old insures his life for $9450; find his annual premium.

7. A man 40 yr. old insures his life, and pays an annual premium of $992.25. Find the amount of his policy.

8. A man 42 yr. old insures his life, and pays an annual premium of $554.32. Find the amount for which he insured.

9. The annual premium on a life insurance policy of $27,100 is $734.41. Find the rate per $1000. Also find the age of the insured.

10. The annual premium on a life insurance policy for $9800 is $784.08. Calculate the rate on $1000 insurance. At what age was the insurance taken out?

11. If the annual premium on a life insurance policy, at the rate of $43 per $1000 insurance, is $107.50, find the amount of the policy.

12. If the annual premium on a life insurance policy, at the rate of $19.92 per $1000 insurance, is $104.58, find the amount of the insurance.

13. If the premium on a life insurance policy, at the rate of $23.30 per $1000 insurance, is $542.89, find the amount of the policy.

14. A man 52 yr. old insures his life; find the amount of the policy he may obtain on payment of an annual premium of $230.40.

PROPERTY INSURANCE

324. The premium on **property insurance** is a rate per cent of the value of the property insured. The computation of premiums involves, therefore, nothing beyond a knowledge of percentage.

The rates charged for insurance against fire vary somewhat throughout the country. The following specifies the rates for fire insurance on dwellings in Galveston, Texas:

Dwelling houses with shingle roofs, 80¢ per $100.
Dwelling houses with slate or metal roofs, 65¢ per $100.

Add 10¢ for each dwelling within 80 ft.
For 3 yr., double annual premium.
For 5 yr., treble annual premium.
Mercantile risks, 1% up.

A frame dwelling house with a shingle roof, and having a dwelling house on each side within a distance of 80 ft. would thus pay an annual premium of $1 per $100, or $2 per $100 for a policy good for 3 yr., or $3 per $100 for a policy good for 5 yr. If 4 dwelling houses adjoined it, the respective premiums for 1 yr., 3 yr., and 5 yr. would be $1.20, $2.40, $3.60 per $100.

Insurance companies as a rule do not insure property at its full value.

325. Insurance companies will insure property against fire, and if before the expiration of the time for which it is insured, the assured desires to have his policy cancelled, the company will pay back a portion of the premium to the assured, on the basis of the table below.

For example, if a person owns a residence insured for one year, and in the course of the year, say in ninety days, sells it and cancels his policy, the company insuring will return to the assured a part of the sum paid for insurance. The custom is, however, to transfer the *policy* with the *deed*.

The following tariff of rates for periods less than 1 yr. is in general use throughout this country and is known as:

SHORT RATE TARIFF

5 days,	7% of the annual premium.
8 days,	9% of the annual premium.
10 days,	10% of the annual premium.
12 days,	12% of the annual premium.
15 days,	14% of the annual premium.

20 days, 17% of the annual premium.
25 days, 19% of the annual premium.
30 days, 20% of the annual premium.
35 days, 23% of the annual premium.
40 days, 26% of the annual premium.
45 days, 27% of the annual premium.
50 days, 28% of the annual premium.
60 days, 30% of the annual premium.
70 days, 37% of the annual premium.
80 days, 38% of the annual premium.
90 days, 40% of the annual premium.
105 days, 45% of the annual premium.
120 days, 50% of the annual premium.
150 days, 60% of the annual premium.
180 days, 70% of the annual premium.
210 days, 75% of the annual premium.
240 days, 80% of the annual premium.
270 days, 85% of the annual premium.
300 days, 90% of the annual premium.
330 days, 95% of the annual premium.
360 days, 100% of the annual premium.

The above table gives the per cent of the annual premium retained by the insurance company. The remainder of the premium is returned to the party who insures.

EXERCISE 191 (Written)

What is the premium on mercantile risks for 1 yr. for the following amounts at the rates specified:

1. $3500 at $1.10 per $100?
2. $4500 at $1.20 per $100?
3. $5800 at $1.20 per $100?
4. $9200 at $1.25 per $100?
5. $8750 at $1.25 per $100?
6. $9500 at $1.10 per $100?
7. $10,500 at $1.50 per $100?
8. $9500 at $1.60 per $100?
9. $20,000 at $1.60 per $100?
10. $18,000 at $1.25 per $100?

EXERCISE 192 (Written)

Find the premium for insuring dwellings against loss by fire for 3 yr. at the rates specified:

1. $2500 at $1.30 per $100.
2. $2000 at $1.15 per $100.
3. $4500 at $1.50 per $100.
4. $3000 at $1.25 per $100.
5. $2500 at $1.90 per $100.
6. $5500 at $1.70 per $100.
7. $6500 at $1.50 per $100.
8. $4000 at $1.80 per $100.
9. $5400 at $1.70 per $100.
10. $3300 at $1.80 per $100.
11. $7500 at $1.60 per $100.
12. $7250 at $1.40 per $100.
13. $10,500 at $1.30 per $100.
14. $19,250 at $1.25 per $100.
15. $16,450 at $1.60 per $100.
16. $7900 at $1.35 per $100.
17. $22,500 at $1.10 per $100.
18. $18,250 at $1.20 per $100.

EXERCISE 193 (Written)

Find the premiums for insuring dwellings for 5 yr. for the following amounts, at the rates specified:

1. $2400 at $1.80 per $100.
2. $9300 at $1.50 per $100.
3. $8500 at $1.60 per $100.
4. $9450 at $1.50 per $100.
5. $6500 at $1.90 per $100.
6. $5400 at $1.60 per $100.
7. $9500 at $1.90 per $100.
8. $12,000 at $1.75 per $100.
9. $18,000 at $1.75 per $100.
10. $200,000 at $1.25 per $100.
11. $15,500 at $1.60 per $100.
12. $16,200 at $1.60 per $100.
13. $1800 at $1.90 per $100.
14. $1750 at $1.75 per $100.

EXERCISE 194 (Written)

1. A cotton factor insures cotton to the amount of $12,000 at $1.10 per $100 for 1 yr. He sells the cotton in 3 mo. and has his policy cancelled. Find amount of premium returned by the insurance company.

2. If a residence is insured for 1 yr. for $5000 at the rate of 90¢ for $100, and in 4 mo. the policy is cancelled, how much is returned by the insurance company?

3. A commission merchant insures a stock of goods for $18,000 for 1 yr. at $1.25 per $100. In 2 mo. he cancels his policy. Find the amount of premium returned by the insurance company.

4. A commission merchant insures cotton to the amount of $25,000 for 1 yr. at $1.25 per $100. In 20 da. he cancels his policy. Find the return premium the insurance company gives on the premium paid.

5. A grain dealer insures a quantity of grain valued at $50,000 at $1 per $100. In 15 da. he cancels his policy. Find the return premium allowed by the insurance company.

6. A commission merchant insures $20,000 worth of coffee at 90¢ on the $100. If he disposes of the coffee in 35 da. and cancels his policy, find his return premium allowed by the insurance company.

7. A commission merchant insures a quantity of cotton for $25,000 at 90¢ per $100. He sells the cotton in 40 da. and cancels his policy. Find his return premium.

8. Find the return premium on an insurance of 80,000 bu. of wheat at 70¢ a bushel, at the rate of 90¢ per $100, if the policy is cancelled in 30 da.

9. Find the return premium on an insurance of 60,000 bu. of barley at 40¢ per bushel, at the rate of $1.10 per $100, if the policy is cancelled in 60 da.

326. *Example* 1. A church is insured for 1 yr. at the rate of 90¢ per $100. If the premium is $225 and the amount of the policy is $\frac{5}{6}$ of the estimated value of the church, find the estimated value of the church.

SOLUTION. 90 ¢ × (number of hundreds in the policy) = $225.

$$\text{No. of hundreds in the policy} = \frac{\$225}{\$.9} = 250.$$

Amount of the policy = $250 × 100 = $25,000.

Five-sixths of the value of the church = $25,000,

∴ The value of the church = $30,000.

Or, analytically:

On $100 the premium is 90 ¢.

$$\text{On } \frac{\$100}{90} \text{ the premium is } 1\,¢.$$

$$\text{On } \frac{\$100}{90} \times 22{,}500 \text{ the premium is } 22{,}500\,¢ \text{ or } \$225.$$

The amount of the policy is $25,000.

Five-sixths of the value of the church is $25,000.

∴ One-sixth of the value of the church is $5000.

∴ Six-sixths of the value of the church is $30,000.

The estimated value is $30,000.

Example 2. If it cost a commission merchant $2.97 to insure cotton for 45 da. at the rate of $1.10 per $100, what was the value of the cotton, if the tariff of rates above given was in operation?

SOLUTION. The commission merchant paid on $100 27% of $1.10 = $.2970.

The insurance rate is $.2970 on $100.

The insurance rate is $.00297 on $1.

$.009297 × (number indicating the dollars in policy) = $2.97.

$$\text{Number indicating dollars in policy} = \frac{\$2.97}{\$.00297} = 1000.$$

Value of cotton = $1000.

Or, analytically:

$$\text{The insurance rate is } \$.297 \text{ on } \$100.$$

$$\text{The insurance rate is } \$.001 \text{ on } \frac{\$100}{297}.$$

$$\text{The insurance rate is } \$1 \text{ on } \$100 \times \frac{1000}{297}.$$

$$\text{The insurance rate is } \$2.97 \text{ on } \$100 \times 1000 \times \frac{2.97}{297} = \$1000.$$

EXERCISE 195 (Written)

Find the amount of insurance in each case if the following premiums were paid at the rates specified:

	PREMIUMS	RATE ON $100		PREMIUMS	RATE ON $100
1.	$22.40,	80¢.	**11.**	$1051.25,	$1.45.
2.	$72.25,	85¢.	**12.**	$720.75,	$1.55.
3.	$132.00,	$1.10.	**13.**	$61.25,	$1.75.
4.	$180.50,	95¢.	**14.**	$316.84,	$1.78.
5.	$144.50,	85¢.	**15.**	$68.95,	$1.85.
6.	$168.75,	75¢.	**16.**	$57.66,	$1.86.
7.	$34.30,	70¢.	**17.**	$39.71,	$1.90.
8.	$396.75,	$1.15.	**18.**	$76.05,	$1.95.
9.	$100.80,	$1.20.	**19.**	$65.34,	$1.98.
10.	$816.75,	$1.65.	**20.**	$101.25,	$2.25.

21. A commission merchant insured his grain at the rate of 85¢ per $100. He cancelled his policy in 90 da. and thereby got a return premium of $45.90. Find the amount of his policy.

22. A merchant insures his stock of goods at the rate of 75¢ per $100; he cancels his policy in 5 mo., and thereby gets a return premium from the insurance company of $12.30. Find the amount of insurance.

23. By insuring a dwelling at the rate of $1.20 per $100, an insurance company loses, on account of the destruction of the building by fire a month later, $9761.44. Find the amount of the policy.

24. For what sum must I insure my library, worth $790, at the rate of $1.25 per $100, so that in case of loss by fire I may get back the value of the library and the premium?

25. Seven months after a stock of goods had been insured the entire stock was sold and the policy cancelled. If the return premium from the insurance company was $60.40 and the amount of the policy was $18,120, find the rate of insurance.

26. The net charge for insuring a dwelling for 1 yr. after giving a return premium at the end of 8 mo. was $23.12. If the dwelling was insured for $3400, find the rate of insurance.

TAXATION

327. The price paid for the support of government is called a **tax**. Taxes on property, whether real estate or personal property, are generally reckoned on the basis of $100. A tax on voters is called a **poll tax**.

EXERCISE 196 (Written)

1. The assessed valuation of the property in Galveston is $15,572,000. The school tax is 20¢ per $100. Find the total school tax.

2. Find the tax on property valued at $9500 at 2 mills on the dollar.

3. Find the tax on property valued at $10,500 at the rate of 40 cents on the $100.

4. The assessed valuation of property in San Diego, Cal., is $11,500,000. The rate of taxation is $1.45 per $100. Find the total tax of San Diego.

5. The tax rate in San Antonio, Tex., is $1.67 on $100. Taking the valuation of property in San Antonio at $33,000,000, find the total property tax.

6. The tax for the Galveston sea wall is 5 mills on $1. The bonded indebtedness of the city on account of the sea wall will be about $1,400,000. If the bonds pay 5% interest, by how much does the sea wall tax exceed the interest on the sea wall bonds?

7. Find the tax on property valued at $7500, at 1.74%.

8. Find the tax on property valued at $8500, at 5 mills on $1.

9. Find the total tax of a property owner whose property is assessed at $9000, the tax rate being $1.79%, if he pays 2 polls at $1.50 each.

10. Find the tax on property valued at $3250, at 8 mills on the dollar.

11. The assessed valuation of property in Providence, R. I., is, in round numbers, $198,000,000. The tax rate is $1\frac{3}{5}$%. Find the total tax collected if 95% of the tax levy is collected.

12. The assessed valuation of property in Duluth, Minn., is nearly $27,000,000, and the bonded indebtedness of the city of Duluth is about $5,100,000. Supposing Duluth city bonds bear interest at the rate of 5%, what tax rate per $100 would equal the interest on the bonds?

13. The assessed valuation of property in Cleveland, O., is about $180,000,000. The tax rate is 3.05%. Find, on the supposition that 95% of the tax levy is collected, the amount raised by taxation on property in Cleveland.

14. The assessed valuation of real estate and personal property in St. Louis is nearly $390,000,000; the public bonded debt is about $24,000,000. If the bonds bear 5% interest, find the tax rate on the dollar necessary to pay this interest.

15. Supposing St. Louis votes to raise by taxation on property $1,950,000, what is the rate of taxation necessary to do this?

16. If $21,280 is raised by taxation at the rate of 50¢ on the $100, find the valuation of taxable property.

17. The tax rate in a city is $1.20 per $100, and 90% of the tax levy is collected. If the amount raised annually by taxation on property is $3,780,000, find the assessed valuation of property. Supposing a man's property in this city was assessed for $8500, and that he paid two polls of $1.50 each, find his tax.

18. If $5280 are raised by a tax of $\frac{1}{4}$% on property, find the valuation of the property.

19. A reduction in the tax rate from 23.8 mills on $1 to 20 mills on $1 reduces the tax of a property owner $273.60. Find the valuation of his property.

20. The valuation of taxable property in Bloomington, Ill., is $5,902,250. The tax rate is $2.38 per $100. Find the tax levy. If 95% of this tax is collected, find the amount collected.

21. The valuation of taxable property in Baltimore is, in round numbers, $440,000,000. The rate of taxation in Baltimore is $1.91 per $100. Find the tax levy in this city.

22. The tax rate in Philadelphia is $1.85 per $100. Find the tax paid on property by a citizen of Philadelphia who owns property assessed at $18,500.

CHAPTER XX

THE LANGUAGE OF MATHEMATICS, RATIO, PROPORTION, PARTNERSHIP

329. By **mathematics** we understand those branches of knowledge which deal with quantity. Arithmetic, algebra, geometry, surveying, etc., are included in the term mathematics.

Mathematics has a **language** of its own. This language we shall now consider.

The word *eight* conveys a definite idea to the mind; the sign or symbol 8 conveys the same idea. The words *eight squared* convey a definite idea to the mind; the symbol 8^2 conveys the same idea. The words *three-fourths of sixteen* convey an idea; the symbols $\frac{3}{4} \times 16$ convey the same idea. Similarly, the words, the quotient of *seventy-two divided by eight*, convey an idea; the symbol $\frac{72}{8}$ conveys the same idea.

330. When we **reason about numbers** and the relations of numbers we make letters represent numbers. Thus, a, b, c, x, y, z, etc., may each represent any number whatever. We indicate the product of the numbers represented by letters by writing the letters in succession, one after the other. Thus, abc implies the continued product of a, b, and c. Also a number written before a letter indicates multiplication. Thus, $5a$ means 5 times a. $5a$ is then a short way of writing $a + a + a + a + a$.

$4b$ is a short way of writing $b + b + b + b$.

$7x$ is a short way of writing $x + x + x + x + x + x + x$.

a^2 is a short way of writing $a \times a$ or aa.

a^3 is a short way of writing $a \times a \times a$ or aaa.

a^4 is a short way of writing $a \times a \times a \times a$ or $aaaa$.

a^5 is a short way of writing $a \times a \times a \times a \times a$ or $aaaaa$.

The number denoting how many times a number is added is called a **coefficient.** The coefficient of the expression $9b$ is 9.

The expression $a + b$ stands for the sum of any two numbers. The expression $a - b$ stands for the number which when added to b gives a, or in other words, the remainder obtained when b is subtracted from a.

331. *Example* 1. If $a = 5$, $b = 3$, what is the value of $a + b$? $a - b = ?$ $4a = ?$ $3b = ?$ $2a - 3b = ?$

SOLUTION. $a + b = 5 + 3 = 8$. $a - b = 5 - 3 = 2$. $4a = 4 \times 5 = 20$. $3b = 3 \times 3 = 9$. $2a - 3b = 2 \times 5 - 3 \times 3 = 1$.

Example 2. If $a = 7$, what is the value of a^2? a^3? $3a^2$? $4a^3$?

$a^2 = a \times a = 7 \times 7 = 49$. $a^3 = a \times a \times a = 7 \times 7 \times 7 = 343$. $3a^2 = 3 \times a \times a = 3 \times 7 \times 7 = 147$. $4a^3 = 4 \times a \times a \times a = 4 \times 7 \times 7 \times 7 = 1372$.

EXERCISE 198 (Oral)

If $a = 4$, what is the value of $4a$? $7a$? $11a$? $13a$? $17a$? $19a$? $27a$? $\frac{1}{2}a$? $\frac{1}{4}a$?

If $a = 5$, what is the value of a^2? a^3? a^4? $2a^2$? $3a^2$? $2a^3$? $4a^2$? $a + a^2$? $a^2 + a^3$?

If $a = 3$, $b = 2$, what is the value of $a + b$? $a - b$? $2a + b$? $a + 2b$? $2a - b$? $2a - 3b$? $3a - 2b$?

If $a = 6$, $b = 3$, what is the value of $6a + 3b$? $3a + 5b$? $6a - 36$? $5a - 10b$? $7a - 5b$? $3a^2$? $a^2 + b^2$? $a^2 - b^2$?

If $x = 5$, $y = 6$, what is the value of xy? $2xy$? $3xy$? x^2y? xy? $\frac{1}{3}x^2$? $\frac{1}{4}y^2$?

If $x = 9$, $y = 4$, what is the value of $2x^2 - y^2$? $x^2 + 2y^2$? $3x^2 + y^2$? $2x^2 + 3y^2$? $y^2 - x^2$?

If $a = 10$, $b = 7$, find the value of ab, $5ab$, $ab + a^2$, $2ab + b^2$, $ab + 2b^2$.

EXERCISE 199 (Oral)

1. What is the sum of two times a number and three times the same number? What is the sum of $2x$ and $3x$?

2. What is the sum of $4x$ and $3x$? of $8a$ and $3a$? of $5b$ and $2b$? of $6b$ and $4b$?

3. What is the difference between $8x$ and $3x$? $6x$ and $2x$? $11b$ and $7b$? $8a$ and a?

4. Add $5x$ and $7x$; $4x$ and $9x$; $9b$ and $6b$; $10y$ and $6y$; $12x$ and $4x$.

5. Subtract $4x$ from $9x$; $8x$ from $14x$; $9x$ from $16x$; $7x$ from $13x$; $5ab$ from $8ab$.

6. Find the difference between $11y$ and $2y$; $5ab$ and ab; $7ab$ and $4ab$; $12ab$ and $2ab$.

332. Every sentence conveys a thought. The sum of 3 and 4 is 7. This sentence is expressed in the language of mathematics as follows: $3 + 4 = 7$. (1) Let us write another sentence: The difference between 18 and 7 is 11. This sentence, written in mathematical language, is $18 - 7 = 11$. (2) Let us write a third sentence: Two thirds of 27 is 18. In mathematical language this sentence is written $\frac{2}{3} \times 27 = 18$. (3) The statements (1), (2), (3), are called equations. An **equation** is a statement in symbols that two expressions are equal to each other.

The part of an equation to the left of the sign of equality is called the **first member** of the equation; the part of an equation to the right of the sign of equality is called the **second member** of the equation.

333. What is the product of a and a? $a \times a = a^2$.

What is the product of a and a^2? $a \times a^2 = a \times a \times a = a^3$.

What is the product of a^2 and a^3? $a^2 = a \times a$; $a^3 = a \times a \times a$.

$\therefore\ a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a^5$.

What is the product of a^4 and a^3?

$$a^4 = a \times a \times a \times a.$$

$$a^3 = a \times a \times a.$$

$$\therefore\ a^4 \times a^3 = (a \times a \times a \times a) \times (a \times a \times a) = a^7.$$

What is the product of $4\,a^2$ and $5\,a^3$?

$$4\,a^2 = 4 \times a \times a.$$

$$5\,a^3 = 5 \times a \times a \times a.$$

$\therefore\ 4\,a^2 \times 5\,a^3 = 4 \times a \times a \times 5 \times a \times a \times a = 4 \times 5 \times a \times a \times a \times a \times a = 20\,a^5$. (Associative Law.)

EXERCISE 200 (Oral)

1. $a \times 2a = ?$
2. $2a \times a^2 = ?$
3. $3a^2 \times a = ?$
4. $3a^2 \times 2a^2 = ?$
5. $4a \times a^3 = ?$
6. $7a^2 \times 2a^2 = ?$
7. $3a \times 5a^3 = ?$
8. $9a^2 \times 2a^3 = ?$
9. $4a^3 \times a^2 = ?$
10. $5x^3 \times 4x = ?$
11. $6x^2 \times x^3 = ?$
12. $5b \times b^2 = ?$
13. $4b^2 \times 3b = ?$
14. $5b^3 \times 4b^3 = ?$
15. $2b^2 \times 5b = ?$
16. $4y^2 \times 3y^3 = ?$
17. $7z^2 \times 5a = ?$
18. $4y \times 8b^3 = ?$

334. What is the quotient when a^3 is divided by a?

$$a^3 \div a = \frac{a^3}{a} = \frac{a \times a \times a}{a} = a \times a = a^2.$$

Here we utilize our knowledge of cancellation.

What is the quotient of $8\,a^3$ by $2\,a$?

$$8\,a^3 \div 2\,a = \frac{8\,a^3}{2\,a} = \frac{\overset{4}{\cancel{8}} \times a \times a \times a}{\cancel{2} \times a} = 4 \times a \times a = 4\,a^2.$$

EXERCISE 201 (Oral)

Find the following quotients:

1. $\frac{8a}{2a}$.
2. $\frac{9a}{3}$.
3. $\frac{12a}{4}$.
4. $\frac{16a^2}{4a^2}$.
5. $\frac{12a^3}{4a}$.
6. $\frac{21a^3}{7a^2}$.
7. $\frac{28a^4}{14a^2}$.
8. $\frac{18a^4}{3a^3}$.
9. $\frac{24a^4}{8a^2}$.
10. $\frac{11a^4}{a}$.
11. $\frac{22a^5}{2a^2}$.
12. $\frac{16a^5}{4a^3}$.
13. $\frac{32a^6}{16a^4}$.
14. $\frac{24a^5}{8a^3}$.
15. $\frac{25a^4}{5a^2}$.
16. $\frac{35x^4}{5x^3}$.
17. $\frac{39x^5}{13x}$.
18. $\frac{42x^4}{6x^3}$.
19. $\frac{45x^4}{9x}$.
20. $\frac{50x^5}{10x^4}$.
21. $\frac{60x^5}{10x^5}$.
22. $\frac{26x^5}{13x}$.
23. $\frac{33b^2}{11b}$.
24. $\frac{48b^4}{16b}$.
25. $\frac{46b^5}{23b^2}$.

RATIO

335. We have seen that the ratio of one number a to another number b, is the quotient obtained by dividing a by b.

The ratio of a to b is written $a : b$. When the quotient $a \div b$ is written $\frac{a}{b}$, we call the expression $\frac{a}{b}$ a fraction.

The ratio $b : a$ is called the inverse ratio of a to b.

EXERCISE 202 (Oral)

1. What is the ratio of 2 ft. to 6 ft.?

2. What is the ratio of 4 in. to 1 yd.? of 3 in. to 1 yd.? of 1 yd. to 1 rd.? of $\frac{1}{2}$ rd. to 1 rd.?

3. What is the ratio of 80 A. to 1 sq. mi.? of 120 A. to 1 sq. mi.? of $\frac{1}{2}$ A. to 2 A.?

4. What is the ratio of the distance traveled by two trains in the same time, if the rate of the first train is 20 mi. per hour, and the rate of the second train is 30 mi. per hour?

5. If A walks at the rate of $2\frac{1}{2}$ mi. per hour, and B walks at the rate of 5 mi. per hour, what is the ratio of A's time to B's time in going any given distance?

6. What is the ratio of the time that 8 men take to do a piece of work to the time that 6 men take to do the same piece of work?

7. If you ride in a carriage at the rate of 7 mi. an hour and walk back the same distance at the rate of 3 mi. an hour, what is the ratio of the time in the carriage to the time walking?

8. What is the ratio of the price of 7 lb. of sugar to the price of 10 lb. of sugar of the same kind?

9. What is the ratio of the work done by 6 men to the work done by 9 men?

10. What is the ratio of the time that 9 men take to do a piece of work to the time that 6 men take to do the same work?

11. I can buy two kinds of matting for 40¢ and 50¢ a yard respectively. If I spend the same amount of money in the purchase of the two kinds of matting, what is the ratio of the number of yards of matting of the first kind to the number of yards of the second kind bought?

12. Divide 15 in the ratio 2 : 3.

13. Divide 20 in the ratio 3 : 7.

14. Divide $1 in the ratio 18 : 7.

15. Divide 1 mi. in the ratio 7 : 9.

16. Divide 1 in the ratio 9 : 11.

17. Divide 1 gal. in the ratio 1 : 3.

18. Divide $1 in the inverse ratio 9 : 16.

19. Divide 22 yd. in the inverse ratio 3 : 8.

20. Divide $1000 in the inverse ratio 3 : 5.

336. A statement indicating that two ratios are equal is called a **proportion**. Thus, the ratio of two to three equals the ratio of four to six. Expressed in the language of mathematics the statement becomes, $2:3=4:6$. (1).

Statement (1) is a proportion. Another example of a proportion is the statement: $9:15=12:20$. (2).

Statement (2) is a proportion because the value of the first ratio is $\frac{3}{5}$, and the value of the second ratio, *i.e.* $12:20$, is also $\frac{3}{5}$.

Statement (2) may read 9 is as large compared with 15 as 12 is compared with 20.

The first and fourth terms of a proportion are called the **extremes**, and the second and third terms are called the **means**, of the proportion.

337. In a proportion the product of the extremes is equal to the product of the means.

Let $a:b=c:d$ be any proportion whatever. Then $ad=bc$.

PROOF. $\frac{a}{b}=\frac{c}{d}$. Multiply each member by bd. We then get $\frac{abd}{b}=\frac{cbd}{d}$. $\therefore$ By cancellation $ad=bc$.

This property of a proportion enables us to find any term of a proportion, if three of the terms of the proportion are known.

The proportion $a:b=c:d$ is sometimes written $a:b::c:d$. The double colon used as a sign of equality is now rapidly becoming obsolete.

338. *Example* 1. Find x in the proportion $x:4=9:6$.

SOLUTION. The product of the extremes is equal to the product of the means.

$$\therefore 6x=36. \qquad x=6.$$

Example 2. Find x in the proportion $10 : 35 = x : 42$.

SOLUTION. Since the product of the means is equal to the product of the extremes,

$$35x = 10 \times 42.$$

$$\therefore \quad x = 10 \times \tfrac{42}{35} = 12.$$

Two numbers which vary directly are said to be **directly proportional.** Two numbers which vary inversely are said to be **inversely proportional.**

EXERCISE 203

If x stands for the unknown term in each of the following proportions, find it:

1. $2 : 3 = 6 : x$.
2. $3 : 4 = 6 : x$.
3. $15 : 25 = 12 : x$.
4. $12 : 20 = 18 : x$.
5. $14 : 21 = x : 27$.
6. $21 : 27 = x : 45$.
7. $35 : 84 = x : 72$.
8. $20 : 48 = x : 96$.
9. $16 : 24 = x : 33$.
10. $20 : 32 = x : 72$.
11. $25 : 45 = x : 99$.
12. $45 : 126 = x : 154$.
13. $57 : 133 = x : 126$.
14. $68 : 85 = x : 75$.
15. $36 : x = 52 : 65$.
16. $28 : x = 36 : 63$.
17. $27 : x = 15 : 50$.
18. $15 : x = 21 : 77$.
19. $28 : x = 36 : 81$.
20. $25 : x = 45 : 72$.
21. $35 : x = 30 : 48$.
22. $x : 81 = 16 : 72$.
23. $x : 99 = 26 : 117$.
24. $x : 65 = 24 : 52$.
25. $x : 112 = 45 : 144$.

339. *Example* 1. If 7 bu. of wheat cost $5.25, find the cost of 11 bu. of wheat at the same rate.

SOLUTION. It is reasonable to assume that the price of 11 bu. of wheat is greater than the price of 7 bu. of wheat in the ratio of 11 to 7. $\therefore$ The price of 11 bu. of wheat $= \frac{11}{7}$ of \$5.25 = \$8.25.

Example 2. If 12 men pave a street in 15 da., how long will it take 9 men to pave a street of the same area?

SOLUTION. It will take 9 men longer than it takes 12 men in the ratio of 12 to 9. ∴ The time 9 men take = $\frac{12}{9}$ of 15 da. = 20 days.

To **solve a problem in proportion,** find first the relation of the answer to the quantity of the same kind as the answer given in the problem. Second: Multiply this quantity by the ratio of the answer to this quantity.

EXERCISE 204 (Written)

1. If 20 men earn $450 in a given time, how much will 30 men earn in the same time?

2. If 15 bu. of corn cost $7.20, what will 48 bu. of corn cost?

3. If 12 A. of land cost $456.90, what will 16 A. of the same land cost?

4. If 4 men can do a piece of work in 15 da., how long will it take 6 men to do an equal amount of work?

5. If 18 head of cattle cost $1450, what will 27 head of cattle cost at the same rate?

6. If a train goes 400 mi. in 12 hr., how long will it take to go 560 mi.?

7. If 8 masons build a wall in 15 da., how long will it take 6 masons to build a wall of the same size?

8. If 18 horses consume 14 bu. of corn in a given time, how much will 24 horses consume in the same time?

9. If 18 horses plow a tract of land in 13 da., how long will it take 26 horses to plow the same tract?

10. If 105 sheep eat a quantity of feed in 30 da., how long will it take 126 sheep to eat a quantity of feed equally large?

11. A garrison consisting of 1200 men has provisions for 16 da. How many men must be sent away so that the provisions may last 24 da.?

12. A garrison consisting of 1400 men has provisions for 27 da. If the garrison is reënforced by 400 men, in how many days will the provisions be consumed?

13. If I can buy a dozen turkeys for $20.50, how many turkeys can I buy for $30.75?

14. If the interest on $750 for 4 mo. is $12.50, what is the interest on $39.60 for the same time?

15. If an arc of 12″ on the 40th parallel of latitude is 933.92 ft., find the length of 1° on the 40th parallel of latitude.

16. If an arc of 30′ on the circumference of a wheel is $1\frac{1}{4}$ in., find the length of the circumference of the wheel.

COMPOUND PROPORTION

340. If the product of the corresponding terms of two or more ratios are taken, the ratio of the resulting products is called the **ratio compounded** of these ratios. For example, the ratio compounded of the ratios 2 : 3, 4 : 5, 7 : 8, is the ratio $2 \times 4 \times 7 : 3 \times 5 \times 8$, or 56 : 120, or 7 : 15.

A proportion in which the final result depends upon a ratio compounded of two or more ratios is called a **compound proportion.**

The equality of one ratio and of two or more ratios which, when compounded, give an equivalent ratio, is called a compound proportion. A concrete example may give a clearer conception of proportion than any formal definition.

341. *Example* 1. If 15 men mow 90 A. in 12 da., how many acres will 12 men mow in 14 da.?

SOLUTION. The 12 men in a given time will mow less than 15 men in the same time. ∴ The 12 men in 12 da. will mow $\frac{12}{15}$ of 90 A. But the 12 men in 14 da. will mow more than this quantity in the ratio 14 : 12. ∴ 12 men in 14 da. will mow $\frac{14}{12}$ of $\frac{12}{15}$ of 90 A. $=\frac{14}{15}$ of 90 A. = 84 A.

Example 2. If 24 men build a house in 18 da. of 10 hr. each, how many men will it take to build the same house in 30 da. of 8 hr. each?

SOLUTION. *Step* 1. It will take fewer men to build a house in 30 da. than it will take to build it in 18 da. of the same length.

∴ The number of men it will take to build the house in 30 da. of 10 hr. each $=\frac{18}{30}$ of 24 men.

Step 2. More men are needed when they work 8 hr. a day than when they work 10 hr. a day.

∴ The number of men, in 30 da. of 8 hr. each, required to build the house $=\frac{10}{8}$ of $\frac{18}{30}$ of 24 men = 18 men.

EXERCISE 205 (Written)

1. If 12 horses plow 84 A. in 6 da., how many acres will 16 horses plow in $4\frac{1}{2}$ da.?

2. If 14 men pave a street 200 ft. long in 8 da., how many feet will 12 men pave in 7 da.?

3. If a man earns $117 in 3 mo. working 6 hr. a day, how much will he earn in 5 mo. working 8 hr. a day?

4. A garrison of 3650 men consumed in 30 da. 82.3 T. of food. How much food would be required for 7500 men for 1 yr. at the same rate?

5. If 8 masons build in 2 da. a wall 40 ft. long and 6 ft. high, what height of wall 30 ft. long can they build in 5 da.?

6. If 21 men complete a piece of work in 8 da. of $7\frac{1}{2}$ hr. each, in how many days of 10 hr. each can 18 men do the same work?

7. A wall is to be built in 10 da. by 30 men. After 2 da. 10 men are dismissed. In what time will the remaining 20 men finish the work?

8. If 4 men or 6 boys dig a trench in 12 da., in what time can 2 men and 9 boys dig it?

9. If 12 men mow 30 A. in 3 da. of 8 hr. each, how many hours a day must 16 men work to mow 48 A. in 4 da.?

10. If the interest on $100 for 1 yr. is $6, find the interest on $840 for 2 yr. 3 mo.

11. If 12 men working 7 hr. a day earn $227.50 in 20 da., how much will 15 men earn in 20 da., working 9 hr. each?

12. If 6 men mow $\frac{3}{5}$ of a meadow in $4\frac{1}{2}$ da., how long will it take 8 men to mow the remainder?

13. In 10 da. of 8 hr. each 9 horses can plow $\frac{2}{5}$ of a field. In how many days of 9 hr. each can the remainder of the field be plowed by 15 horses?

14. A marble block 3 ft. by 4 ft. and 5 ft. in length weighs 5.1 T. Find the weight of a marble block 7 ft. by 3 ft. and 10 ft. long.

15. A mason can build 3 yd. of a wall in 15 hr. How long will it take 9 masons to build 24 yd. of a wall which is one-third higher than the other wall?

5. What property of a proportion enables you to find any term if three terms are given?

6. When are numbers directly proportional? When are they inversely proportional?

7. What is a compound ratio?

8. What is a compound proportion?

9. What is partnership?

[illegible] questions in partnership [illegible]

342. *Example.* A, B, and C enter into partnership. A puts in $840, B puts in $350, and C puts in $2000. A withdraws from the concern in 5 mo., C in 7 mo., and at the end of 8 mo. the profits are divided. If the entire profit is $450, how shall this be divided among A, B, and C?

SOLUTION. A has $840 in the concern for 5 mo. This is equivalent to $4200 for 1 mo.

B has $350 in the concern for 8 mo. This is equivalent to $2800 for 1 mo.

C has in the concern $2000 for 7 mo. This is equivalent to $14,000 for 1 mo.

The profits will be divided in proportion to the numbers 4200, 2800, 14,000, or in proportion to the numbers 3, 2, 10.

$$3 + 2 + 10 = 15.$$

$$\therefore \text{ A's share} = \tfrac{3}{15} \text{ of the profits} = \tfrac{3}{15} \text{ of } \$450 = \$90.$$

$$\text{B's share} = \tfrac{2}{15} \text{ of the profits} = \tfrac{2}{15} \text{ of } \$450 = \$60.$$

$$\text{C's share} = \tfrac{10}{15} \text{ of the profits} = \tfrac{10}{15} \text{ of } \$450 = \$300.$$

EXERCISE 206 (Written)

1. A, B, and C enter into partnership with capitals of $3000, $3750, and $4500 respectively. At the end of the year they divide among themselves a profit of $3000. Find each person's share.

CHAPTER XXI

INVOLUTION, SQUARE ROOT, AREAS

343. In the language of mathematics, $3x$ means three times the number x; *i.e.* $3x$ may stand for three times any number whatever.

$a+b$ stands for the sum of any two numbers, a and b. $a-b$ stands for the difference of two numbers, a and b. In other words, $a-b$ is the number which when added to b gives a for sum.

ab stands for the product of two numbers, a and b. $\frac{a}{b}$ stands for the quotient obtained by dividing a by b. In other words, $\frac{a}{b}$ is that number which when multiplied by b gives a for product.

a^2 stands for the square of a. a^3 stands for the cube of a, $(a+b)^2$ stands for the square of the sum of two numbers, a and b.

$(a-b)^2$ stands for the square of the difference of two numbers, a and b.

$7ab$ stands for seven times the number ab, which is itself the product of two numbers, a and b.

$\frac{a+b}{2}+\frac{a-b}{2}$ stands for one-half the sum of two numbers plus one-half their difference.

$(a+b)x$ stands for the product of a number, x, by the sum of two numbers, a and b.

$(a-b)x$ stands for the product of a number, x, by the difference of two numbers, a and b.

EXERCISE 207 (Oral)

Add:

1.	**2.**	**3.**	**4.**	**5.**	**6.**
$2x$	$7y$	$4a$	$9b$	$4z$	$11a$
$3x$	$8y$	$9a$	$7b$	$7z$	$7a$
$\underline{5x}$	$\underline{9y}$	$\underline{11a}$	$\underline{8b}$	$\underline{9z}$	$\underline{5a}$

7.	**8.**	**9.**	**10.**	**11.**
$4ab$	$9xy$	$4bc$	$12ab$	$9ab$
$5ab$	$7xy$	$7bc$	$15ab$	$4ab$
$\underline{7ab}$	$\underline{8xy}$	$\underline{11bc}$	$\underline{10ab}$	$\underline{5ab}$

12.	**13.**	**14.**	**15.**	**16.**	**17.**
$7bc$	ax	$10x$	$11b$	$12m$	$11x$
$8bc$	$\underline{bx}$	$\underline{ax}$	$\underline{ab}$	$\underline{am}$	$\underline{bx}$
$\underline{9bc}$					

EXERCISE 208 (Oral)

1. $7x-4x=?$

2. $6x-x=?$

3. $11x-8x=?$

4. $12x-2x=?$

5. $5x-3x=?$

6. $12a-a=?$

7. $15a-11a=?$

8. $17a-12a=?$

9. $18a-13a=?$

10. $12ab-3ab=?$

11. $11ab-4ab=?$

12. $5ab-ab=?$

13. $2xy-xy=?$

14. $14xy-10xy=?$

15. $17xy-9xy=?$

16. $19xy-7xy=?$

17. $ax-bx=?$

18. $cx-dx=?$

19. $mx-nx=?$

344. *Example* **1.** Multiply $4x$ by $5x$.

SOLUTION. $4x\times 5x=4\times x\times 5\times x=4\times 5\times x\times x=20x^2$.

Example **2.** Multiply $7x^2y$ by $3xy^2$.

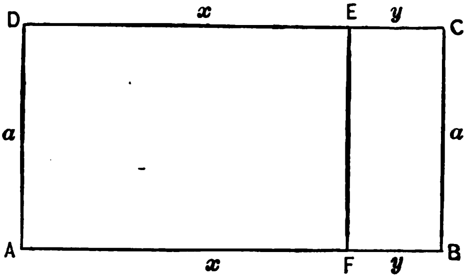
SOLUTION. $7x^2y\times 3xy^2=(7\times x\times x\times y)\times(3\times x\times y\times y)$ $=7\times x\times x\times x\times y\times y\times y=21x^3y^3$. (Associative Law.)

Example 3. Multiply $x + y$ by a.

SOLUTION. $$\begin{array}{l} x+y \\ a \\ \hline ax+ay \end{array}$$

We multiply x by the multiplier and write the result, ax. We next multiply y by a and write the product, ay. This process may be graphically represented as follows:

Let $DE = x$,
$EC = y$,
$BC = a$.



The rectangle $ABCD$ has for its dimensions $x + y$, and a. Its area $= (x+y)a$. Also the rectangle $ABCD$ $=$ the rectangle $AFED +$ the rectangle $FBCE = ax + ay$.

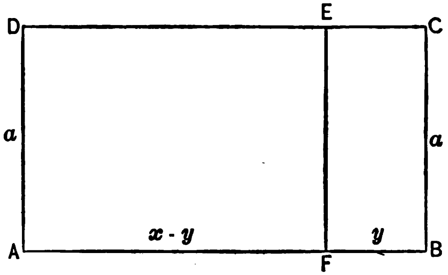
$(x + y)a = ax + ay$. Hence, we have the following statement:

The product of a third number and the sum of two other numbers equals the sum of the products of the third number and each of the two other numbers. (1)

Example 4. Multiply $(x - y)$ by a.

Let $AB = x$,
$FB = y$,
$BC = a$.

Then $AF = x - y$.



The rectangle $AFED =$ the rectangle $ABCD -$ the rectangle $FBCE$.

But the rectangle $AFED = (x - y)a$,

$$\begin{aligned} &\text{The rectangle } ABCD = ax, \\ &\text{The rectangle } FBCE = ay, \\ \therefore \quad &(x-y)a = ax - ay. \end{aligned}$$

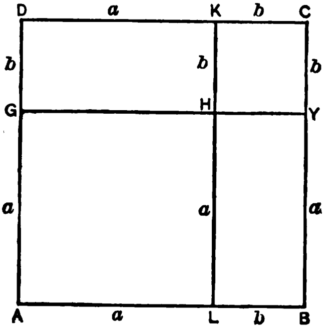
Hence, we have the following statement:

The product of the difference of two numbers and a third number equals the difference of the products of the third number and each of the other two numbers. (2)

Example 5. What is the square of $a+b$?

$$\begin{array}{llll} a & + & b & \\ a & + & b & \\ \hline a^2 & + & ab & \\ & + & ab & + b^2 \\ \hline a^2 & + & 2\,ab & + b^2 \end{array}$$

We multiply $a+b$ by a, and write the result, a^2+ab. We next multiply $a+b$ by b and write the result, $ab+b^2$. We then add. The first two steps are a consequence of statement (1). The following is a graphical representation of the result:



$$\begin{aligned} \text{Let } AL &= a, \\ LB &= b. \\ \text{Then } AB &= a+b. \end{aligned}$$

Let $ABCD$ be the square on AB, $ALHG$ be the square on AL. Then $HYCK$ is the square on b, and $GHKD = LBYH$, because their dimensions are equal to each other. Now, $ABCD = ALHG + GHKD + HYCK + LBYH = ALHG + HYCK + 2\,GHKD$.

$\therefore (a+b)^2 = a^2 + b^2 + 2ab$. Hence, we have the following important conclusion:

The square of the sum of two numbers equals the square of the first number plus the square of the second number plus twice the product of the two numbers. (3)

345. The process of finding the power of a number is called **involution.**

346. *Example* **1.** Square $x+9$. From Statement (3) we have $(x+9)^2 = x^2 + 9^2 + 2 \times 9 \times x = x^2 + 81 + 18x$, or $x^2 + 18x + 81$.

Example **2.** What is the square of 43?

SOLUTION. $43^2 = (40+3)^2 = 40^2 + 3^2 + 2 \times 40 \times 3 = 1600 + 9 + 240 = 1849$.

EXERCISE 209 (Written)

Find by Statement (3) the square of:

1. $10+x$.
2. $20+x$.
3. $30+x$.
4. $40+x$.
5. $50+x$.
6. $60+a$.
7. $70+b$.
8. $80+c$.
9. $90+y$.
10. 14.
11. 15.
12. 24.
13. 29.
14. 37.
15. 47.
16. 54.
17. 62.
18. 68.
19. 72.
20. 76.
21. 78.
22. 79.
23. 84.
24. 87.
25. 89.
26. 92.
27. 93.
28. 67.
29. 98.
30. 55.

EXERCISE 210 (Written)

1. Find the square of .1, .2, .3, .4, .5, .6, .7, .8, .9.

2. Find the square of the reciprocals of the numbers from 1 to 20 inclusive.

3. Find the squares of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{2}{7}$, $\frac{9}{10}$, $\frac{11}{12}$, $\frac{10}{13}$, $\frac{9}{16}$.

4. Find the squares of $\frac{5}{8}$, $\frac{7}{3}$, $\frac{9}{7}$, $1\frac{1}{4}$, $1\frac{3}{4}$, $1\frac{7}{8}$, $2\frac{1}{5}$, $6\frac{1}{4}$.

5. How does the square of a fraction compare in value with the fraction itself?

6. Find the cubes of the reciprocals of the numbers from 1 to 20 inclusive.

7. Find the value of $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3 + 8^3 + 9^3 + 10^3$.

8. Find the value of $1^4 + 2^4 + 3^4 + 4^4 + 5^4 + 6^4 + 7^4 + 8^4 + 9^4 + 10^4$.

9. Square each of the following numbers and give the product correct to four decimal figures: (*a*) 1.732, (*b*) 9.256, (*c*) 5.401, (*d*) 8.129, (*e*) .6834, (*f*) .7609, (*g*) 9.482, (*h*) .7071, (*i*) .7746, (*j*) .9487.

10. Find the value of $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2 + 9^2 + 10^2$.

11. Find the cube of .1, .2, .3, .4, .5, .6, .7, .8, .9.

12. Find the value of $(1.04)^4$, $(1.05)^4$, $(1.06)^4$, $(1.07)^4$.

EVOLUTION

SQUARE ROOT

347. By the **square root** of a number is meant that number which when squared produces the given number. Thus, 4 is the square root of 16, since $4^2 = 16$. 7 is the square root of 49 for a similar reason.

The square root is defined also as one of the two equal factors of a number. Thus, $7 \times 7 = 49$. One of the factors is the square root of 49.

The symbol for square root, $\sqrt{\ }$, is called the *radical sign*. It is a degenerate form of the first letter of the word *radix*, the Latin word for root. The exponent $\frac{1}{2}$ is also used as a sign for the square root. $\sqrt{49}$, $49^{\frac{1}{2}}$ are the two ways of indicating the same process, namely, the extraction of the square root of 49.

The number written under the radical sign is called the *radicand*.

348. *Example* 1. What is the square root of 5329?

Solution. By trial we find the square root of 5329 is more than 70 and less than 80.

$$\therefore \sqrt{5329} = 70 + x.$$

$$\therefore 5329 = 4900 + 140\,x + x^2. \quad \text{Statement (3).}$$

Subtracting 4900, we get,

$$140\,x + x^2 = 429.$$

$$\therefore (140 + x)x = 429.$$

Since 140 is contained in 429 3 times, we try 3 as a value of x.

$$(140 + 3)3 = 429.$$

$$\therefore \sqrt{5329} = 70 + 3 = 73.$$

Example 2. $\sqrt{9025} = ?$

Solution. $\sqrt{9025} = 90 + x.$

$$\therefore 9025 = 8100 + 180\,x + x^2. \quad \text{Statement (3).}$$

$$\therefore 180\,x + x^2 = 925.$$

$$\therefore (180 + x)x = 925.$$

Since 180 is contained in 925 5 times, we try 5 for the next figure of the root.

$$(180 + 5)5 = 925.$$

$$\therefore \sqrt{9025} = 90 + 5 = 95.$$

$$\begin{array}{r|l} \multicolumn{1}{r}{} & \multicolumn{1}{l}{9\;\;\,5} \\ \multicolumn{1}{r}{} & \multicolumn{1}{l}{90\;25} \\ \multicolumn{1}{r}{} & \multicolumn{1}{l}{81} \\ \cline{2-2} 185 & 9\;25 \\ & 9\;25 \\ \hline \end{array}$$

In practice, the work is contracted as follows: Beginning at the decimal point, point off the figures of the number in periods of two figures each. By trial find the greatest digit whose square is contained in the number denoted by the period to the left. Write it as the first figure of the root, and write also its square. Subtract the latter from the period to the left and bring down the next period. Double the part of the root just found for trial divisor. Find next the number of times the trial divisor is contained in the number denoted by the remainder and the period brought down. Write this result in the quotient and in the divisor and then multiply.

EXERCISE 211 (Written)

Find the square root of:

1. 169.
2. 441.
3. 625.
4. 961.
5. 1024.
6. 1849.
7. 2809.
8. 3844.
9. 4225.
10. 5329.
11. 5776.
12. 6724.
13. 7056.
14. 7569.
15. 8464.
16. 9216.
17. 9604.
18. 9801.
19. 7921.
20. 6084.
21. 4761.
22. 3481.
23. 2401.
24. 3364.

349. *Example* $\sqrt{127449} = ?$

$$\begin{array}{r|r} & 3\ \ \ 5 \\ & 12\ 74\ 49 \\ & 9 \\ \hline 65 & 3\ 74 \\ & 3\ 25 \\ \hline & 49\ 49 \end{array}$$

SOLUTION. We divide the figures of the number into periods of two as in the previous exercises. We then proceed to extract the square root of the number denoted by the two periods to the left.

The answer is obviously 350 + some number. Let x represent this number; we have then

$$350 + x = \sqrt{127,449}.$$
$$\therefore (350 + x)^2 = 127,449.$$
$$\therefore 122,500 + 700\,x + x^2 = 127,449.$$
$$\therefore 700\,x + x^2 = 4949.$$
$$\therefore (700 + x) = 4949.$$

Since 700 is contained in 4900 7 times, we try 7 as a value of x.

$$(700 + 7)7 = 4949.$$
$$\therefore \sqrt{127449} = 350 + 7 = 357.$$

Notice the trial divisor is twice the part of the root found. Therefore in a problem in square root where the radicand is an integer consisting of five or six figures, we may proceed in exactly the same way as we have done in a problem consisting of four figures. Beginning once more, the solution in its contracted form stands as follows:

$$\begin{array}{r|rrr} & 3 & 5 & 7 \\ & 12 & 74 & 49 \\ & 9 & & \\ \cline{2-3} 65 & 3 & 74 & \\ & 3 & 25 & \\ \cline{2-4} 707 & & 49 & 49 \\ & & 49 & 49 \\ \cline{2-4} \end{array}$$

The trial divisor is always twice the part of the root already found.

EXERCISE 212 (Written)

Extract the square root of:

1. 100,489.
2. 110,224.
3. 120,409.
4. 171,396.
5. 190,096.
6. 199,809.
7. 229,441.
8. 277,729.
9. 310,249.
10. 354,025.
11. 391,876.
12. 456,976.
13. 474,721.
14. 501,264.
15. 654,481.
16. 772,641.
17. 819,025.
18. 826,281.

350. To extract the square root of a decimal, begin *at the decimal point,* and proceeding to the right, point off the figures in periods of two. Next proceed as if the number were an integer. Thus, in taking the square root of .0225, we first point off in periods of two figures each. We then have .02 25. Next extract the root of the number denoted by the figures 225. The result is 15. Hence, the required root is .15.

To extract the square root of a number part integer and part decimal, begin at the decimal point and proceeding to the left, point off the integral part in periods of two figures each; next point off the decimal part in periods of two figures each, beginning *at the decimal point.* If there are not enough figures in the decimal part to make an exact number of periods, annex a cipher, or as many ciphers as are necessary to make the required number of periods.

351. *Example* 1. Extract the square root of 1.7.

$$\begin{array}{r|l} \multicolumn{2}{l}{1.\ 3\ 0\ 3\ 8\ 4} \\ \multicolumn{2}{l}{1.\ 70\ 00\ 00\ 00\ 00} \\ \multicolumn{2}{l}{1.} \\ \hline 23 & 70 \\ & 69 \\ \hline 2603 & 10000 \\ & 7809 \\ \hline 26068 & 219100 \\ & 208544 \\ \hline 260764 & 1055600 \\ & 1043056 \\ \hline \end{array}$$

We double 1 for the first trial divisor; we double 13 for the next trial divisor. We then find the next figure of the root is 0. We write it in the root and in the trial divisor. We then annex two more ciphers, and find the next figure of the root is 3, and so on.

EXERCISE 213 (Written)

Extract the square root of :

1. .150932.
2. .246016.
3. .3448.
4. .2909.
5. .2632.
6. .4616.
7. .5319.
8. .61575.
9. .784.
10. .083.
11. .062.
12. .0037.

To extract the square root of a fraction when its numerator and denominator are perfect squares is a simple matter. Thus the square root of $\frac{25}{49}$ is $\frac{5}{7}$, the square root of $1\frac{17}{64}$, or $\frac{81}{64}$ is $\frac{9}{8}$, or $1\frac{1}{8}$.

To get the square root of a fraction, we take the square root of the numerator, and the square root of the denominator, and then write the former result for numerator and the latter result for denominator. The fraction thus found is the required square root.

Example 1. What is the square root of $\frac{17}{36}$?

SOLUTION. $\sqrt{17} = 4.123$; $\sqrt{36} = 6$.

$$\therefore \sqrt{\tfrac{17}{36}} = \frac{4.123}{6} = .687.$$

Example 2. What is the square root of $\frac{23}{39}$?

SOLUTION. $\sqrt{23} = 4.796$; $\sqrt{39} = 6.245$.

$$\therefore \sqrt{\tfrac{23}{39}} = \frac{4.796}{6.245} = .768.$$

This is a roundabout way to take the square root of $\frac{23}{39}$. A shorter and better way is to reduce the fraction to an equivalent decimal, and then to extract the square root of this decimal.

To extract the square root of a fraction whose denominator is not a perfect square, reduce the fraction to an equivalent decimal and then extract the square root of this decimal.

EXERCISE 214 (Written)

Extract, to three decimal figures, the square root of :

1. 1.2.
2. 4.25.
3. 1.1.
4. 5.2.
5. 3.3.
6. $5\frac{1}{4}$.
7. $3\frac{1}{8}$.
8. $9\frac{1}{6}$.
9. $1\frac{3}{8}$.
10. $4\frac{1}{2}$.
11. $2\frac{7}{8}$.
12. $\frac{7}{12}$.
13. $\frac{2}{3}$.
14. $\frac{2}{5}$.
15. $\frac{7}{11}$.

EXERCISE 215

PROBLEMS INVOLVING SQUARE ROOT

1. The area of a square field is 1 A. Find the length in yards of one of its sides.

2. The area of a square field is 12 A. Find the length in yards of one of its sides.

3. The dimensions of a rectangle are 289 yd. and 196 yd. Find the side of an equivalent square.

4. The dimensions of a rectangle are $1\frac{1}{2}$ mi. and .7 mi. Find, correct to four decimal figures, the side of an equivalent square.

5. Find in rods the perimeter of a square field whose area is $\frac{1}{8}$ of a square mile.

6. The area of a rectangle whose length is twice its breadth is 10 A. Find its dimensions in yards.

HINT. Draw a diagram, divide it into two equal parts by a line parallel to its width. Notice what each part is.

7. The area of a rectangle whose length is three times its width is 20 A. Find its dimensions in yards.

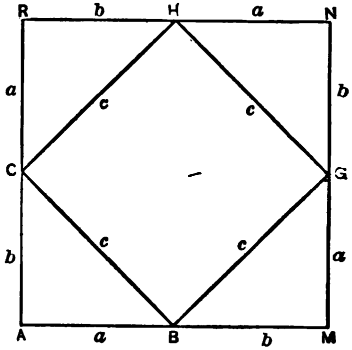
8. A square and a rectangle have the same area, namely 40 A. If the length of the rectangle is twice its width, find, in rods, the difference between their perimeters.

352. A triangle which has a right angle is called a **right triangle.**

Let *ABC* (see figure on page 437) be a right triangle, angle *A* being the right angle.

The side of a right triangle opposite the right angle is called the **hypothenuse.** The other two sides are called the **legs** of the right triangle. One of the legs is called the **base** of the right triangle, and the other leg is called the **altitude** of the right triangle.

353. In a right triangle the square on the hypothenuse is equal to the sum of the squares on the two legs. This is the famous Pythagorean Theorem. It is so called because Pythagoras, a Grecian philosopher of the sixth century B.C., first proved that this relationship always exists between the sides of a right triangle. The Pythagorean Theorem is one of the most important theorems of elementary mathematics.



Designate the sides of the right triangle *ABC* by the letters a, b, c. $(a+b)^2 = a^2+b^2+2\,ab$ (Statement 3). But $(a+b)^2 = c^2 + 4$ triangles, each having for its base and altitude a and b. Fig. $1 = c^2 + 4 \times \frac{1}{2}\,ab = c^2 + 2\,ab$ (Associative Law).

$$a^2+b^2+2\,ab = c^2+2\,ab.$$
$$\therefore\ a^2+b^2=c^2.$$

354. *Example.* In a right triangle the legs are 7 and 24. Find the hypothenuse.

SOLUTION.
$$a^2+b^2=c^2.$$
$$\therefore\ 7^2+24^2=c^2.$$
$$\therefore\ 49+576=c^2.$$
$$\therefore\ c^2=625.$$
$$\therefore\ c=\sqrt{625}=25.$$

EXERCISE 216

1. In a right triangle, given $a=6$, $b=8$, find c.
2. In a right triangle, given $a=5$, $b=12$, find c.
3. In a right triangle, given $a=8$, $b=15$, find c.
4. In a right triangle, given $a=20$, $b=21$, find c.
5. In a right triangle, given $a=56$, $b=90$, find c.

6. In a right triangle, given $a = 20$, $b = 99$, find c.

7. In a right triangle, given $a = 17$, $b = 144$, find c.

8. In a right triangle, given $a = 39$, $b = 80$, find c.

9. In a right triangle, given $a = 51$, $b = 140$, find c.

10. In a right triangle, given $a = 44$, $b = 52.5$, find c.

11. In a right triangle, given $a = 87$, $b = 416$, find c.

12. In a right triangle, given $a = 136$, $b = 273$, find c.

13. In a right triangle, given $a = 145$, $b = 408$, find c.

14. In a right triangle, given $a = 207$, $b = 224$, find c.

15. A ladder is placed 14 ft. from a wall 48 ft. high. How long must the ladder be to reach to the top of the wall?

16. Find the length of the diagonal of a square if one side of the square is 10 rods.

17. Find the length of the diagonals of a rectangle the dimensions of the rectangle being 17 rd. and 25 rd.

355. *Example.* If the hypothenuse of a right triangle is 493 and one leg is 468, find the other leg.

SOLUTION. Let the required leg be a. Then $a^2 + 468^2 = 493^2$. $468^2 = 219{,}024$. $493^2 = 243{,}049$.

$\therefore\ a^2 + 219{,}024 = 243{,}049$. Subtract 219,024 from each member of the equation.

$$\therefore\ a^2 = 24{,}025.$$

$$\therefore\ a = \sqrt{24025} = 155.$$

EXERCISE 217

1. Hypothenuse = 377, base = 345, find the altitude.

2. Hypothenuse = 545, base = 513, find the altitude.

3. Hypothenuse = 449, base = 351, find the altitude.

4. Hypothenuse = 5.05, base = 4.56, find the altitude.

5. Hypothenuse = .461, base = .38, find the altitude.
6. Hypothenuse = .481, alt. = .36, find the base.
7. Hypothenuse = .641, alt. = .609, find the base.
8. Hypothenuse = .773, alt. = .195, find the base.
9. Hypothenuse = .697, alt. = .528, find the base.

AREAS OF PLANE TRIANGLES

356. The following rule gives the **area of any triangle**:

1. Add the three sides and take half the sum.

2. Subtract each side separately from the half sum.

3. Find the continued product of the three remainders and the half sum.

4. The square root of this product is the area.

The proof of this rule is too difficult to be given in an elementary arithmetic. This rule enables one to find the area of a tract of land such as a farm.

357. *Example*. Find the area of a triangle whose sides are 34 ch., 65 ch., and 93 ch.

SOLUTION:

$$\begin{array}{rrrrr}
 & 34 & 96 & 96 & 96 \\
 & 65 & \underline{34} & \underline{65} & \underline{93} \\
 & \underline{93} & 62 & 31 & 3 \\
2) & \underline{192} & & & \\
 & 96 & & &
\end{array}$$

$$\text{Area} = \sqrt{96 \times 62 \times 31 \times 3} = \sqrt{553535} = .44.$$

$$\therefore\ \text{Area} = 744 \text{ sq. ch.} = 74.4 \text{ A.}$$

EXERCISE 218

Find the area of each of the following triangles:

1. Given the sides, 13, 20, 21.
2. Given the sides, 13, 30, 37.
3. Given the sides, 33, 34, 65.

4. Given the sides, 35, 52, 73.
5. Given the sides, 29, 60, 85.
6. Given the sides, 140, 143, 157.
7. Given the sides, 507, 603, 721.
8. Given the sides, 46 rd., 75 rd., 109 rd.
9. Given the sides, 40 rd., 51 rd., 77 rd.
10. Given the sides, 3.5 ch., 10 ch., 11.7 ch.
11. Given the sides, 5.6 ch., 6.1 ch., 7.5 ch.
12. Find area of triangle, each side being 10 rd.
13. Find area of triangle, each side being 50 rd.

REVIEW QUESTIONS

1. How is the product of one number and the sum of two others obtained?

2. How is the product of one number and the difference of two other numbers obtained?

3. To what is the square of the sum of two numbers equal?

4. What is involution?

5. What is meant by the square root of a number?

6. What are the two symbols for square root?

7. Define radicand.

8. How many figures are in the square root of a perfect square consisting of five figures?

9. Give a rule for extracting the square root of integers. For extracting the square root of decimal numbers.

10. How is the square root of a fraction obtained when the denominator is not a perfect square?

11. State the Pythagorean Theorem.

12. State the rule for finding the area of a triangle when the three sides are given.

CHAPTER XXII

MENSURATION OF THE CIRCLE, ETC.

358. Take a string and find the length of the circumference of a circle. Take another string and find the length of the diameter of the circle. Divide the former result by the latter to get the ratio of the circumference of the circle to its diameter.

We shall let c represent the length of the circumference of a circle, d the length of the diameter, r the length of the radius, and C the area of the circle.

The ratio of the circumference of a circle to its diameter is approximately 3.14159265. This ratio is denoted by the Greek letter π (Pi). In cases where the numbers involved are not very large, or where extreme accuracy is not demanded, $3\frac{1}{7}$ is a sufficiently accurate approximation of π. The ratio 355 : 113 is a close approximation to the value of π. 3.1416 is generally taken as the value of π. In this chapter we shall consider $\pi = 3.1416$.

Since $\frac{c}{d} = \pi$, $\therefore\ c = \pi d = \pi\ (2\,r) = 2\,\pi r$.

EXERCISE 219

Find the circumference when :

1. The diameter is 22.
2. The diameter is 46.
3. The diameter is 150.
4. The diameter is 164.
5. The diameter is 196.
6. The radius is 67.
7. The radius is 86.
8. The radius is 3.6.
9. The radius is 5.9.
10. The radius is 7.3.

11. Find d when $c = 320.44$.

12. Find d when $c = 477.52$.

13. Find d when $c = 24.50$.

14. Find d when $c = 41.7$.

15. Find r when $c = 377$.

16. Find r when $c = 53.41$.

17. Find r when $c = 60.319$.

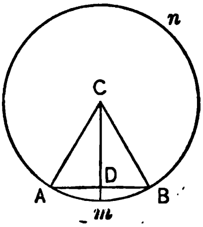
18. Find r when $c = 42.097$.

19. The diameter of the front wheel of a carriage is 3 ft. 6 in. How many times does the wheel revolve in going 1 mi.? Take $3\frac{1}{7}$ as the value of π.

20. The driving wheel of a locomotive is 4 ft. $11\frac{1}{2}$ in. in diameter. If the locomotive travels 34 mi. an hour, how many times does the wheel revolve per minute?

AREA OF A CIRCLE

359. A **sector** of a circle is a portion of a circle bounded by two radii and their included arc. $AmBC$ is a sector.



360. To get the area of the circle ABn we shall first get the area of the sector $AmBC$.

Let CD be perpendicular to AB. The area of the triangle ABC is equal to $\frac{1}{2} AB \times DC$. The triangle ABC is obviously less than the sector $AmBC$. Suppose now the circle ABn to be very large, say as large as the equator of the earth, and let the arc AB be small, say $\frac{1}{6}$ of a minute. Then CD differs from the radius of the earth by about $\frac{1}{14}$ of an inch, a quantity very small when compared with the length of the radius which is nearly 4000 mi. When, therefore, the arc AB is small, the area of the sector and the area of the triangle ABC differ by a quantity which is small. We may, therefore, take the area of the sector as equal to one-half the radius times the arc. If we make another sector, its area is also equal to one-half its arc multiplied by its radius. By dividing the circle into sectors we finally arrive at the conclusion that the sum of the areas of the sectors is equal to one-half the radius times the sum of the arcs of those sectors, and, hence, the area of the circle equals $\frac{1}{2}$ the radius times the circumference.

In the language of mathematics this conclusion is as follows:

$$C = \tfrac{1}{2}cr. \qquad (1)$$

Since $$c = 2\pi r,$$

$$\therefore C = \tfrac{1}{2} \times 2\pi r \times r = \pi r^2. \qquad (2)$$

Also since $$r = (\tfrac{1}{2}d);$$

$$\therefore r^2 = (\tfrac{1}{2}d)^2 = \tfrac{1}{4}d^2.$$

$$\therefore \pi r^2 = \frac{\pi}{4}d^2.$$

$$\therefore C = \frac{\pi}{4}d^2. \qquad (3)$$

We have, therefore, **three rules for finding the area of a circle:**

(1) Multiply one-half the circumference by the radius.

(2) Multiply the square of the radius by π.

(3) Multiply the square of the diameter by $\frac{1}{4}\pi$ or .7854.

There is a **fourth rule** for finding the area of a circle. We may derive it as follows:

$$c = 2\pi r.$$

$$\therefore c^2 = 2\pi r \times 2\pi r = 4\pi^2 r^2.$$

$$\therefore c^2 \times \frac{1}{4\pi} = \frac{4\pi^2 r^2}{4\pi} = \pi r^2 = C.$$

$$\frac{1}{4\pi} = .07958, \text{ very nearly.}$$

That is, the area of a circle equals the square of its circumference multiplied by .07958.

EXERCISE 220 (Written)

1. Given $r = 14$, find C.
2. Given $r = 22$, find C.
3. Given $r = 36$, find C.
4. Given $r = 4.7$, find C.
5. Given $r = 6.5$, find C.
6. Given $r = 8.6$, find C.
7. Given $r = 9.7$, find C.
8. Given $d = 78$, find C.
9. Given $d = 64$, find C.
10. Given $d = 96$, find C.
11. Given $d = 74$, find C.
12. Given $d = 92$, find C.
13. Given $c = 100$, find C.
14. Given $c = 78$, find C.
15. Given $c = 83$, find C.
16. Given $c = 93$, find C.
17. Given $c = 8.7$, find C.
18. Given $c = 6.9$, find C.
19. Given $c = 9.8$, find C.
20. Given $c = 10.8$, find C.

361. *Example* 1. Given the area of a circle equal to 535.08, find the radius of the circle.

SOLUTION.
$$\pi r^2 = C.$$
$$\therefore 3.1416\, r^2 = 535.08.$$
$$\therefore r^2 = \frac{535.08}{3.1416} = 170.31.$$
$$\therefore r = 13.05, \text{ nearly.}$$

Example 2. Given the area of a circle equal to 658.98, find the circumference of the circle.

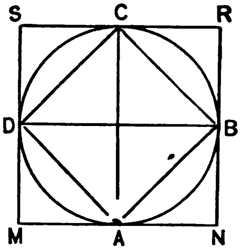
SOLUTION.
$$.07958\, c^2 = C.$$
$$.07958\, c^2 = 658.98.$$
$$\therefore c^2 = \frac{658.98}{.07958} = 8281, \text{ nearly.}$$
$$\therefore c = 91, \text{ nearly.}$$

EXERCISE 221 (Written)

1. Given $C = 3019.1$, find r.
2. Given $C = 907.9$, find r.
3. Given $C = 3421.2$, find r.
4. Given $C = 5541.8$, find r.
5. Given $C = 21.24$, find r.

6. Given $C = 40.715$, find d.
7. Given $C = 66.476$, find d.
8. Given $C = 75.43$, find d.
9. Given $C = 109.36$, find d.
10. Given $C = 141.03$, find d.
11. Given $C = 4.5964$, find c.
12. Given $C = 6.883$, find c.
13. Given $C = 779.94$, find c.
14. Given $C = 57{,}495$, find c.
15. Given $C = 33{,}621$, find c.

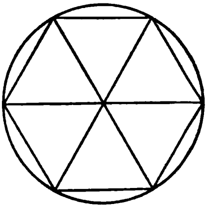
16. How long must a rope be so that by tying one end of it to a stake driven into the ground and fastening the other end to a cow's horn, the cow may graze over 1 A?



17. *ABCD* is a square described in a circle; *MNRS* is a square described about the circle. If the radius of the circle is 12, find the areas of *ABCD*, *MNRS*, and of the circle.

18. If the diameter of a circle is 34 in., find the difference between the area of the circle and of the square described in the circle.

19. If the diameter of a circle is 88 in., find the difference between the area of the square described about the circle and the area of the circle.



20. A regular hexagon is a figure of six sides each of the same length, and its six angles are equal to one another. A hexagon may be broken up into six equilateral triangles.

If the side of a regular hexagon is 10, find its area.

21. If a regular hexagon is described in a circle, its side equals the radius of the circle. If the radius of a circle is 16 in., find the difference between the areas of the circle and the regular hexagon described in the circle.

22. A regular hexagon and a square have each a perimeter of 60 in. Find their areas.

23. The circumference of a circle equals the perimeter of a square. Find which has the larger area.

362. If we take any angle, A, formed by two radii in a circle, and take another angle, B, formed by two radii of the same circle, then designate the subtended arcs by arc A and arc B, we have $A : B = \text{arc } A : \text{arc } B$. The reason for this is: If A were twice B, then arc A would be twice arc B. If A were three times B, then arc A would be three times arc B, and so on for any number of times (Fig. 1).

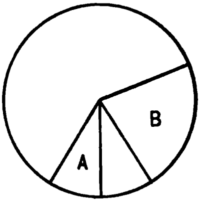


FIG. 1.

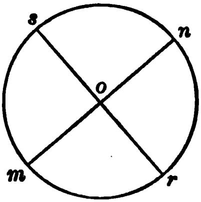


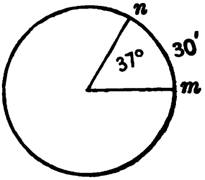
FIG. 2.

$$\frac{A}{B} = \frac{\text{arc } A}{\text{arc } B}.$$

$$\therefore\ A : B = \text{arc } A : \text{arc } B. \qquad (a)$$

If two diameters, *mn*, *sr* (Fig. 2), are drawn at right angles, the four angles at the center are all right angles. Therefore, the sum of the angles formed at a point by drawing lines from that point equals four right angles, or 360°. If we divide each of the right angles at O into 90 equal parts, we have 360 angles, each equal to 1°. The arcs these angles intercept on the circumference are all equal, and as the 360th part of the circumference is called a degree on the circumference, hence (*a*) the number of degrees in an angle at the center is equal to the number of degrees in its arc on the circumference. This fact is generally stated as follows: A **central angle** is measured by its intercepted arc.

363. *Example* 1. The radius of a circle is 15 in. Find the length of an arc of 37° 30′ of the circumference of this circle.



SOLUTION. $360^\circ : 37^\circ\ 30' =$ length of the circumference : length of the arc *mn*.

$$c = 2\pi r.$$

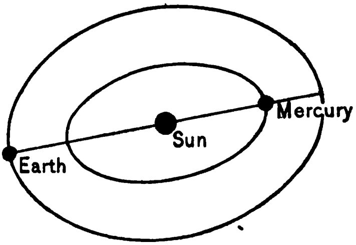
$$\therefore\ c = 2 \times 3.1416 \times 15 \text{ in.} = 94.248 \text{ in.}$$

$$\therefore\ 360^\circ : 37\tfrac{1}{2} = 94.248 \text{ in.} : \text{arc } mn.$$

$$\therefore\ 360^\circ \times \text{arc } mn = 94.248 \times 37\tfrac{1}{2} \text{ in.}$$

$$\therefore\ \text{arc } mn = \frac{94.248 \times 37\frac{1}{2} \text{ in.}}{360} = 9.82 \text{ in., nearly.}$$

Example 2. To an observer on the earth's surface what angle does the planet Mercury subtend when Mercury is at its greatest distance from the earth?



SOLUTION. *Step* 1. Draw a diagram.

Step 2. Find Mercury's distance from the earth when it is in a straight line with the earth and the sun. This is 36,000,000 + 92,900,000, *i.e.* 128,900,000 mi.

Step 3. Find the angle which an arc of 3030 mi. subtends at the center of a circle whose radius is 128,900,000 mi.

$$c = 2\pi r.$$

$$\therefore\ c = 2\pi \times 128{,}900{,}000 \text{ mi.}$$

$$2\pi \times 128{,}900{,}000 : 3030 = 360^\circ : x.$$

$$\therefore\ (2\pi \times 128{,}900{,}000)\,x = 360^\circ \times 3030.$$

$$\therefore\ x = \frac{360^\circ \times 3030}{2\pi \times 128{,}900{,}000} = 5'', \text{ nearly.}$$

EXERCISE 222 (Written)

1. The radius of a circle is 87 in. Find the length of an arc of 72° of this circle.

2. The radius of a circle is 94 in. Find the length of an arc of 30° of this circle.

3. What is the length of an arc of 1° on a circle whose radius is 58 ft.?

4. What angle does an arc of 40.212 ft. subtend at the center of a circle whose radius is 64 ft.?

5. What angle does an arc of 6.032 ft. subtend at the center of a circle whose radius is 96 ft.?

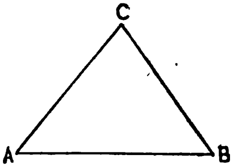
6. The distance of the moon from the earth is 239,000 mi., and the diameter of the moon is 2170 mi. To an observer on the earth, what angle does the moon's diameter subtend?

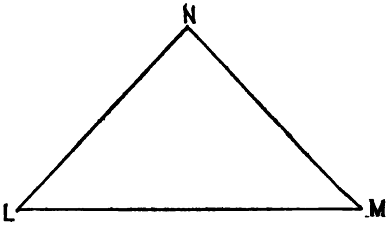
7. From the data in Example 2, solved above, determine the angle which Mercury makes to an observer on the earth when Mercury attains its maximum brilliancy.

8. What angle does the sun's diameter make at the earth? The sun's diameter is 866,400 mi., and its distance from the earth is 92,900,000 mi.

SIMILAR FIGURES

364. **Similar figures** are figures having the same form.





Examples. The triangles ABC, LMN are similar triangles.

All regular polygons of the same number of sides are similar figures. The drawing which a surveyor makes of a tract of land is similar to the tract of land.

It is shown in geometry that corresponding dimensions of similar figures have the same ratio; also that the areas of similar figures are to each other as the squares of their corresponding dimensions.

365. *Example* 1. When a pole 6 ft. high casts a shadow 5 ft., how high is a steeple whose shadow is 90 ft.?

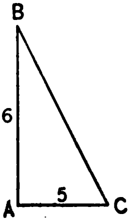
SOLUTION. Let AB represent the pole, AC its shadow. Let x represent the steeple, and MR its shadow. We have then

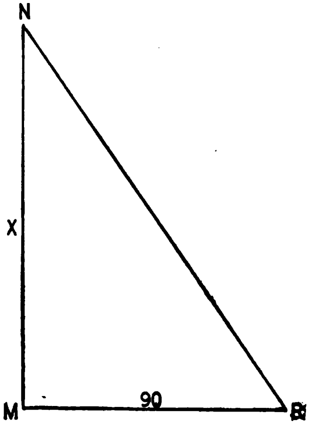
$$5 : 90 = 6 : x.$$

$$5x = 90 \times 6.$$

$$x = \frac{90 \times 6}{5} = 108.$$

Ans. 108 ft.





Example 2. The area of a triangle, one of whose sides is 5 rd., is 11 sq. rd.

Find the corresponding side of a similar triangle whose area is three times as great.

SOLUTION. Let X equal the required side. We know: The area of the first triangle: the area of the second triangle = square of the side of the first triangle : square of the side of the second triangle.

$$11 : 33 = 5^2 : X^2.$$
$$\therefore\ 11X^2 = 33 \times 5^2.$$
$$\therefore\quad X^2 = \frac{33 \times 5^2}{11} = 3 \times 5^2 = 75.$$
$$\therefore\quad X = \sqrt{75} = 8.662. \quad \textit{Ans.}\ 8.662 \text{ rd.}$$

EXERCISE 223 (Written)

1. When a tree 90 ft. high casts a shadow 75 ft. long, find the length of the shadow cast by a pole 24 ft. high.

2. How high is an object which casts a shadow 110 ft. when a pole 8 ft. high casts a shadow 5 ft.?

3. A map is drawn to a scale of 40 mi. to 1 in. On this map two cities are $2\frac{3}{4}$ in. apart. How many miles are there between these cities?

4. In a map of a city two public buildings are $9\frac{1}{2}$ in. distant. If the map is drawn to the scale of 1 in. to $\frac{3}{4}$ of a mile, how far is it from one of these buildings to the other?

5. The area of a triangle is 15 sq. ft., and one of its sides is 10 ft. Find the corresponding side of a similar triangle five times as large.

6. The altitude of a triangle is 10 ft. If the triangle is divided into two equal parts by a line parallel to its base, how far from the vertex must this line be drawn?

7. Corresponding sides of two similar quadrilaterals are in the ratio of 4 to 11. Find the ratio of their areas.

8. The diameters of two circles are 12 and 18 in. Find the ratio of their areas.

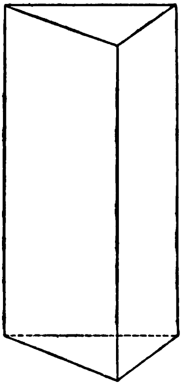
9. The distance between two cities is 90 mi., and on a map containing both cities their positions are distant $5\frac{5}{6}$ in. What area is represented by a circle of $\frac{1}{2}$ in. radius on this map?

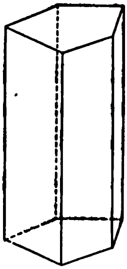
10. Assuming that a 5-dollar gold piece is one-half as thick as a double eagle, what is the ratio of the diameter of the double eagle to the diameter of the 5-dollar piece?

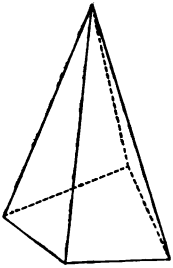
SURFACES OF THE PRISM, PYRAMID, CYLINDER, CONE, AND SPHERE

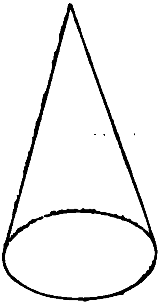
366. A **right prism** is a solid, two of whose faces are equal and parallel polygons, and whose other faces are rectangles.

The upper and lower faces are called the *bases*, and the other faces are called *lateral faces*


Triangular Prism


Prism


Pyramid


Cone

367. A **right pyramid** is a solid whose base is a regular polygon, and whose other faces are triangles equal in area.

368. A **cone** is a solid made by the revolution of a right triangle about one of its legs.

369. A **cylinder** is a solid made by the revolution of a rectangle about one of its sides when that side is fixed.

370. A **sphere** is a solid formed by the revolution of a semicircle about its diameter when the diameter is fixed.

371. The **lateral surface of a right prism** equals the product of the perimeter of its base by the height of the prism.

372. The **lateral surface of a right pyramid** equals one-half the perimeter of its base by the altitude of one of its lateral faces.

373. The **convex surface of a cone** equals one-half the circumference of its base by its slant height.

374. The **convex surface of a cylinder** equals the circumference of its base by its height.

375. The **surface of a sphere** equals four times the area of one of its great circles, *i.e.* $4\pi r^2$.

EXERCISE 224 (Written)

1. Find the lateral surface of a quadrangular prism the dimensions of whose base are 16 ft. by 8 ft. and whose height is 12 ft.

2. Find the area of the walls of a room, having given the dimensions of the floor as 18 ft. by 16 ft., and the height as 10 ft.

3. Find the height of a triangular prism, the sides of its base being 5, 6, and 7 ft., and its lateral area being 190 sq. ft.

4. Find the height of a pentagonal prism whose lateral area is 300 sq. ft., and each side of whose base is 8 ft.

5. The base of a square pyramid is 40 ft. long, and the altitude of each of its triangular faces is 16 ft. Find its lateral area.

6. Each side of a hexagonal pyramid is 14 ft. and its slant height is 15 ft. Find the area of its lateral surface.

7. A pyramidal tent whose base is a square 22 ft. on a side has a slant height of 30 ft. Find the cost of the canvas for the tent, at 18¢ per square yard.

8. Find the number of square yards in the lateral surface of a triangular pyramid, each side of the base being 21 ft. and the slant height being 42 ft.

9. The radius of the base of a right cone is 49 in. and the slant height is 50 in. Find its convex surface in square feet.

10. Find the lateral surface of a right circular cone, the diameter of whose base is 64 in. and whose slant height is 40 in.

11. How many yards of canvas are required to make a conical tent, the diameter of the base being 24 ft. and the slant height 16 ft.?

12. Find the convex surface of a cylinder the diameter of whose base is 19 ft. and whose height is 50 ft.

13. Find the convex surface of a cylinder, the radius of the base being 41 in., and the height, 60 in.

14. A standpipe has a diameter of 30 ft. and is 150 ft. high. Find the cost of painting it at 25¢ per square yard.

15. Find the surface of a sphere whose radius is 98 in.

16. Find the surface of a sphere if its diameter is 42 in.

17. The diameter of the planet Mercury is 3030 mi.; find the area of the planet.

18. The diameter of the planet Venus is 7700 mi.; find the area of the planet.

19. The diameters of the major planets are, respectively, 86,000, 73,000, 32,000, 33,000 mi. Find the number of million square miles in the area of each of these planets.

20. The surface of a sphere is 10,568 sq. in. Calculate its diameter.

21. The base of a cylinder is 20 in. in diameter, and its height is 20 in. Find the area of the cylinder. Find the area of a sphere 20 in. in diameter.

22. The lateral surface of a square pyramid each side of whose base is 10 in., is 1100 sq. in. Find the slant height.

23. Find the diameter of a sphere which has an area of 7854 sq. yd.

VOLUMES OF SOLIDS

376. The **volume of a rectangular prism** is equal to the area of its base multiplied by its height.

The **volume of a cylinder** is also equal to the product of its base by its height.

377. The **volume of a pyramid** and that of a cone are each equal to one-third the product of the area of the base and the height. In each case we understand by height, the perpendicular distance from the vertex to the base.

378. The **volume of a sphere** is equal to $\frac{4}{3}\pi r^3$.

Since $$r = \tfrac{1}{2}d,$$

$$\therefore r^3 = \tfrac{1}{8}d^3,$$

$$\therefore \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi \times \tfrac{1}{8}d^3 = \frac{\pi}{6}d^3 = .5236\,d^3$$

EXERCISE 225 (Written)

1. Find the volume of a triangular prism, the sides of the base being 11, 25, 30 in., respectively, and the height of the prism being 40 in.

2. Find the volume of a square pyramid, if the sides of the base are each equal to 10 in., and the height is 21 in.

3. Find the volume of a cone, the radius of its base being 12 in., and its height being 27 in.

4. Find the volume of a cone, if the radius of the base is 25 in., and the height is 24 in.

5. Find the volume of a hexagonal pyramid, each side of its base being 10 in., and its height being 30 in.

6. Find the volume of a sphere whose radius is 20 in.

7. Find the volume of a sphere, the radius being 8 ft.

8. How many gallons does a cylindrical cistern hold, the diameter of its base being 9 ft. 4 in., and its height 8 ft.?

9. How many gallons does a cylindrical cistern contain, if the diameter of its base is 11 ft. and its height is 6 ft. 5 in.?

10. The diameter of the base of a cylinder is 10 in., and its height is 10 in. Find the ratio of the volume of this cylinder to the volume of a sphere 10 in. in diameter.

11. The diameter of the base of a cone is 1 ft. and its height is 1 ft. Find the ratio of the volume of this cone to the volume of a sphere whose diameter is 1 ft.

12. The surface of a cube contains 84 sq. ft. 54 sq. in. Find its volume.

13. The base of a pyramid is a triangle whose sides are 1 ft. 1 in., 3 ft. 1 in., 3 ft. 4 in., and whose volume is 1 cu. ft. 1152 cu. in. Find its height.

14. The surface of a sphere equals 1257 sq. in. Find its volume.

15. The surface of a hemispherical dome is 2513.5 sq. ft. Find its diameter.

16. The volumes of similar solids are to each other as the cubes of their corresponding dimensions. How many times as large as the earth is the sun? The diameter of the sun is nearly 888,000 mi., and the diameter of the earth is nearly 8000 mi.

17. Find how many times as large as the moon is the earth. The moon's diameter is 2200 mi., nearly.

18. How many times as large as the earth is Saturn? The diameter of Saturn is 73,000 mi.

19. How many times as large as the earth is Jupiter? The diameter of Jupiter is 88,000 mi., nearly.

REVIEW QUESTIONS

1. Give three approximations of the ratio of the circumference of a circle to the diameter.

2. How do you find the circumference when the radius is given?

3. How do you find the circumference when the diameter is given?

4. How do you find the radius when the circumference is given?

5. Give four rules for finding the area of a circle.

6. How do you find the area of an equilateral triangle?

7. How do you find the area of a regular hexagon?

8. How is the length of an arc obtained, if the radius and the number of degrees in the arc are given?

9. Explain, a central angle is measured by its intercepted arc.

10. What are similar figures?

11. How do the areas of similar figures vary?

12. How do corresponding dimensions of similar figures vary?

13. Define right pyramid, cone, cylinder, sphere.

14. How are the lateral surfaces of a right pyramid, right prism, cylinder, and cone found?

15. Give the rule for finding the surface of a sphere.

16. Show that the surface of a sphere equals the product of the circumference of a great circle of the sphere by its diameter.

17. Give the rule for finding the volume of a rectangular prism, cylinder, cone, sphere.

CHAPTER XXIII

THE METRIC SYSTEM OF WEIGHTS AND MEASURES

379. In the last decade of the eighteenth century, Louis XVI, on the recommendation of the Constituent Assembly, by a decree invited the nations of the world to confer respecting an international system of weights and measures. No adequate response to this invitation being received, the Constituent Assembly committed the consideration of this subject to a committee of learned men who devised what has since been known as the **Metric System of Weights and Measures.**

This system is now used by upward of forty countries and it is the only system used in text-books of science. Upward of twenty nations contribute to the support of the International Bureau of Weights and Measures in Paris. For these reasons the Metric System deserves to be called the International System.

The only great nations which have not adopted the Metric System are the United States and Great Britain. In the United States the system is legalized, and none other is used in the Philippines and Porto Rico. In Great Britain the adoption of the system is permissive, and probably in a few years it will be obligatory.

The Metric System is so-called because the meter is the basis of the system. The **meter** is the standard unit of linear measure. Its length is the ten-millionth part of the distance from the equator to the north pole measured on the meridian of Paris. It is a natural unit of measure as unchangeable as the world itself.

The standard meter from which all others are derived is a bar made of an alloy of platinum and iridium, kept in the International Bureau of Weights and Measures in Paris. Duplicates of this standard meter have been furnished to all the nations of the world. These duplicates are made of the hardest, most durable, and least expansible substances known.

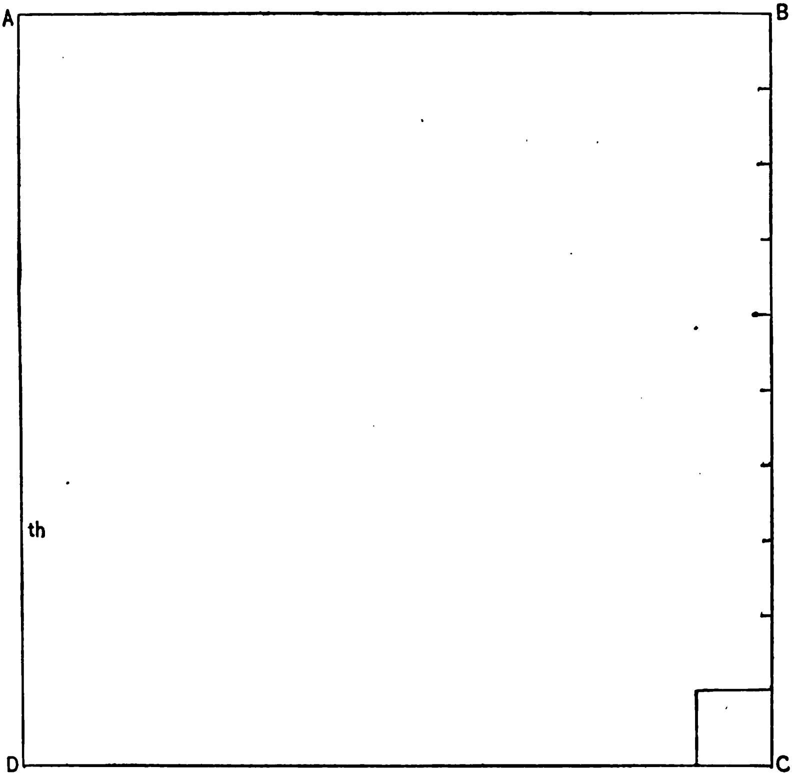
The legal equivalent of the meter at zero degrees, Centigrade, on the continent of Europe is 39.370432 English inches. The legal equivalent in England is 39.3707904 English inches. For practical purposes, 39.37 in. is a close approximation to the value of the meter. This is the value recommended by the United States Bureau of Standards.

The Metric System is a decimal system. In it there are no compound rules. It is the simplest and the most perfect system ever devised.

The names for the multiples of the standard unit in the Metric System are formed from the names of the standard unit by means of prefixes derived from the Greek words meaning ten, one hundred, one thousand, and ten thousand, *i.e. deka*, *hekaton*, *chilioi*, *murioi*. The names for the submultiples of the standard units are formed in a similar manner from the Latin words meaning ten, one hundred, one thousand, *i.e. decem*, *centum*, *mille*. Thus:

Dekameter means ten meters.
Hektometer means one hundred meters.
Kilometer means one thousand meters.
Myriameter means ten thousand meters.
Decimeter means one-tenth of a meter.
Centimeter means one-hundredth of a meter.
Millimeter means one-thousandth of a meter.

The **standard units** are the *meter*, the *liter*, and the *gram*. The gram being a very small weight, the kilogram is most used in ordinary affairs. In the Metric System the meter, the liter, and the kilogram are used in exactly the same manner as the yard, the quart measure, and the pound Avoirdupois, in our system of weights and measures.



The above illustration gives a clear idea of the units of the Metric System.

Each side of the square *ABCD* is a decimeter, or one-tenth of a meter.

Each side of the small square is a centimeter, or one-hundredth part of a meter.

The *liter* is the volume of a cube which has the square *ABCD* as one of its faces.

The *gram* is the weight of distilled water at 4° Centigrade, which fills a cubical vessel, one of whose faces is the size of the small square.

380. LINEAR MEASURE

10 millimeters (mm.) = 1 centimeter
10 centimeters (cm.) = 1 decimeter
10 decimeters = 1 meter
10 meters (m.) = 1 dekameter
10 dekameters = 1 hektometer
10 hektometers = 1 kilometer
10 kilometers (km.) = 1 myriameter

The units in common use are the millimeter, centimeter, meter, and kilometer. To express very small linear measurements a unit called a mikrometer, or micron is sometimes used. It is equal to the millionth part of a meter. The abbreviation for micron is the Greek letter μ.

381. SURFACE MEASURE

100 square millimeters (qmm.) = 1 square centimeter
100 square centimeters (qcm.) = 1 square decimeter
100 square decimeters = 1 square meter
100 square meters (qm.) = 1 ar
100 ars = 1 hektar
100 hektars (ha.) = 1 square kilometer (qkm.)

An ar is the area of a square whose side is ten meters. 1 qm. is therefore equal to a centar (ca.). The areas of small tracts of land, such as gardens, are expressed in ars.

The areas of larger tracts, such as farms, are expressed in hectars. The areas of still larger tracts, such as countries, are expressed in square kilometers.

CUBIC MEASURE

382. 1000 cubic millimeters (cmm.) = 1 cubic centimeter
1000 cubic centimeters (ccm.) = 1 cubic decimeter, or liter
1000 liters (l.) = 1 cubic meter (cbm.)

The liter corresponds to our quart. The hektoliter (hl.) corresponds to our bushel. The millionth part of a liter is called a mikroliter (λ). The cubic meter is also called a stere.

WEIGHT

383. 1000 milligrams (mg.) = 1 gram
1000 grams (g.) = 1 kilogram, or kilo
1000 kilograms (kg.) = 1 tonneau (T.), or ton

The millionth part of a gram is called a mikrogram(γ).

A gram represents the weight of 1 ccm. of distilled water at 4° C.

A kilogram is the weight of 1 cu. dm. of distilled water at 4° C.

A tonneau, or ton, is the weight of 1 cbm. of distilled water at 4° C.

A quintal is 100 kg.

REDUCTION

384. *Example* 1. Reduce 75.623 m. to centimeters, and also to millimeters.

Solution. (*a*) Since there are 100 cm. in 1 m., we therefore reduce meters to centimeters by multiplying the number of meters by 100. This we do by moving the decimal point two places to the right.

(*b*) Since there are 1000 mm. in 1 m., we therefore multiply the number of meters by 1000. This we do by moving the decimal point three places to the right.

Answers. (*a*) 7562.3 cm.
(*b*) 75,623 mm.

Example **2.** Reduce 85,679 mm. to meters.

SOLUTION. This we do by dividing by 1000, *i.e.* by moving the decimal point three places to the left.

Ans. 85.679 m.

EXERCISE 226 (Written)

1. Reduce 234.982 m. to centimeters.

2. Reduce 132.98745 m. to centimeters.

3. Reduce 1653.98231 m. to centimeters.

4. Reduce 98,765.23456 m. to centimeters.

5. Reduce 101,907.9701 m. to millimeters.

6. Reduce 342.67539 m. to millimeters.

7. Reduce 106,598.95467 m. to kilometers.

8. Reduce 90,187,943.9 mm. to kilometers.

9. Reduce 986,543.9789 cm. to kilometers.

10. Reduce 345.9018765 km. to meters.

11. Reduce .987615 km. to centimeters.

12. Reduce 374,321 qcm. to square meters.

13. Reduce 234,895 qcm. to square meters.

14. Reduce 98,654.3 ca. to ars.

15. Reduce 92,345,723 ca. to hektars.

16. How many square meters are in 1 qkm.?

17. How many square meters are in a square hektometer?

18. How do you reduce square hektometers to square kilometers?

19. How many square decimeters are in 1 ar?

20. How many ars are in 1 qkm.?

21. How many square millimeters are in 1 ca.?

22. How many square millimeters are in 1 ar?

23. How many cubic millimeters are in 1 cbm.?

24. How many cubic centimeters are in 1 cbm.?

25. Reduce 98234.87 l. to cubic millimeters.

26. Reduce 27.236 cbm. to cubic centimeters.

27. Express 9873 ccm. as liters.

28. Express 92342784.2 cmm. as liters.

29. Reduce 54972 mg. to grams.

30. Reduce 19,821,096,543 g. to tonneaux.

31. How many milligrams are in 1 T.?

EXERCISE 227 (Written)

Add:

1.	**2.**	**3.**
96458.23 m.	92435.9 cm.	924357.9 mm.
23458.98 m.	23456.8 cm.	8765.1 mm.
14329.09 m.	98239.2 cm.	986201.8 mm.
920184.60 m.	98209823.9 cm.	87623.7 mm.
1908.02 m.	1980098.76 cm.	987.9 mm.
90327.92 m.	6543097.1 cm.	23456.2 mm.
1872095.72 m.	7609.9 cm.	10098.3 mm.
902376.253 m.	980098.3 cm.	90098.9 mm.
		999999.9 mm.

4. A man walks on four successive days. The first day he walks 11.7 km.; the second day, 984 m.; the third day, 2950 m.; the fourth day, 12.8 km. How far does he travel? Give the answer in meters.

5. From 25.724 km. take 26,270 m.

6. Multiply 11.732 m. by 12.

7. How many times is 25 mm. contained in 24 m.?

8. How many times is 7.03 m. contained in .0494209 km.?

9. How many times is 12 ca. contained in 6 ha.?

10. How many farms of 8 ha. each can be made out of a square whose side is 3 km.?

11. A vessel contains 250 ccm. How many such vessels would hold 5 cbm.?

12. How many times is 800 ccm. contained in 50 l.?

13. How many times is 450 ccm. contained in 22.5 hl.?

14. How many mikroliters are in 1 ccm.? How many mikroliters are in 1 cbm.?

15. How often is 1521 mg. contained in 5.9319 kg.?

385. *Example.* Find the area of a rectangle, if its length is 425.8 m. and its breadth is .3256 km.

SOLUTION. First reduce .3256 km. to meters.

.3256 km. = 325.6 m.

Second, multiply in the usual manner and get for product 138,640.48 qm. We reduce this to ars by dividing by 100, and we reduce the ars to hektars by dividing by 100. Both operations can be performed at once by moving the decimal point four places to the left. *Ans.* 13.864048 ha.

$$\begin{array}{r}
425.8 \\
\underline{325.6} \\
25548 \\
21290 \\
8516 \\
\underline{12774} \\
138640.48 \text{ qm.}
\end{array}$$

EXERCISE 228 (Written)

Find the area of each of the following rectangles:

	LENGTH	WIDTH		LENGTH	WIDTH
1.	625 m.	125 m.	**4.**	369.4 m.	184.7 m.
2.	305 m.	61 m.	**5.**	488.9 m.	244.45 m.
3.	338 m.	169 m.	**6.**	5767 m.	11.534 m.

7. Find the volume of a rectangular solid whose dimensions are 1.2 m., .9 m., and .75 m.

8. Find the area of a triangle whose sides are 14 m., 48 m., and 50 m.

9. Find the area of a circle whose radius is .78 m.

10. Find the volume of a sphere whose radius is 18 cm.

11. The legs of a right triangle are 56 cm. and 105 cm. Find the hypothenuse.

12. Find the volume of a rectangular prism, the dimensions of the base being 14 m. by 12 m., and height 9.5 m.

A COMPARISON OF OUR SYSTEM OF WEIGHTS AND MEASURES WITH THE METRIC SYSTEM

386. In order to show the practical utility of the Metric System, we submit the following solutions:

Example 1. Find the weight of water which fills a rectangular tank 14 in. by 12 in. by $7\frac{7}{8}$ in.

SOLUTION. (*a*)

$$\begin{array}{r} 14\\ \underline{12}\\ 168\\ \underline{7\frac{7}{8}}\\ 1176\\ \underline{147}\\ 1323 \end{array}$$

(*b*) $$\frac{1323}{1728}\times 1000\text{ oz.}=\frac{1323000}{1728}\text{ oz.}$$

(*c*)

$$\begin{array}{l} 1728)1323000(765\frac{5}{8}\text{ oz.}\\ \begin{array}{r} \underline{12096}\\ 11340\\ \underline{10368}\\ 9720\\ \underline{8640}\\ 1080 \end{array} \end{array}$$

(*d*)

$$\begin{array}{r} 16\underline{)765\frac{5}{8}}\text{ oz.}\\ 47\text{ lb. }13\frac{5}{8}\text{ oz. } \textit{Ans.} \end{array}$$

To solve this example, we have four steps. (*a*) Take the product of the three dimensions. (*b*) Divide the result by 1728 and multiply the quotient by 1000. (*c*) We perform the actual division. This gives us the answer in ounces. (*d*) Reduce the ounces to higher denominations.

Now take a similar problem in the Metric System.

Find the weight of water which will fill a rectangular tank 14 cm. by 12 cm. by $7\frac{7}{8}$ cm. deep.

SOLUTION. (*a*)

$$\begin{array}{r} 14 \\ \underline{12} \\ 168 \\ \underline{7\frac{7}{8}} \\ 1176 \\ \underline{147} \\ 1323 \end{array}$$

Ans. 1323 ccm. = 1.323 l.

Example 2. Find the area of a rectangle whose length is 1136 yd. and width 284 yd.

SOLUTION. 1 sq. yd. × 1136 × 284 = 322,624 sq. yd.

$$\begin{array}{r} 4840)322624(66.68 \\ \underline{29040} \\ 32224 \\ \underline{29040} \\ 31840 \\ \underline{29040} \\ 38000 \\ \underline{38720} \end{array}$$

Ans. 66.68 A., nearly.

Take a similar problem in the Metric System.

Find the area of a rectangle whose length is 1136 m., width 284 m.

SOLUTION. 1 qm. × 1136 × 284 = 322,624 qm. = 32.2624 ha. *Ans.*

EQUIVALENTS OF COMMON UNITS IN METRIC UNITS

1 inch = 25.4001 mm.
1 foot = .304801 m.
1 yard = .914402 m.
1 mile = 1.60935 km.
1 quart = .94636 l.
1 gallon = 3.78543 l.
1 bushel = .35239 hl.

1 sq. inch = 6.452 qcm.
1 sq. foot = .0929 qm.
1 sq. yard = .836 qm.
1 A. = .4047 ha.
1 cu. inch = 16.387 ccm.
1 cu. foot = .02832 cbm.
1 cu. yard = .765 cbm.

1 lb. avoir. = .45359 kg.

EQUIVALENTS OF METRIC UNITS IN COMMON UNITS

1 meter = 39.37 in.
1 kilometer = .62137 mi.

1 sq. centimeter = .155 sq. in.
1 sq. meter = 1.196 sq. yd.

1 hektar = 2.471 A.
1 cu. centimeter = .061 cu. in.
1 cu. meter = 35.314 cu. ft.

1 liter = 1.0567 qt.
1 hektoliter = 2.8733 bu.

1 milligram = .01543 grains
1 kilogram = 15,432.36 grains
1 kilogram = 2.20462 lb.
1 tonneau, or ton = 2204.6 lb.

REVIEW QUESTIONS

1. Where did the Metric System originate?

2. Why may this system be called international?

3. What great nations have not yet adopted the Metric System?

4. Why is the Metric System so called?

5. What is the value of the meter in our system?

6. Give reasons why the Metric System is the simplest of all systems of weights and measures.

7. What are the three standard units of the Metric System?

8. To what common units do they correspond?

9. What is a liter? A kilogram? A tonneau, or ton?

10. What is an ar?

11. What other name is given to a square meter?

12. How is reduction ascending accomplished in the Metric System?

13. How are problems in reduction descending worked?

14. What unit in the Metric System corresponds to the common acre?

15. In terms of what metric unit are the areas of countries given?

16. In what metric unit are the heights of mountains expressed?

17. What other name is given to the cubic millimeter?

18. Name the linear units in common use in France.

19. Name the units of square measure in common use in France.

20. Is there a table of Troy weights in the Metric System?

21. By what measure is sugar sold in countries where the Metric System is used?

22. By what measure is muslin bought and sold in countries where the Metric System is used?

MISCELLANEOUS EXAMPLES (A)

(Arranged by Topics)

NOTATION

1. Write in figures, ten million ten.

2. Write in figures, seven million two hundred five thousand.

3. Write in figures, one billion one million one.

4. Write in figures, five million fifty thousand.

5. Write in figures, ten billion ten million one hundred.

6. Write decimally, twenty-five tenths. One hundred twenty hundredths. One hundred and fifty-five hundredths. Ten thousand ten hundred thousandths. One million one ten-millionths. Fifty-five thousand two hundred eighteen ten-thousandths. One million two hundred thousand four ten-millionths. Five and five hundredths. Six hundred six hundredths. Seven hundred ten thousandths. Nine hundred and fifteen thousandths. Nine million and nine millionths.

ADDITION AND SUBTRACTION

7. The following table of railroad mileage in the United States is taken from the report of the Interstate Commerce Commission:

Year	Mileage	Year	Mileage
1890	163,597.05	1897	184,428.47
1891	168,402.74	1898	186,396.32
1892	171,563.52	1899	189,294.66
1893	176,461.07	1900	193,345.78
1894	178,708.55	1901	197,237.44
1895	180,657.47	1902	201,839.00
1896	182,776.63		

Find the number of miles built during each year, beginning with 1890, up to 1902. Take the sum of the results, and see whether it agrees with the increase in railroad mileage from 1890 to 1902.

8. From 9000 take .009.

9. From 275 take .000275.

10. Add:
657,987,324,011
119,008,675,987
199,887,564,999
999,555,777,888
987,345,234,876
987,234,678,100
923,524,896,987
987,567,342,959
725,926,846,368
929,935,829,349
929,563,768,968

MULTIPLICATION

11. Multiply 10,500 by 60,600.

12. Multiply 15,010 by 50,080.

13. Multiply 1.01 by 7.07.

14. Multiply 9010.90706 by 90.400609.

15. A train travels at the rate of 30.25 mi. an hour. How far will it go in 5½ hr.?

16. Find the price of 87½ A. of land at $43.75 an acre.

17. Find the price of 4225 bu. of wheat at 84½¢ per bushel.

18. Find the value of .1 × .2 × .3 × .4 × .5 × .6 × .7.

19. Find the value of $(1.03)^4$.

20. The number of bales of cotton produced in Texas in the season 1901–02 was 2,993,000, and in 1900–01, 3,550,000. Allowing 500 lb. to a bale, how many more pounds of cotton were produced in the latter year than in the former?

DIVISION

21. A steamer's cargo consisted of 120,000 bu. of corn, valued at $57,000; 13,541 bbl. of flour, valued at $47,499; 3050 bales of cotton, valued at $158,224; 9688 staves, valued at $2355; 609 pieces of redwood valued at $869. Find the value of a bushel of corn, a barrel of flour, a bale of cotton, a stave, and a piece of redwood.

22. Divide 26.78508 by .072 (not by long division).

23. The annual consumption of sugar in a certain state was, in 1890, 702,201 T., which was found to be 49.93 lb. per head of population. Find the population.

24. Make a column of eight numbers, the first of which is 73,214, the second is 1.5 times less than the first, the third is 1.5 times less than the second, and so on for the other numbers.

25. How many miles are in 278,784,000 ft.?

26. Divide the difference of the squares of 1.125 and .784 by their sum.

27. Divide the difference of the cubes of .75 and .26 by their difference.

28. Divide 14.302 by 83.92 correct to four places.

29. Divide 24.619 by 56,000.

30. The length of a degree on the earth's surface is approximately 69.15 mi. Two places are on the same meridian, one being 1000 mi. from the other. Find, in degrees, the difference in latitude.

31. Two places on the 60th parallel of latitude are 300 mi. apart. Find the difference of their longitudes.

32. A bankrupt's liabilities are $47,875, his assets are $38,650. How many cents on the dollar can he pay?

33. The product of two numbers is 642,978, and one of the numbers is 5.67. Find the other number.

34. In a division problem the quotient is 24,400, the remainder is 15, and the dividend is 6,100,015. Find the divisor.

35. The total amount of money in circulation in the United States on March 1, 1903, was $2,353,738,834. The per capita circulation in the United States on the same day was $29.41. Find the population of the United States.

36. Divide the square of 1001 by the product of 77 and 169.

37. When 450 lb. of sugar cost $20.25, find the price of 84 lb.

38. Find the value of a rectangular plot of ground 726 yd. long and 240 yd. wide, at $50 an acre.

39. £100 sterling are equivalent to $486.65. Find, in United States currency, the value of £79 sterling.

40. When 1.75 yd. of silk cost $3.85, find the cost of 14 yd.

41. Divide 39.328 by .0032.

42. If .6 of a yd. of cloth cost 27¢, find the cost of 45 yd.

43. Divide 1 by 1.732.

44. Divide the cube of 11.1 by 27 times 1369.

45. What is the ratio of 25 A. to 640 A.?

G. C. M. AND L. C. M.

46. Find the G. C. M. of 288 and 432. Find also the L. C. M.

47. Find all the common measures of 36 and 54.

48. Find the common divisors of 288, 360.

49. Express 1110, 777, and 1001 as the products of prime numbers. Find their L. C. M.

50. Find the G. C. M. of 208, 572, and 1326.

51. Find the L. C. M. of 26, 28, 48, 70, and 117.

52. Find the G. C. M. of 6549 and 7067.

53. Find the G. C. M. and L. C. M. of 7560, 10,296, and 4851.

54. Find the prime factors of 80,850.

55. Two numbers have for their G. C. M. 101, and for L. C. M. 27,573. Find the product of these numbers.

56. Resolve 61,776 into its prime factors.

57. Two tracts of land, containing 1225 acres and 1675 acres, are divided into farms each containing the same number of acres. What is the largest possible acreage of each farm?

58. Telephone poles are 231 ft. apart. What is the smallest number of poles which will correspond to an exact number of half miles?

FRACTIONS, DECIMALS, AND DENOMINATE NUMBERS

59. Arrange in order of magnitude $\frac{3}{4}$, $\frac{7}{9}$, $\frac{11}{15}$.

60. Find the difference between the greatest and the least of the fractions $\frac{2}{3}$, $\frac{5}{8}$, $\frac{17}{21}$, and $\frac{19}{24}$.

61. Add: $2\frac{1}{2}$, $3\frac{1}{4}$, $5\frac{5}{8}$, $3\frac{7}{12}$.

62. Reduce to its lowest terms $\frac{999}{1110}$.

63. Express as decimals $\frac{35}{32}$, $\frac{7}{64}$, $\frac{16}{125}$.

64. Reduce to common fractions .0375, .0175, .03125.

65. Simplify $\dfrac{1\frac{3}{4}-\frac{2}{3}}{8\frac{3}{4}-3\frac{1}{3}} \times 15\frac{3}{5}$.

66. Simplify $2\frac{5}{6}$ of $\dfrac{4\frac{1}{5}}{4\frac{2}{3}} - (\frac{1}{7}$ of $17\frac{1}{2}$ of $\frac{2}{5}$ of $1\frac{1}{2})$.

67. Reduce 198 ft. to the decimal of $1\frac{1}{2}$ mi.

68. Reduce 2° 30′ to the decimal of 90°.

69. Reduce 3 pt. to the decimal of 5 gal.

70. Reduce .375 of 16*s.* 8*d.* + $\frac{2}{3}$ of 15*s.* 6*d.*, to the decimal of £5.

71. Show that if our calendar had 8 leap years in every 33 yr., it would be more correct than it now is.

72. If our calendar were so arranged that 31 leap years would occur in 128 yr., how many years would elapse before the error would amount to 1 da.?

73. The following distances have been run by trains in the times indicated. Find in each case the rate per hour:

Route	Distance in Miles	Time
Jersey City to Oakland	3311	83 hr. 45 min.
New York to Chicago	964	19 hr. 57 min.
Chicago to New York	962	17 hr. 45 min.
London to Aberdeen	539.75	8 hr. 32 min.
Chicago to Buffalo	510.1	8 hr. 1 min. 7 sec.
Albany to Syracuse	147.84	2 hr. 10 min.
Erie to Buffalo Creek	86	1 hr. 10 min. 45 sec.
Camden to Atlantic City	58.3	45 min. 45 sec.
Liberty Park to Absecon	49.8	37 min. 30 sec.
Berlin to Absecon	35.6	25 min. 45 sec.
New York to Philadelphia . . .	90	1 hr. 17 min.

74. The price of oats in December, 1901, was $46\frac{3}{4}$¢ per bushel. Find the value of 25,000 bu. of oats in December, 1901.

75. The price of oats in June, 1900, was $26\frac{1}{4}$¢ per bushel, and in August of the same year it was 21¢ per bushel. If a speculator lost $1050 by buying oats at the former price and selling at the latter, how many bushels did he buy?

76. A speculator in Chicago bought 10,000 bu. of corn in February, 1901, at $38\frac{5}{8}$¢ per bushel, and sold it in December, 1901, at $69\frac{1}{4}$¢ per bushel. Find his profit.

77. The total number of bales of cotton exported from the United States for the season of 1901–02 was 6,715,793, valued at $284,779,190. Find the average price per bale, correct to the cent.

78. The total number of farms in Alabama is 223,220; the total acreage of these is 20,685,427. Find, correct to two decimal places, the average number of acres to a farm.

79. The total sugar production of California was, in 1902, 356,500 T., valued at $15,500,000. Find the average price per 100 lb.

80. According to the census of 1900, the number of persons employed in manufacturing industries in Florida was 1778, and the salaries paid amounted to $1,295,139. Find the average salary received by each person, correct to the cent.

81. The total enrollment in the elementary and secondary schools in the United States in 1901 was 15,603,451, and the total number of teachers was 430,004. Find the average number of pupils to a teacher.

82. The total expenditure for higher education in Canada in a recent year was $1,014,254. This expenditure was 19.5¢ per capita of the total population. Find the population of Canada.

83. The total expenditure for higher education in Germany in a recent year was $7,450,366. The per capita expenditure was 14.3¢. Find the population of Germany.

The cost of higher education in Great Britain and Ireland for a recent year was given as $8,353,655. The per capita expenditure was 21.7¢. Find the population of Great Britain and Ireland.

84. The total amount of money in circulation in the German Empire is, estimated in our currency, $1,080,100,000. The per capita circulation is $19.53. Find the population of Germany.

85. Express in feet .002357 of a mile.

86. Find the value of $\frac{5}{16}$ of a ton + $\frac{7}{8}$ of a hundredweight. Give your answer in pounds.

87. How many times is 12 lb. 8 oz. contained in 2 T.?

88. Light travels at the rate of 185,000 mi. a second. How long does it take a ray of light to pass from the earth to the moon, a distance of 239,000 mi.?

89. How many cubic yards of sand are required to fill a street $1\frac{1}{2}$ mi. long, 40 ft. wide, to the depth of 5 in.?

90. Express $\frac{3}{4}$ of a day as a decimal of a common year.

91. If $\frac{5}{16}$ of an acre of land is worth $23, find the value of 85 A. of land.

92. If .375 of an acre of land is worth $22, find the value of 57 A.

93. Multiply 68.4 by .0027, and divide the product by $\frac{3}{4}$ of .96.

94. Find the value of .1875 of a guinea + $\frac{2}{3}$ of £1 + .25 of 7*s.* 8*d.* Give your answer in pounds, shillings, and pence.

95. Reduce 12*s.* 6*d.* to the decimal of £4 sterling.

96. (*a*) Divide $1293.46 by $.00\frac{7}{8}$. (*b*) Divide $147.32 by $.00\frac{2}{3}$. (*c*) Divide $3473.85 by $.00\frac{3}{4}$. (*d*) Divide $3295 by $.00\frac{2}{5}$.

97. (*a*) Divide $1456.77 by $.00\frac{7}{8}$. (*b*) Divide $3947.85 by $.00\frac{3}{4}$.

98. How many acres in a field 160 ch. long, 40 ch. wide?

99. A wheel is $12\frac{1}{2}$ ft. in circumference. How many revolutions will it make in going 6 mi. 80 rd.?

100. How many bushels will a bin 7 ft. by 5 ft. and 4 ft. deep hold?

101. Reduce $\frac{11}{16}$ to a fraction having 12 for denominator.

102. .08 of a boy's money is $6. How much money has the boy?

103. .875 of a man's property is valued at $21,700. What is the value of the man's property?

104. How many acres are in a square field whose side is 40 rd.?

105. A and B can mow a field in 7 da. A, B, and C can mow the same field in 5 da. for $50. What should C receive?

106. Write decimally three-eighths of one hundredth, and reduce it to a simple decimal.

107. Reduce $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{12}$, and $\frac{1}{16}$ to equivalent fractions having 100 for denominator.

108. Find the difference between $\frac{.16}{16}$ and $\frac{16}{.16}$.

109. Reduce $16\frac{3}{4}$ to an improper fraction having 16 for a denominator.

110. A rectangular field which is 18 rd. wide contains 6 A. How much will it cost to fence the field at 75¢ a rod?

111. What decimal of 4 ft. 2 in. is 9 ft. 6 in.?

112. Find the least fraction which added to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ will make the sum an integer.

113. Divide 27.8 of a yard by .00125 of a foot.

114. 8 cwt. 20 lb. of sugar cost $41.42. What will 1 T. cost at the same rate?

115. Find the least length which is a multiple of 1 ft. 3 in., 1 ft. 8 in., 2 ft. 1 in., and 2 ft. 6 in.

116. Twelve-tenths of a number equals 42. Find the number.

117. Divide 54,218 by 64, using the factors of 64.

118. A train 165 yd. long passes a telegraph pole in 12 sec. Find the rate of the train in miles per hour.

119. A city lot 42 ft. by 120 ft. is sold for $840. At this rate find the value of 1 A. of land in that city.

120. Find the greatest number which, when divided into 1958 and 2741, will give for remainders 8 and 11 respectively.

121. By buying eggs at 25¢ per dozen and selling them at 60¢ a score, a dealer makes a profit of $10.01. How many eggs does he sell?

122. Reduce $\frac{24871}{29887}$ to its lowest terms.

123. If a sum of money which will pay A's wages for $41\frac{3}{4}$ da. will pay B's wages for $55\frac{7}{8}$ da., for how long will it pay both?

124. If gold weighs 19.3 times as much as water, and copper 8.9 times as much as water, how much heavier than water is an alloy consisting of 16 parts of gold and 3 of copper?

125. A rectangular tank is 18 ft. $8\frac{3}{4}$ in. long, 11 ft. $3\frac{5}{6}$ in. wide, and contains 41 cu. yd., 6 cu. ft., and $34\frac{1}{8}$ cu. in. Find its depth. Find the area of each of its faces.

126. A tennis court is 42 yd. long and 20 yd. wide. It has a walk around it 6 ft. wide. Find the cost of paving the walk at $1.25 per square yard.

127. Telegraph poles along a certain railroad are 132 ft. apart. Find the rate of a train, in miles per hour, which passes 18 poles in 24 sec.

LONGITUDE AND TIME

128. A train leaves New York City at 9 A.M., Apr. 1, 1903, and arrives in Carson City, Nev., in 109 hr. 15 min. Find the hour of the day, and day of the month, Standard time, that it reaches its destination.

129. The time of mail transit between Chicago and Santa Fé, N. M., is 60 hr. 55 min. "The California Limited" leaves Chicago at 10 P.M. At what time, by the clocks in Santa Fé, should "The California Limited" pass Santa Fé?

130. The longitude of Cairo, Egypt, is 31° 21′ E., and the longitude of Savannah, Ga., is 81° 5′ 30″ W. Find the difference in time.

131. The longitude of Toulon is 5° 56′ E. The time difference between Toulon and Halifax, N. S., is 4 hr. 38 min. 4 sec. Find the longitude of Halifax.

132. The time difference between Toulon and Point Barrow, Alaska, is 10 hr. 48 min. 44 sec. Find the longitude of Point Barrow.

133. The time difference between Osaka and Point Barrow is 19 hr. 26 min. 48 sec. Find the longitude of Osaka. (See previous problem.)

134. (*a*) The difference between Standard and local time of Portland, Me., is 19 min. Find the longitude of Portland, Me.

(*b*) The difference between Standard and local time of Fort Wayne, Ind., is 20 min. Find the longitude of Fort Wayne.

(*c*) Cleveland, O., uses Central time; the difference between its local and Standard time is 33 min. Find the longitude of Cleveland, O.

PERCENTAGE

135. The total sugar production of the world for the year 1902 was 9,635,000 T. The amount of sugar consumed in the United States the same year was 2,372,000 T. What per cent of the world's production was the amount consumed in the United States?

136. The foreign-born population of New Orleans, according to the census of 1900, was 30,325; of this number, 1262 came from England, 4428 from France, 8733 from Germany, 5398 from Ireland. What per cent of the foreign-born population of New Orleans came from England? from France? from Germany? from Ireland?

137. The number of Canadians in Detroit, according to the census of 1900, was 25,400; this number was 26.3% of the foreign-born population. Find, correct to 100, the number of foreign-born population of Detroit.

138. A horse is sold, at a loss of 15%, for $127.50. Find the cost of the horse.

139. By selling silk at $1.60 per yard, a dealer makes a profit of 25%. What would the selling price be if he made a profit of 12½%?

140. When cloth is sold for $1.04 per yard, a clothier makes a profit of 30%. What would his profit be if he sold the cloth at 96¢ per yard?

141. A wholesale dealer makes a profit of 10% on canned goods. The retail dealer makes a profit of 25%. Find the original cost of canned goods which cost the consumer $11.

142. A coal merchant buys coal by the long ton at $4.50 a ton, and sells it at the rate of $5 a short ton. Find his gain per cent.

143. How much water must be added to a 25% wine mixture to make it a 20% mixture?

144. A sells goods to B at a profit of 20%; B sells them to C at a profit of 20% on his outlay; C sells them to D for $180, thereby losing 16⅔%. How much did the goods cost A?

145. A merchant buys goods at 20%, and 10% off list price, and sells them at the list price. Find his per cent of gain.

146. When 20 lb. of tea are sold for what 22½ lb. cost, what is the gain per cent?

147. (*a*) A vessel contains 31 gal. of wine and 17 gal. of water. What per cent of the mixture is wine and what per cent is water? (*b*) How many gallons of water must be added to this mixture to make a mixture containing 60% wine?

148. The following table gives the distances from Atlantic to Pacific ports by the present routes:

New York to San Francisco	13,244 mi., nautical
New York to Sydney	14,560 mi., nautical
Charleston to San Francisco	13,180 mi., nautical
Charleston to Valparaiso	8,296 mi., nautical
New Orleans to San Francisco	13,644 mi., nautical
New Orleans to Melbourne	15,535 mi., nautical
Galveston to San Francisco	13,826 mi., nautical
Galveston to Wellington	14,182 mi., nautical
Liverpool to San Francisco	13,844 mi., nautical
Hamburg to Callao	10,702 mi., nautical
Bordeaux to San Francisco	13,691 mi., nautical

The following table gives the distances from Atlantic to Pacific ports via the Panama Canal route:

New York to San Francisco	5299 mi., nautical
New York to Sydney	9852 mi., nautical
Charleston to San Francisco	4898 mi., nautical

Charleston to Valparaiso	4229 mi., nautical
New Orleans to San Francisco	4698 mi., nautical
New Orleans to Melbourne	9826 mi., nautical
Galveston to San Francisco	4800 mi., nautical
Galveston to Wellington	8392 mi., nautical
Liverpool to San Francisco	8038 mi., nautical
Hamburg to Callao	6527 mi., nautical
Bordeaux to San Francisco	7938 mi., nautical

What per cents of the distances by the old routes are saved by the Panama Canal route?

INTEREST

149. Find the simple interest on $78 for 93 da. at 8%.

150. Find the simple interest on $98 for 63 da. at 7%.

151. Find the amount of $179 for 123 da. at 6%.

152. Find the simple interest on £324 7*s.* 9*d.* from June 12 to Dec. 7 following at 5%.

153. Find the simple interest on £1169 6*s.* 8*d.* from Jan. 25 to June 18 following at 9%.

154. What principal will produce $19.50 in 1 yr. at 6½%?

155. What principal will produce $180 interest in 3 mo. at 5%?

156. What principal will amount to $412.50 in 7½ mo. at 5%?

157. What principal will amount to $1219 in 3 mo. 5 da. at 6%?

158. What principal will produce $29.17 in 5 mo. at 7%?

159. At what rate will $1000 produce $23.33 interest in 4 mo.?

160. What principal will produce 75¢ interest in 9 da. at 6%?

161. Find the exact interest on $73.15 from June 18 to Aug. 1 at 7%.

162. A note for $3500 bearing interest at 8% and dated Jan. 2, 1900, was indorsed as follows: June 7, 1900, $450;

Aug. 2, 1900, $208; Jan. 2, 1901, $500; July 7, 1901, $800; Oct. 4, 1901, $500; Jan. 11, 1902, $300; Aug. 4, 1902, $700. Calculate, by the United States Rule for partial payments, the amount due on this note on Jan. 1, 1903.

163. A demand note dated Jan. 5, 1902, and drawn for $575 was paid 6 mo. 18 da. later. Find the date of payment and the amount of the note, the rate of interest being 7%.

BANK DISCOUNT

164. A note for 60 da. is drawn on Jan. 10, 1903. Find the proceeds of this note, if its face is $150, the date of discount Feb. 5, and the rate 6%. (Neglect days of grace.)

165. A note for $900, dated Mobile, Ala., Jan. 8, 1903, and drawn for 90 da., is discounted March 1. Find the proceeds.

166. A 60-day note bearing interest at 8%, drawn Feb. 1, 1903, for $1000, is discounted Feb. 27 at 9%. Find the proceeds of this note.

167. A demand note was drawn Oct. 1, 1902, for $800, and paid 5 mo. 10 da. later. Find the date of payment and the amount of the note; rate of interest, 7%.

168. The proceeds of a note is $450 when the term of discount is 93 da., and the rate of interest is 8%. What is the maturity value of the note?

MENSURATION

169. Find the area of a parallelogram if its base is 100 yd. and its altitude is 75 yd.

170. Find the area of a trapezoid if its parallel sides are 60 yd. and 80 yd., and its altitude is 50 yd.

171. A tract of land is sold for $3943.84; the land cost as many dollars per acre as there were acres in the tract. Find the cost per acre.

172. Find the number of square yards in the walls and ceiling of a room 36 by 23, and 16 ft. high.

173. Find the perimeter of a square which contains 40 A.

174. A tract of land in the shape of a rectangle contains 320 A.; its length is twice its width. Find its dimensions.

175. Find the area of an equilateral triangle one side of which is 100 ft.

176. Find the area of a regular hexagon each side of which is 50 ft.

177. Find the circumference of a circle whose radius is 56.5 in.

178. Find the area of a circle if its diameter is 20 in.

179. Find the surface of a sphere whose diameter is 20 in.

180. Find the volume of a cube one of whose dimensions is 1 ft. 3 in.

181. The surface of a cube is 221.0694 sq. in. Find the length of one edge of this cube.

MISCELLANEOUS EXAMPLES (B)

(TAKEN FROM VARIOUS EXAMINATION PAPERS)

1. What fractional part of $\frac{5}{7}$ of a gallon is $\frac{3}{28}$ of a pint?

2. The difference in time between two places is 2 hr. 33 min. Find the difference in longitude.

3. A bicycle wheel measuring 88 in. in circumference must make how many revolutions a minute to run eighteen miles an hour?

4. A coal bin $16\frac{1}{2}$ ft. long and 8 ft. 9 in. wide must be how deep to contain 10 T. of coal, if one ton of coal occupy 40 cu. ft. of space?

5. Reduce 2 yr. 21 da. to years and decimals of a year.

6. Reduce .09625 bbl. to integers of lower denominations.

7. Find the value of a piece of land 64 ch. by $13\frac{1}{2}$ ch. at \$ $48\frac{1}{3}$ an acre.

8. Required the cost of 18 $2\frac{1}{2}$ in. plank 16 ft. long and 10 in. wide, and 33 pieces of scantling 2 in. by 4 in. 16 ft. long, at \$22 per M, board measure.

9. The average yield per bushel of wheat is 14 bu. 1 pk. What will 7 bu. 3 pk. 2 qt. yield?

10. What is the difference in weight, expressed in Avoirdupois pounds, between 100 lbs. Troy and 100 lbs. Avoirdupois?

11. Reduce to simplest form $\dfrac{\frac{2}{3}+\frac{3}{7}}{7-\frac{4}{5}} \div \dfrac{\frac{2}{9}}{2+\frac{1}{2}}$.

12. If it cost \$510 to fence a rectangular field 98 rd. by 72 rd., what will it cost to fence a square field of the same area?

13. Express $\frac{9}{7}$ as a decimal fraction.

14. What is the ratio of 32 ft. to 6 yd.? Express the result decimally.

15. What is the length of a plank $1\frac{1}{2}$ in. thick, 1 ft. 6 in. wide, containing 36 board feet?

16. When it is 12 M. in New York City (74° W.), what is the time in Manila (120° E.)?

17. If the value of $\frac{1}{7}$ of $\frac{3}{8}$ of an estate is $4500, what is the value of $\frac{1}{14}$ of $\frac{1}{12}$ of it?

18. At $16 per M, board measure, find the cost of 20 plank 2 in. by 8 in. 18 ft. long, and 30 plank $1\frac{1}{2}$ in. by 6 in. 10 ft. long.

19. A can do a piece of work in 6 da. and B can do the same work in 8 da. How long will it take B to finish after they have worked together two days?

20. $20\frac{3}{4}$ is the product of three factors. Two of these factors are $1\frac{3}{5}$ and $4\frac{5}{8}$. Find the other factor.

21. How many bushels of wheat will a box 6 ft. by $3\frac{1}{2}$ ft. by 2 ft. 8 in. hold?

22. How many yards are in .04675 mi.?

23. If the dividend is 807 and the quotient $34\frac{1}{3}$, what is the divisor?

24. How many rods of fence will inclose a square field whose area is 20 acres?

25. Coal sells at $5.75 per ton. What will be the cost of 2315 lb. at this rate?

26. How many gallons of water will a tank 5 ft. by 2 ft. by 2 ft. hold?

27. What is the length of one side of a square piece of land whose area is 538,756 sq. rd.?

28. A room is 27 ft. by 22 ft. 6 in. How many yards of carpet 27 in. wide will be required to carpet this room?

29. A man is hired to dig a cellar 20 ft. by 15 ft. by 5 ft. How much money will he receive at 30¢ per cu. yd.?

30. How many days are there between Aug. 14 and Dec. 29?

31. Find the value of a car load of wheat, estimated at 21,643 lb., at 92¢ per bushel.

32. Two persons travel in opposite directions from the same point at the rate of $4\frac{1}{3}$ and $7\frac{3}{4}$ mi. per hour, respectively. How far apart are they after traveling $37\frac{1}{2}$ hrs.?

33. A man was born Nov. 22, 1861. What is his age to-day?

34. Factor the following numbers and from these factors determine the G. C. M.: 42, 112, 140, 308.

35. What will 75 boards 2 in. by 4 in. by 16 ft. long cost at $12 per M board measure?

36. 160 rd. of fence will inclose how many acres in the form of a square?

37. The difference in longitude between two places is 7° 42′ 30″. Find the difference in time.

38. How wide is a rectangular field containing 5 A., the length of the field being 7 ch. 25 l.?

39. A pavement is $5\frac{1}{8}$ rd. long and 8 ft. 6 in. wide. What did it cost at $1.40 per sq. yd.?

40. Three men, A, B, and C do a piece of work; A works 3 da. of 5 hr. each, B, 2 da. of 6 hr. each, and C, 7 da. of 3 hr. each. At the same rate of wages, how should they divide $120, the total amount received for doing the work?

41. A miller charges $\frac{1}{16}$ for toll. How many bushels of wheat must one take to mill to get 12 bbl. of flour, each containing 196 lb., if a bushel of wheat makes 40 lb. of flour?

42. The annual rainfall in a certain locality is 30 in. How many tons of water fall on an acre of land in this locality, if a cubic foot of water weighs 1000 oz.?

43. How much does a man gain or lose on the sale of two houses at $1200 each, if he gains $\frac{1}{3}$ of the cost price on one, and loses $\frac{1}{5}$ of the cost price on the other?

44. What is the ratio of 7 lb. Troy weight to 10 oz. Avoirdupois?

45. The divisor is 357, the quotient is $6\frac{3}{7}$; what is the dividend?

46. A farmer had 28 A. of land left after selling $\frac{1}{4}$ of his farm to one neighbor, $\frac{2}{5}$ of it to another, and $\frac{3}{7}$ of the remainder to another. How large was his farm?

47. Multiply 8.035 by .0035, add 3, and divide the sum by .000625.

48. Divide $459.25 into three parts that shall be to one another as $\frac{3}{4}$, $\frac{5}{8}$, and 3 respectively.

49. When it is two o'clock P.M. in Jerusalem, what is the time in Cincinnati? The longitude of Jerusalem is 35° 12′ E., and of Cincinnati, 84° 26′ W.

50. Find the exact number of days between Dec. 23, 1902, and to-day.

51. A man's farm is mortgaged for $\frac{2}{3}$ of its cost; he sells it for $6000 which is 25% above its cost. How much money will he have after paying the mortgage?

52. A note for $600, dated Oct. 24, 1902, and due in 8 mo., with interest at 6% per annum, is discounted at bank Dec. 20, 1902. Find the proceeds.

53. A man sold two lots each for $600, gaining 20% on one, and losing 20% on the other. What was his gain or loss?

54. A man buys a book the list price of which is $7.20, at a discount of $16\frac{2}{3}$%, and sells it for $7.50. What is his gain per cent?

55. What principal at interest for 1 yr. 3 mo. will amount to $506, the rate of interest being 8% per annum?

56. Twelve per cent of 90 is what per cent of 100?

57. At the following rates per annum of simple interest, what time is required for the accruing interest to equal the principal: 6%, 8%, $9\frac{1}{11}$%?

58. What is the exact interest on $10,000 from Jan. 18, 1903, to May 6, 1904, at 3½% ?

59. A 30-day note, without interest, is discounted at a bank at 8% for $350. What is the face of the note?

60. Bonds bearing 5% interest are bought at 120. What is the rate of income on these bonds?

61. An agent buys sugar at 4¾¢ per pound; his commission at ¼% is $25. How many pounds of sugar does he buy ?

62. The discount of a note, discounted at bank, for 3 mo. 18 da. at 5% is $4.20. Find the proceeds.

63. What single discount is equivalent to trade discounts of 25%, 10%, and 5% on the list price of an article?

64. The property in a school district is assessed at $196,000. What rate of taxation would be required to provide about $800 annually for the improved maintenance of the schools? What annual tax would a man pay on this account whose property is assessed at $1200?

65. A man sold two horses at $80 each. On one he gained 20%, on the other he lost 20%. Find the gain or loss.

66. What must I ask for an article worth $36 that, after falling 20%, I may gain 25% on the value?

67. A school district advertised for bids to build a schoolhouse, the lowest bid being $21,049. If it costs 3% to collect the money, how large a levy should be made, supposing 29% of it to be non-collectible?

68. What must a man pay for 4% stock to get 5% on his investment?

69. If you buy United States 3's at 110, what per cent per annum would your investment pay ?

70. A merchant's expenses average 10% of his sales. At what per cent advance on cost must he sell his goods to clear 20% profit?

71. A merchant sold goods to the amount of $760.95, thereby losing 11%. What did he pay for the goods?

72. A ship is insured for half its value for $374. If the rate is 2¾%, what is the value of the ship?

73. A carriage dealer sold 16 buggies at $200 each; on one half he gained 10%, and on the other half he lost 10%. Find his net gain or loss.

74. What principal will amount to $1253.86 in 2 yr. 11 mo. 13 da., interest at 5%?

75. How do you find the rate per cent per annum when the principal, interest, and time are given?

76. How do you find the principal when the rate per cent per annum, time, and interest are given?

77. How do you find the time when the principal, rate per cent per annum, and interest are given?

78. The list price of office desks is $15, but 12 desks are sold for $126. What rate of discount is allowed?

79. In a certain time $650 will amount to $713.05 at 6% simple interest. Find the time.

80. A note for $500, due in 3 months, is discounted at bank at 6%. Find the proceeds.

81. 236½ is what per cent of 78⅚?

82. A man insures his life, paying a premium of $28, which is at the rate of ⅞% on the amount of his insurance. Find the face of the policy.

83. If 25% of the selling price of an article is profit, what is the per cent of gain on its cost?

84. A man fails in business; his assets amount to $2100, his liabilities to $6000. What per cent will his creditors receive?

85. What is the interest on $475 for 1 yr. 3 mo. 24 da. at 6%?

86. A man bought four loads of hay, each weighing 2750 lb., at $20 per ton; he gave in payment his note, without interest, at 60 da. What are the proceeds of this note, discounted at a bank at 6%?

87. What per cent of ½ is ⅛?

88. An agent's commissions at 5% amount to $37.65. Find the amount of his sales.

89. The tax on property assessed at $8500 is $48.37. What is the rate on $1000?

90. Find the date of maturity of a note made and dated Sept. 11, 1902, and payable 90 da. after date.

91. Find the cost of 87 shares of stock at 76¼, brokerage ⅛ per cent.

92. A New York sight draft was sold in Atlanta, Ga., for $3542, exchange being at ¾% premium. What was the face of the draft?

93. What per cent of 5 lb. is 3 oz. Avoirdupois?

94. An agent's commission for renting a house is $13.25; his rate of commission is 2½%. What is the yearly rent of the house?

95. A man pays a premium of $150 for insuring his house for ⅔ of its value; the rate of premium is 1½ per cent per annum. What is the value of the house?

96. A building worth $6000 is insured for ¾ of its value at 75¢ on the $100. In case of the destruction of the building by fire, what will be the owner's loss, including premium?

97. What per cent of 1 bu. is 3 qt.?

98. A merchant can buy flour on six months' credit at $8 per barrel, or for cash at $7.50 per barrel. He buys 100 bbl., paying cash, but borrows the money at 8% to pay for it. Is this better than to buy on credit, and how much better?

99. A man sells 16 shares of bank stock at 127¾, brokerage ⅛%. How much does he receive for his stock?

100. A speculator buys stocks at 20% premium and sells the same stocks at 10% discount. What per cent of his investment does he lose?

101. A hardware merchant's books show sales during the month of March amounting to $1000. One half of his sales are at a profit of 25% on the cost, and the other half, a loss of 16⅔% on the cost. Find the cost of the goods sold during the month.

102. A merchant failing in business paid his creditors $3874.75, which was at the rate of 55¢ on every dollar of his indebtedness. Find his indebtedness.

103. The list price of a mower is $38; the retail dealer is allowed discounts of 20%, 5%, and 3%. What does he pay for mowers? If the retailer sells these mowers at a profit of 50%, what does the farmer pay for these mowers?

104. A certain stock, selling at 121¾, pays a semiannual dividend of 4%. What is the rate per cent per annum on an investment in this stock?

APPENDIX

THE succeeding portion of the book is intended primarily for teachers. The topics are treated in a very elementary manner, but, doubtless, this very elementary treatment is too difficult for the average grammar-grade student.

Special attention is directed to treatment of Compound Interest, Annuities, and Present Worth. So far as the authors know, the methods of attack in the solutions of the problems in Annuities and Partial Payments are new.

SERIES

387. A **series** is a succession of numbers, each one after the first being formed according to some law, *e.g.*:

I. $2+4+6+8+10+12$, etc.

II. $2+4+8+16+32+64$, etc.

III. $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$, etc.

In (I) each of the numbers after the first is formed from the preceding by the addition of 2 to the preceding. In (II) the numbers increase twofold as we go to the right. In (III) the numbers of the series are the reciprocals of the natural number series.

The numbers of a series are called its **terms**.

A series in which the difference between each two successive terms is the same is called an **arithmetic series**, or **arithmetic progression**.

We shall use the letters A. P. to denote an Arithmetic Progression.

Examples of Arithmetic Progression are

$3+5+7+9+11$, etc.

19, 16, 13, 10, 7, 4, etc.

NOTATION

388. We shall use A_1 to denote the 1st term of an A. P.
A_2 to denote the 2d term of an A. P.
A_3 to denote the 3d term of an A. P.
A_4 to denote the 4th term of an A. P.
A_n to denote the nth term of an A. P., *i.e.* any term of an A. P.

Then $A_1+A_2+A_3+A_4+\cdots A_n$ may denote any A. P. whatever.

We shall further use,

S_2 to denote the sum of 2 terms of an A. P.
S_3 to denote the sum of 3 terms of an A. P.
S_4 to denote the sum of 4 terms of an A. P.
S_5 to denote the sum of 5 terms of an A. P.
S_n to denote the sum of n terms of an A. P.
d to denote the common difference between two terms.

389. Given the 1st term and the common difference to find any other term.

$$A_2=A_1+d.$$
$$A_3=A_2+d=(A_1+d)\quad+d=A_1+2\,d.$$
$$A_4=A_3+d=(A_1+2\,d)+d=A_1+3\,d.$$
$$A_5=A_4+d=(A_1+3\,d)+d=A_1+4\,d.$$
$$A_6=A_5+d=(A_1+4\,d)+d=A_1+5\,d.$$
$$A_7=A_6+d=(A_1+5\,d)+d=A_1+6\,d.$$
$$A_n=A_1+(n-1)d\cdots(\text{I}).$$

The nth, or last term equals the first term plus one less than the number of terms multiplied by the common difference. In case each term gets smaller as we go to the right

$$A_n=A_1-(n-1)d.$$

Example 1. Find the 12th term of the A. P. $3+5+7+9+\cdots$.

SOLUTION. $$A_n=A_1+(n-1)d.$$

$$A_{12}=3+(12-1)2=3+11\times 2=25.$$

Example 2. Find the 17th term of the A. P. $97+94+91+\cdots$.

SOLUTION. $A_n = A_1 - (n-1)d.$

$$A_{17} = 97 - (17-1)3 = 97 - 16 \times 3 = 97 - 48 = 49.$$

EXERCISE 229

1. Find the 10th term of the A. P. $1+5+9+13+\cdots$.
2. Find the 12th term of the A. P. $3+8+13+18+\cdots$.
3. Find the 20th term of the A. P. $5+11+17+23+\cdots$.
4. Find the 18th term of the A. P. $100+98+96+94+\cdots$.
5. Find the 30th term of the A. P. $150+147+144+\cdots$.

SUM OF AN A.P.

390. Since

$$\begin{aligned} A_1 &= A_1, \\ A_2 &= A_1 + d, \\ A_3 &= A_1 + 2d, \\ A_4 &= A_1 + 3d, \\ A_5 &= A_1 + 4d, \\ A_n &= A_1 + (n-1)d. \end{aligned}$$

I. $S_n = A_1 + (A_1 + d) + (A_1 + 2d) + (A_1 + 3d) + (A_1 + 4d) + \cdots + A_1 + (n-1)d.$

Writing the sums beginning with the last term, we have,

II. $S_n = A_n + (A_n - d) + (A_n - 2d) + (A_n - 3d) + (A_n - 4d) + \cdots A_n - (n-1)d.$

Adding I and II, member by member, we have

$2S_n = (A_1 + A_n) + (A_1 + A_n) + (A_1 + A_n) + (A_1 + A_n) + \cdots$ to n terms.

$$\therefore \quad 2S_n = n(A_1 + A_n).$$

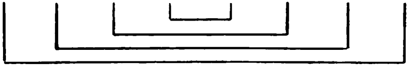
$$\therefore S_n = \frac{n(A_1 + A_n)}{2} = \frac{n}{2}(A_1 + A_n) \cdots (2).$$

The sum of an A. P. equals one-half the number of terms multiplied by the sum of the first and last terms.

This rule states in words what (2) states in the language of mathematics.

We may derive this rule from concrete instances as follows:

$$7 + 10 + 13 + 16 + 19 + 22 + 25 + 28.$$



The sum of the first and last terms is 35.
The sum of the next pair is also 35.
The sum of the next pair is also 35.
The sum of the next pair is also 35.

The sum of all the terms equals 35 × the number of pairs, *i.e.* equals the sum of the first and last terms multiplied by half the number of terms.

To find the sum of an A. P., **find first the last term, or A_n, then multiply the sum of the first and last terms by half the number of terms.**

EXERCISE 230

1. Find the sum of 10 terms of the A. P. $4 + 6 + 8 + \cdots$.
2. Find the sum of 12 terms of the A. P. $6 + 10 + 14 + \cdots$.
3. Find the sum of 15 terms of the A. P. $7 + 12 + 17 + \cdots$.
4. Find the sum of 18 terms of the A. P. $90 + 86 + 82 + \cdots$.
5. Find the sum of all the odd numbers less than 50.
6. Find the sum of all the even numbers less than 51.
7. Find the sum of the numbers exactly divisible by 3 which are each less than 60.

GEOMETRIC SERIES

391. A series in which each term after the first is obtained from the preceding term by multiplying it by a constant factor called the **common ratio** is called a **geometric series**, or **geometric progression** (G. P.) *e.g.:*

$$3 + 6 + 12 + 24 + 48 + 96.$$
$$32 + 16 + 8 + 4 + 2 + 1 + \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \cdots.$$

392. We shall designate the 1st term by A_1; the 2d term by A_2; the 3d term by A_3; the 4th term by A_4; the nth term by A_n.

We shall designate the common ratio by r, and the sum of any number of terms by S_n.

$$A_2 = A_1 r.$$
$$A_3 = A_1 r^2.$$
$$A_4 = A_1 r^3.$$
$$A_5 = A_1 r^4.$$
$$A_6 = A_1 r^5.$$
$$\therefore\ A_n = A_1 r^{n-1} \cdots. \tag{1}$$

(1) states in mathematical language the fact that the last term, or the nth term, in a G. P. equals the first term multiplied by the common ratio raised to the power indicated by one less than the number of terms.

393. Since

$$S_n = A_1 + A_2 + A_3 + A_4 + A_5 + \cdots A_n,$$
$$S_n = A_1 + A_1 r + A_1 r^2 + A_1 r^3 + A_1 r^4 + \cdots A_1 r^{n-1}. \tag{a}$$

Multiply each side of our equation by r. We have

$$rS_n = A_1 r + A_1 r^2 + A_1 r^3 + A_1 r^4 + \cdots A_1 r^n. \tag{b}$$

But $S_n = A_1 + A_1 r + A_1 r^2 + A_1 r^3 + A_1 r^4 + \cdots A_1 r^{n-1}$. (a)

Subtracting (a) from (b), we have

$$rS_n - S_n = A_1 r^n - A_1, \text{ or}$$
$$\therefore\ (r-1)\, S_n = A_1\,(r^n - 1).$$
$$\therefore\ S_n = A_1 \times \frac{r^n - 1}{r - 1} \cdots. \tag{2}$$

To find the sum of a number of terms of a G. P.:

Raise the ratio to the power indicated by the number of terms, from this subtract one, divide by the ratio minus one, and multiply the quotient by the first term.

Example. Find the sum of 10 terms of the G. P. $3 + 6 + 12 + \cdots$.

SOLUTION. $S_n = A_1 \times \left(\frac{r^n - 1}{r - 1}\right)$.

$$S_{10} = 3 \times \frac{2^{10} - 1}{2 - 1} = 3 \times \frac{1024 - 1}{2 - 1} = 3 \times \frac{1023}{1} = 3069.$$

EXERCISE 231

1. Find the sum of the G. P. of 6 terms, $2 + 4 + 8 + \cdots$.
2. Find the sum of the G. P. of 8 terms, $4, 12, 36 + \cdots$.
3. Find the sum of the G. P. of 5 terms, $8, 12, 18 + \cdots$.
4. Find the sum of the G. P. of 7 terms, $27, 18, 12 + \cdots$.
5. Find the sum of the G. P. of 6 terms, $1 + 2\frac{1}{2} + 6\frac{1}{4} + \cdots$.

394. Consider the G. P. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \cdots$ without end.

Here each succeeding term becomes smaller and smaller. If we take a very large number of terms, the last term becomes nearly equal to zero, and the sum of the series approaches in value some fixed number which is called the limiting value of the sum of the series.

When the common ratio is less than one,

$$S_n = A_1 \times \frac{1 - r^n}{1 - r}.$$

This we get by subtracting (*b*) from (*a*) (§ 393).

Any proper fraction raised to a very large power becomes nearly equal to zero. For instance $(\frac{2}{3})^{10000}$ is so small that if we were to take that part of the entire earth the result would be less in volume than the millionth part of a single grain of sand. Therefore, if n is very large and r less than 1 we may neglect r^n.

$$\therefore\ S_n = A_1 \times \frac{1}{1 - r} = \frac{A_1}{1 - r}.$$

Example. Find the limiting value of $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$.

SOLUTION. $S_n = \dfrac{A_1}{1 - r} = \dfrac{1}{(1 - \frac{1}{2})} = \dfrac{1}{(\frac{1}{2})} = 2.$

EXERCISE 232

Find the limiting value of each of the following series as the number of terms becomes great:

1. $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \cdots$.
2. $9 + 6 + 4 + 2\frac{2}{3} + \cdots$.

3. $\frac{6}{10}+\frac{6}{100}+\frac{6}{1000}+\frac{6}{10000}+\cdots.$

4. $\frac{36}{100}+\frac{36}{10000}+\frac{36}{1000000}+\cdots.$

5. $\frac{63}{100}+\frac{63}{10000}+\frac{63}{1000000}+\cdots.$

6. $\frac{9}{10}+\frac{9}{1000}+\frac{9}{100000}+\cdots.$

7. $\frac{4}{5}+\frac{1}{125}+\frac{1}{12500}+\cdots.$

8. $\frac{2}{5}+\frac{1}{250}+\frac{1}{25000}+\cdots.$

9. $\frac{9}{10}+\frac{9}{100}+\frac{9}{1000}+\cdots.$

CIRCULATING DECIMALS

395. If we reduce $\frac{2}{9}$ to a common fraction, we get

$$\tfrac{2}{9} = .222222 + \cdots. \qquad (m)$$

If we reduce $\frac{15}{88}$ to a common fraction, we get

$$\tfrac{15}{88} = .17045454545 + \cdots. \qquad (n)$$

In (*m*) the figure 2 is repeated without end. In (*n*) the figures 4, 5, are repeated without end.

Decimals of this character, *i.e.* decimals containing a boundless number of figures, some or all of which are repeated, are called **circulating**, or **recurring decimals**. The figure or figures repeated are called the **circulating period**. A decimal in which the circulating period begins at the decimal point is called a pure circulating decimal. A decimal containing figures between the decimal point and the circulating period is called a mixed circulating decimal. Thus, $\frac{15}{88}$ gives rise to a mixed circulating decimal.

A circulating period is indicated by writing a dot over the first and last figures of the period. Thus, $.\dot{2}$ means .22222222222, etc. $.\dot{4}\dot{3}$ means .434343434343 ⋯, etc. $.01\dot{2}3\dot{4}$ means .01234234234, etc.

396. *Example* 1. Express as a decimal fraction $.\dot{2}$.

Solution. Denote the fraction by f.

$$\text{Then} \qquad f = .2222222222222222\cdots, \text{ etc.} \qquad (c)$$

$$10f = 2.2222222222222222\cdots, \text{ etc.} \qquad (d)$$

Subtracting (*c*) from (*d*) member by member,

$$9f = 2. \qquad \therefore f = \tfrac{2}{9}.$$

Example 2. Express as a common fraction $.170\dot{4}\dot{5}$.

Denoting the fraction by f, we have,

$$f = .17045454545 + \cdots.$$
$$100000 f = 17045.454545 + \cdots.$$
$$1000 f = 170.454545.$$
$$\therefore\ 99000 f = 16875.$$
$$f = \tfrac{16875}{99000} = \tfrac{135000}{792000} = \tfrac{135}{792} = \tfrac{15}{88}.$$

Example 3. Reduce to a common fraction $.\dot{0}3\dot{6}$.

$$f = .036036036 + \cdots.$$
$$1000 f = 36.036036036 + \cdots.$$
$$\therefore\ 999 f = 36.$$
$$f = \tfrac{36}{999} = \tfrac{4}{111}.$$

By considering examples such as Example 2 and Example 3, we are led to the following conclusion:

To reduce a pure circulating decimal to a common fraction:

Write as an integer the figure or figures of the circulating period for the numerator of the fraction, and for the denominator write as many nines as there are figures in the circulating period.

To reduce a mixed circulating decimal to a common fraction:

I. **Write the integer represented by the figures to the end of the first circulating period.**

II. **Write the integer represented by the figures to the beginning of the first circulating period.**

III. **Subtract II from I, and write the remainder as the numerator of the fraction, and for its denominator write as many nines as there are figures in the circulating period, and as many ciphers as there are figures in the non-circulating part of the decimal.**

EXERCISE 233

1. Reduce to circulating decimals:

(*a*) $\frac{2}{3}$, $\frac{5}{9}$, $\frac{7}{11}$, $\frac{19}{111}$, $\frac{17}{72}$, $\frac{15}{44}$, $\frac{102}{666}$, $\frac{1}{7}$, $\frac{9}{13}$, $\frac{76}{225}$.

(*b*) Prove your answer in each case by finding the fraction which gave rise to the circulating decimal.

2. Find the limiting value of:

$$.\dot{3}\dot{6}, \qquad .\dot{6}\dot{3}, \qquad .\dot{9}\dot{0}, \qquad .\dot{8}\dot{0}.$$

(These examples are given under G. P., Exercise 232.)

3. What fractions give rise to the following circulating decimals:

$$.0784\dot{6}, \qquad .\dot{4}2857\dot{1}, \qquad .00\dot{6}, \qquad .001\dot{3}, \qquad .02\dot{1}\dot{8}.$$

4. Add $.\dot{8}\dot{1}$, $.25$, $.11\dot{3}\dot{7}$, and give the result as both a common fraction and a circulating decimal.

5. Find, correct to six decimal places, the value of the series:

$$1-\frac{1}{1\cdot2\cdot3}+\frac{1}{1\cdot2\cdot3\cdot4\cdot5}-\frac{1}{1\cdot2\cdot3\cdot4\cdot5\cdot6\cdot7}+\cdots.$$

(Dots between numbers indicate multiplication.)

Find, correct to six figures, the value of the series:

6. $1-\frac{1}{1\cdot2}+\frac{1}{1\cdot2\cdot3\cdot4}-\frac{1}{1.2\cdot3\cdot4\cdot5\cdot6}+\cdots$, etc.

7. $1+\frac{1}{1}+\frac{1}{1\cdot2}+\frac{1}{1\cdot2\cdot3}+\frac{1}{1\cdot2\cdot3\cdot4}+\frac{1}{1\cdot2\cdot3\cdot4\cdot5}+\cdots$, etc.

8. $1+\frac{1}{2}-\frac{1}{2^2\cdot2}+\frac{1\cdot3}{2^3\cdot2\cdot3}+\frac{1\cdot3\cdot5}{2^4\cdot2\cdot3\cdot4}-\frac{1\cdot3\cdot5\cdot7}{2^5\cdot2\cdot3\cdot4\cdot5}+\cdots.$

9. $$\frac{1}{11}-\frac{1}{3}\left(\frac{1}{11}\right)^3+\frac{1}{5}\left(\frac{1}{11}\right)^5-\cdots-\frac{1}{7(\frac{1}{11})^7}.$$

10. $$\frac{1}{20}-\frac{1}{3}\left(\frac{1}{20}\right)^3+\frac{1}{5}\left(\frac{1}{20}\right)^5-\frac{1}{7}\left(\frac{1}{20}\right)^7-\frac{1}{7(\frac{1}{20})^7}.$$

COMPOUND INTEREST

397. Savings banks and other banks which give interest on deposits add the interest semiannually or annually to the amount deposited. This interest added bears interest until the next date of balancing the depositor's account book. The next interest is added in the same way.

Suppose a person deposited in a savings bank $250 and allowed it to remain there for 10 yr. The question might be asked, how much will principal and interest amount to at the end of that time? The accruing interest in this case is called **compound interest.**

398. We shall designate principal by p, rate of interest by r, time in years by n, and amount by a.

When the rate of interest on \$1 is \$$r$ per year, the amount of \$1 at the end of 1 yr. is \$$(1+r)$.

We have then,

$$\text{The amount of } \$1 \text{ in 1 yr.} = \$(1+r),$$
$$\text{The amount of } \$p \text{ in 1 yr.} = \$p(1+r).$$

To get the amount of any principal for 1 yr. or for 6 mo., or for any other period at simple interest, we multiply the principal by the amount of \$1 for that time.

Take now \$$p(1+r)$ as our principal, then the amount of \$$p(1+r)$ in 1 yr. $= \$p(1+r)(1+r) = p(1+r)^2$.

The amount of \$$p(1+r)^2$ in 1 yr. $= \$p(1+r)^2(1+r) = \$p(1+r)^3$.

Similarly, the amount of \$$p(1+r)^3$ in 1 yr. $= \$p(1+r)^4$.

At compound interest,

\$$p$ amounts in 1 yr. to \$$p(1+r)$.
\$$p$ amounts in 2 yr. to \$$p(1+r)^2$.
\$$p$ amounts in 3 yr. to \$$p(1+r)^3$.
\$$p$ amounts in 4 yr. to \$$p(1+r)^4$.
\$$p$ amounts in 5 yr. to \$$p(1+r)^5$.
\$$p$ amounts in 20 yr. to \$$p(1+r)^{20}$.
Generally, \$$p$ amounts in n yr. to \$$p(1+r)^n$.

At compound interest,

The amount of \$1 at 4% for 20 yr. is \$$(1.04)^{20}$.
The amount of \$1 at 5% for 20 yr. is \$$(1.05)^{20}$.
The amount of \$1 at 6% for 20 yr. is \$$(1.06)^{20}$.

The labor of raising 1.06 to the 20th power is quite considerable.

A student who has a working knowledge of logarithms can do this by the aid of a table of logarithms in less than a minute.

The following table gives the amount of $1 at compound interest:

YRS.	2%	2½%	3%	4%	5%	6%
1	1.020000	1.025000	1.030000	1.040000	1.050000	1.060000
2	1.040400	1.050625	1.060900	1.081600	1.102500	1.123600
3	1.061208	1.076891	1.092727	1.124864	1.157625	1.191016
4	1.082432	1.103813	1.125509	1.169859	1.215506	1.262477
5	1.104081	1.131408	1.159274	1.216653	1.276282	1.338226
6	1.126162	1.159693	1.194052	1.265319	1.340096	1.418519
7	1.148686	1.188686	1.229874	1.315932	1.407100	1.503630
8	1.171659	1.218403	1.266770	1.368569	1.477455	1.593848
9	1.195093	1.248863	1.304773	1.423312	1.551328	1.689479
10	1.218994	1.280085	1.343916	1.480244	1.628895	1.790848
11	1.243374	1.312087	1.384234	1.539454	1.710339	1.898299
12	1.268242	1.344889	1.425761	1.601032	1.795856	2.012196
13	1.293607	1.378511	1.468534	1.665073	1.885649	2.132928
14	1.319479	1.412974	1.512590	1.731676	1.979932	2.260904
15	1.345868	1.448298	1.557967	1.800943	2.078928	2.396558
16	1.372786	1.484506	1.604706	1.872981	2.182875	2.540352
17	1.400241	1.521618	1.652847	1.947900	2.292018	2.692773
18	1.428246	1.559659	1.702433	2.025817	2.406619	2.854339
19	1.456811	1.598650	1.753506	2.106849	2.526950	3.025599
20	1.485947	1.638616	1.806111	2.191123	2.653298	3.207136

399. *Example* 1. Find the amount of $2500 at compound interest for 12 yr. at 5%.

SOLUTION. $a = \$2500\ (1.05)^{12} = \$2500 \times 1.795856 =$ $4489.64.

Example 2. Find the amount of $5000 for 8 yr. at 4% compound interest, the interest being compounded semi-annually.

SOLUTION. $\$5000 \times (1.02)^{16} = \$5000 \times 1.372786 = \$6863.93.$

ANNUITIES

400. A payment made annually or semiannually is called an **annuity**. Under this head come the premiums paid on life insurance policies.

How much will a premium of \$30 paid on a life insurance policy amount to in 20 yr., reckoning compound interest at 6%?

Premiums are paid in advance. The first premium bears interest for 20 yr., the next bears interest for 19 yr., the next for 18 yr., the last premium bears interest for 1 yr.

Beginning with the last payment and reckoning up to the first payment, we have for the value of \$1 of premium:

$$\$\,(1.06) + \$\,(1.06)^2 + \$\,(1.06)^3 + \ldots \$\,(1.06)^{20}.$$

This is evidently a geometric series in which the first term is \$1.06 and the common ratio 1.06.

The sum of a geometric series is $A_1\frac{r^n-1}{r-1}$. In our problem $A_1 = \$1.06$, $r = 1.06$.

The amount of $\$1 = \$1.06 \cdot \frac{(1.06)^{20}-1}{(1.06)-1} = \$1.06 \cdot \frac{3.20714-1}{.06}$ $= \$1169.78$.

The amount of an annuity of \$1 paid annually in advance is given by the rule $(1+r)\cdot\frac{(1+r)^n-1}{r}$. (*a*)

The amount of an annuity of \$1 paid annually at the end of a year is $\frac{(1+r)^n-1}{r}$. (*b*)

In (*b*) each payment bears interest for 1 yr. less than in (*a*), and therefore their sum is $(1+r)$ times less than the sum in (*a*).

EXERCISE 234

1. Find the amount of an annuity of \$50 payable in advance, reckoning compound interest at 5% for 20 yrs.

2. Find the amount of a premium of \$100 paid to an insurance company for 20 yrs. at 4%.

3. A person deposits in a savings bank $100 at the end of each 6 mo. for 10 yr. Reckoning interest at 2% semiannually, how much is he worth?

4. A person deposits in a bank $100 at the end of each year for 12 yr. Reckoning interest at 3%, what is the amount of his bank account?

PRESENT WORTH

401. Suppose money is worth 5%. If, therefore, you invested $100, it would amount to $105 in 1 yr. If, on the other hand, you had $105 due you at the end of a year, its present value would not be $105. It would be worth no more than $100 when money is worth 5%.

The present cash value of a sum of money due at some time hence is that sum of money which, put at interest now, would amount to the given sum in the given time. This present cash value is called **present worth.**

402. *Example* 1. What is the present worth of $50, due 6 mo. hence, at 8%?

Ans. That sum which now invested at 8% would amount to $50 in 6 mo.

Designating the sum by S, we have

$$S(1.04) = \$50$$

$$S = \frac{\$50}{1.04} = \$48.01.$$

Example 2. When money is worth 5%, what is the value of a legacy of $1000 due in 8 yr.?

SOLUTION. The answer is that sum which if invested now would amount to $1000 in 8 yr. at 5% compound interest.

$$S(1.05)^8 = \$1000.$$

$$S = \frac{\$1000}{(1.05)^8} = \frac{\$1000}{1.4775}, \text{ nearly}, = \$676.80, \text{ nearly}.$$

Example 3. What is the present value of an annuity of $ 200 which is to run for 20 yr., reckoning interest at 4% ?

To solve this question there are two steps:

I. Find the value of the annuity at the end of 20 yr.

II. Find, as in Example 2, the present worth of this sum.

SOLUTION:

$$\$\,200 \times \frac{(1.04)^{20}-1}{.04} = \$\,200 \times \frac{2.19112-1}{.04} = \$\,5955.60. \quad \text{(I)}$$

$$S \times 2.19112 = \$\,5955.60. \quad \text{(II)}$$

$$S = \frac{\$\,5955.60}{2.19112} = \$\,2718.06.$$

Example 4. A man borrows $ 3000 and pays it in 6 equal annual installments. Find the amount of each installment, rate of interest being 6%.

A little consideration will make this problem quite simple. If each installment is put out to interest as soon as it is received, the party lending the money will have $ 3000 invested for 6 yr. The party who borrowed the money pays to the lender an annuity for 6 yr. The value of this annuity at compound interest must exactly balance $ 3000 at compound interest for 6 yr.

SOLUTION. Value of $ 3000 at compound interest for 6 yr.

$$= \$\,3000 \times (1.06)^6 = \$\,3000 \times 1.41852 = \$\,4255.56.$$

$$\text{Installment} \times \frac{(1.06)^6-1}{.06} = \$\,4255.56.$$

$$\text{Installment} \times 6.9753 = \$\,4255.56.$$

$$\text{Installment} = \frac{\$\,4255.56}{6.9753} = \$\,610.10.$$

EXERCISE 235

1. Reckoning simple interest, what is the present worth of $ 1505 due in 1 yr. 3 mo. at 6% ?

2. If money is worth 6%, which is the more profitable, to sell a suit of clothes for $ 15 cash, or for $ 15.50 payable in 6 mo.?

3. When money is worth 6%, what should be deducted from a bill for $ 522.50 due 9 mo. hence, if the bill is paid now?

4. If $ 4500 is accepted in payment of a debt of $ 4680, money being worth 6%, when is the debt due?

5. Find the present value of a legacy of $ 1225 due in 5 mo. at 5%.

6. A banker discounts a note for $ 500 for 6 mo. at 8%. What rate of interest does the banker receive for his money?

7. On what sum, due 1 yr. hence, would the present worth at 6% exceed the present worth at 8% by $ 5?

8. The difference between the present worth of a sum of money due 3 mo. hence and the proceeds of a note for the same sum of money discounted at a bank for 3 mo. is 45¢. If money is worth 6%, find the face of the note.

9. A banker discounts a note when the rate of interest is $7\frac{1}{2}$%, and thereby makes 8% on his money. Find the term of discount.

10. Find the present worth of $ 5000 due in 10 yr. at 6%.

11. Find the present worth of $ 4000 due in 8 yr. at 5%.

12. Find the present worth of an estate which brings an annual rental of $ 1000, money being worth 5%.

HINT. The interest on present worth should equal the rental.

13. Find the present worth of an annuity of $ 400 for 15 yr. at 5%.

14. A man borrows $ 1000 and pays it back in 5 equal annual installments. If money is worth 5%, find the amount of each installment.

15. (*a*) A person borrows from a loan company $ 3000, and agrees to pay in 50 monthly installments at 8% interest. (Given $(1.00\frac{2}{3})^{50} = 1.39428$, find each installment.)

(*b*) If he paid in 60 monthly payments, find each payment. (Given $(1.00\frac{2}{3})^{60} = 1.48984$.)

16. If \$2500 is borrowed at $7\frac{1}{2}\%$, and \$200 is paid every 4 mo. for 3 yr., how much is due at the end of the third year?

17. If \$4000 is borrowed at 8% interest, and payments of \$300 are made semiannually, how much remains due at the end of 7 yr.?

To solve the last four examples study Example 3, § 402.

CUBE ROOT

403. The number which when raised to the third power produces a given number is called the **cube root** of that number. In other words, the cube root of a number is one of its three equal factors; *e.g.* 4 is the cube root of 64, since $4^3 = 64$.

The *symbol for cube root* is $\sqrt[3]{\ }$, or the exponent $\frac{1}{3}$. Thus, $\sqrt[3]{125}$, $125^{\frac{1}{3}}$ both mean the cube root of 125.

404. Before we undertake to extract the cube root of numbers, it is necessary to get an expression for the cube of the sum of two numbers in terms of those numbers.

Take any two numbers, a and b.

$(a+b)(a+b) = a^2 + 2\,ab + b^2.$

$(a+b)(a+b)(a+b) = (a^2 + 2\,ab + b^2)(a+b) = a(a^2 + 2\,ab + b^2) + b(a^2 + 2\,ab + b^2) = a^3 + 2\,a^2b + ab^2 + a^2b + 2\,ab^2 + b^3 = a^3 + 3\,a^2b + 3\,ab^2 + b^3.$

$\therefore\ (a+b)^3 = a^3 + 3\,a^2b + 3\,ab^2 + b^3 = a^3 + (3\,a^2 + 3\,ab + b^2)b.$

The **cube of the sum of two numbers** equals the cube of the first number plus the second number multiplied by (three times the square of the first number, plus three times the first number by the second number, and the square of the second number).

405. If we raise 1, 10, 100, 1000 to the third power, we get

$$1^3 = 1,$$
$$10^3 = 1000,$$
$$100^3 = 1{,}000{,}000.$$
$$1000^3 = 1{,}000{,}000{,}000.$$

Hence, the cube root of all numbers greater than 1 and less than 1000 is greater than 1 and less than 10.

The cube root of all numbers greater than 1000 and less than 1,000,000 is greater than 10 and less than 100. The cube root of all numbers greater than 1,000,000 and less than 1,000,000,000 is greater than 100 and less than 1000. In other words, the cube roots of numbers whose integral parts have 4, 5, or 6 figures consist of two integral figures, and of numbers whose integral parts have 7, 8, or 9 figures, of three figures.

Example. What is the cube of 24?

SOLUTION. $24^3=(20+4)^3=20^3+(3\times 20^2+3\times 20\times 4+4^2)4$ $=8000+(1200+240+16)4=13{,}824.$

406. Consider next the inverse operation, $\sqrt[3]{13{,}824}=?$

By trial, we find the $\sqrt[3]{13{,}824}$ is greater than 20 and less than 30. It is, therefore, equal to 20 plus some number which we denote by x.

$$(20+x)^3=13{,}824.$$
$$\therefore\ 20^3+(3\times 20^2+3\times 20\times x+x^2)x=13{,}824.$$
$$\therefore\ 8000+(1200+60x+x^2)x=13{,}824.$$
$$\therefore\ (1200+60x+x^2)x=5824.$$

Since 1200 is contained in 5824 4 times, we try 4 as a possible value of x.

$$(1200+60\times 4+4^2)4=(1200+240+16)4=5824.$$
$$\therefore\ \sqrt[3]{13{,}824}=24.$$

We shorten the work as follows:

Mark off the figures of the number in periods of three figures, beginning at the decimal point. Find the greatest cube contained in the period to the left. Write it as the first figure of the required root. Cube it and subtract. Take down three figures.

$$\begin{array}{rr} & 2\;\;4 \\ 3\times 20^2=1200 & \underline{13{,}824} \\ 3\times 20\times 4=240 & 8\phantom{{,}824} \\ 4^2=\underline{16} & 5824 \\ 1456 & \underline{5824} \end{array}$$

For trial divisor take 3 times the square of the part of the root found, *i.e.* $3\times 20^2=1200$. Divide it into 5824 and write 4 as the next figure of the root. Then take 3 times 20 times 4. Next, take 4^2. Add the three results and multiply the sum by 4.

EXERCISE 236

Find the cube roots of:

1. 2197.
2. 10,648.
3. 15,625.
4. 39,304.
5. 74,088.
6. 91,125.
7. 157,464.
8. 250,047.
9. 300,763.
10. 438,976.
11. 592,704.
12. 681,472.
13. 884,736.
14. 970,299.
15. 551,368.

407. *Example.* Extract the cube root of 94,818,816.

Mark off the figures in periods of three figures each. We then proceed to extract the root of the number denoted by the figures in the two periods to the left. The required root is 450 plus some number. Denote this number by x.

$$\begin{array}{rr}
 & 4\quad 5\quad 6 \\
 & 94{,}818{,}816 \\
 & \underline{64\phantom{{,}818{,}816}} \\
3 \times 40^2 = 4800 & 30{,}818\phantom{{,}816} \\
3 \times 40 \times 5 = 600 & \underline{27{,}125\phantom{{,}816}} \\
5^2 = \underline{25} & 3{,}693.816 \\
5425 &
\end{array}$$

$$(450 + x)^3 = 94{,}818{,}816.$$
$$450^3 + (3 \times 450^2 + 3 \times 450\, x + x^2)x = 94{,}818{,}816.$$
$$91{,}125{,}000 + (607{,}500 + 1350\, x + x^2) = 94{,}818{,}816.$$
$$(607{,}500 + 1350\, x + x^2)x = 3{,}693{,}816.$$

Trying 6 as a possible value of x, we get $(607{,}500 + 8100 + 36)6 = 3{,}693{,}816$.

$$\therefore\ \sqrt[3]{94{,}818{,}816} = 456.$$

Beginning once more, the work stands as follows:

$$\begin{array}{rr}
3 \times 40^2 = 4800 & \\
3 \times 40 \times 5 = 600 & 4\quad 5\quad 6 \\
5^2 = \underline{25} & 94{,}818{,}816 \\
5425 & \underline{64\phantom{{,}818{,}816}} \\
 & 30{,}818\phantom{{,}816} \\
3 \times 450^2 = 607{,}500 & \underline{27{,}125\phantom{{,}816}} \\
3 \times 450 \times 6 = 8100 & 3{,}693{,}816 \\
6^2 = \underline{36} & \underline{3{,}693{,}816} \\
615{,}636 &
\end{array}$$

To extract **the cube root of a decimal**, we mark off the figures in periods of three figures each, beginning at the decimal point and proceeding to the right. The mechanical work of extracting the root is then performed in the same manner as if the number were an integer.

To extract **the cube root of a fraction** whose denominator is not a perfect cube, reduce the fraction to a decimal, and then extract the cube root of this decimal to any required degree of approximation.

EXERCISE 237

Extract the cube roots of:

1. 14,348,907.
2. 17,173,512.
3. 482,285,444.
4. 100,544,625.
5. 121,287,375.

6. Extract the cube root to three decimal figures of:

(*a*) .2.
(*b*) .7.
(*c*) .9.
(*d*) $1\frac{1}{9}$.
(*e*) $1\frac{2}{3}$.
(*f*) 1.1.
(*g*) 7.3.
(*h*) $\frac{1}{4}$.
(*i*) $\frac{3}{5}$.

It is not possible to express exactly by any sequence of figures the square root of a number which is not a perfect square. The square root of 2, the square root of 3, the square root of 5, etc., give rise to decimals which neither terminate nor circulate.

Similarly, the cube root of 2 gives rise to a decimal which neither terminates nor circulates.

408. Any integer, any decimal consisting of a definite number of figures, or any circulating decimal, is called a **rational number.**

Any indicated root which does not give rise to a rational number is called a **surd**, or **irrational number.**

That a surd neither terminates nor circulates may be easily shown in individual instances, *e.g.* take the $\sqrt{2}$. If the $\sqrt{2}$ equals a terminating decimal or a circulating decimal, then we can reduce this decimal to a common fraction, whose terms are both integral. Let $\frac{a}{b}$ be such a fraction in its lowest terms.

Then, $$\frac{a}{b}=\sqrt{2}.$$

$$\therefore \frac{a^2}{b^2}=2.$$

$$\therefore b^2 \text{ is a measure of } a^2.$$

$\therefore$ b is a measure of a. But this is impossible since b is prime to a.

That the square root of an integer which is not the square of some integer can never equal a rational number is proved in a simple and elegant manner in "Dedekind's Continuity and Irrational Numbers," translation by Professor Beman.

CONTRACTED MULTIPLICATION

409. Multiply 1.123456 by 2.327413, and give the result correct to four decimal figures.

(1) Multiply in the usual way:

$$\begin{array}{r|l}
1. & 123456 \\
\underline{2.} & \underline{327413} \\
3 & 370368 \\
11 & 23456 \\
449 & 3824 \\
7864 & 192 \\
22469 & 12 \\
337036 & 8 \\
\underline{2246912} & \underline{} \\
2614746 & 099328
\end{array}$$

Ans. 2.6147.

(2) CONTRACTED PROCESS

$$\begin{array}{r}
1.123456 \\
\underline{3.147232} \\
2\,246912 \\
337037 \\
22469 \\
7864 \\
449 \\
11 \\
\underline{3} \\
2.614745
\end{array}$$

Write the units' figure of the multiplier in the sixth place, and the other figures as indicated. Multiply by 2 and write the product. Then multiply the next figure, 3 by 5, and carry 2, because 3 sixes is nearer to 20 than to 10. The other figures to the left are multiplied by 3 in the usual manner.

Multiply by 2, 7, etc., in a similar manner. Reject the last two figures of the final product.

This method is old. It is due to Oughtred (1574–1660).

OTHER SYSTEMS OF NOTATION

410. Roman System of Notation. In the Roman system

I stands for	1	LXX stands for	70
X stands for	10	LXXX stands for	80
C stands for	100	CX stands for	110
M stands for	1000	CXX stands for	120
V stands for	5	CXXX stands for	130
L stands for	50	CXL stands for	140
D stands for	500	CL stands for	150
IV stands for	4	CLX stands for	160
XL stands for	40	CLXX stands for	170
CD, or CCCC, stands for	400	CLXXX stands for	180
IX stands for	9	CXC stands for	190
XC stands for	90	CC stands for	200
CM stands for	900	CCC stands for	300
XX stands for	20	DC stands for	600
XXX stands for	30	DCC stands for	700
LX stands for	60	DCCC stands for	800

In the Roman system, the value of a letter remains the same. There is no decimal point nor any place value. To get the value of a number when it is expressed in this system, add the values of the numbers represented by the letters of the number, remembering that IV, IX, XL, XC, CD, CM are regarded as single characters, standing respectively for 4, 9, 40, 90, 400, 900.

In Roman notation, I, X, M may be repeated three times, C may be repeated four times. Thus, CCCC stands for 400. V, L, D are never repeated. It is quite often stated that, in this system, when a letter of less value precedes a letter of greater value, the value of the former is subtracted from that of the latter. This is so as regards IV, XL, XC, CD, CM. In no other case is it permissible to write a letter of less value before one of greater value. A bar over a letter multiplies the original value of the letter one thousand fold. Thus $\overline{\text{V}}$ stands for 5000, $\overline{X}$ stands for 10,000. The Romans themselves as a rule used no system of notation in writing numbers above a few thousand. Large numbers were not written in symbols; these they wrote in words.

To express a number in the Roman system, we write first the character of less value than the number and nearest to the number in value. Take the remaining part of the number and treat it in the same manner. Treat the next remaining part in the same manner, and so on.

Write 8 in the Roman system.

Write first V. Take the remaining portion, 3, and for it write III. *Ans.* VIII.

Write in the Roman system 98.

We first write XC for 90. We then take the remainder, and writing V for 5 and III for 3, we have XCVIII for 98.

Write 1903.

Write first M for 1000, then CM for 900, and III for 3. We have then for 1903, MCMIII.

SCALES OF NOTATION

411. In the decimal system of notation, 10 is the basis of the system, or, in technical language, the radix of the system. In the decimal system, the figures 27,385 stand for 2 ten-thousands, 7 thousands, 3 hundreds, 8 tens, and 5, or

$$2 \times 10^4 + 7 \times 10^3 + 3 \times 10^2 + 8 \times 10^1 + 5.$$

There is no reason why the system of notation in general use should have 10 for its radix. People have used this system, and that is the reason for it. As a matter of fact, 10 is a very good radix; 12, however, would make a better radix. If 8 were the radix of a system of notation, the figures 3256 would stand for

$$3 \times 8^3 + 2 \times 8^2 + 5 \times 8 + 6.$$

412. In a scale in which 6 is the radix, what is the value of 23,645, expressed in the decimal system?

Solution:

$$2 \times 6^4 + 3 \times 6^3 + 2 \times 6^2 + 4 \times 6 + 5 = 2592 + 648 + 72 + 24 + 5$$
$$= 3341.$$

Since 6 units of one order equal 1 unit of the next higher order, we may work this example as we have worked problems in reduction of compound quantities.

$$\begin{array}{r}
23245 \\
\underline{6} \\
15 \\
\underline{6} \\
92 \\
\underline{6} \\
556 \\
\underline{6} \\
3341
\end{array}$$

A problem to convert an integer from any other scale to the decimal scale is worked in the same manner.

413. Take a number in the decimal system, for instance, 7293, and divide it as often as possible by 6.

$$\begin{array}{r|r}
6 & 7293\phantom{\text{, rem. 3}} \\ \hline
6 & 1215\text{, rem. } 3 \\ \hline
6 & 202\text{, rem. } 3 \\ \hline
6 & 33\text{, rem. } 4 \\ \hline
 & 5\text{, rem. } 3
\end{array}$$

The first remainder, 3, consists of 3 units of the same kind as the units of 7293.

The second remainder, 3, consists of 3 units of the same kind as the units of 1215, *i.e.* groups of 6 each.

The third remainder, 4, consists of 4 units of the same kind as the units of 202, *i.e.* groups of 36, or 6^2 each.

The fourth remainder, 3, consists of 3 units of the same kind as the units of 33, *i.e.* groups of 216, or 6^3 each.

The last quotient, 5, consists of groups of 1296, or 6^4 each.

Writing the last quotient and the remainders in succession, we have

53,433. The 5 represents 5×6^4.
The 3 next to 5 represents 3×6^3.
The 4 next to 3 represents 4×6^2.
The 3 next to 4 represents 3×6.
The 3 to the right represents 3.

The combination 53,433 is a number in the scale whose radix is 6.

The conversion of any other integer from the decimal system to a system having some other number than 10 for radix is done in a similar manner.

In the decimal scale, $.345 = \frac{3}{10} + \frac{4}{10^2} + \frac{5}{10^3}$.

In a scale whose radix is 6, $.345 = \frac{3}{6} + \frac{4}{6^2} + \frac{5}{6^3}$.

414. Transform .345 from the scale of 6 to the decimal scale.

$$.345 = \frac{3}{6} + \frac{4}{6^2} + \frac{5}{6^3} = \text{some fraction.}$$

Denote this fraction by F.

$$F = \frac{3}{6} + \frac{4}{6^2} + \frac{5}{6^3}.$$

$$\therefore \quad 6F = 3 + \frac{4}{6} + \frac{5}{6^2}.$$

$$\therefore \quad 36F = 3 \times 6 + 4 + \tfrac{5}{6}.$$

$$\therefore \quad 216F = 3 \times 6^2 + 4 \times 6 + 5.$$

(Notice, we can multiply .345 in the scale of 6 by 6^3 by moving the decimal point three places to the right.)

$$216F = 137.$$

$$F = \tfrac{137}{216}.$$

We can reduce this fraction to a decimal in the usual way.

415. Reduce .725 to a decimal in the scale of 8.

$$\begin{array}{ll}
.725 & \\
\underline{8} & \\
5.800 & \text{8ths.} \\
\underline{8} & \\
6.4 & \text{64ths.} \\
\underline{8} & \\
3.2 & \text{512ths.} \\
\underline{8} & \\
1.6 & \text{4096ths.} \\
\underline{8} & \\
4.8 & \text{32768ths.} \\
\underline{8} & \\
6.4 \cdots &
\end{array}$$

$$\therefore\ 725 = \frac{5}{8} + \frac{6}{8^2} + \frac{3}{8^3} + \frac{1}{8^4} + \frac{4}{8^5} + \cdots = .5\dot{6}31\dot{4}.$$

Reduce $\frac{5}{9}$ to a decimal in the scale of 6.

$$\tfrac{5}{9} \times 6 = \tfrac{30}{9} = 3\tfrac{3}{9} \text{ sixths.}$$

$$\tfrac{3}{9} \times 6 = \tfrac{18}{9} = 2 \text{ thirty-sixths.}$$

$$\therefore\ \tfrac{5}{9} = \tfrac{3}{6} + \tfrac{2}{36} = .32 \text{ in the scale of 6.}$$

416. In a scale whose radix is 12,

$\frac{1}{2} = .6.$

$\frac{1}{3} = .4.$

$\frac{1}{5} = .\dot{2}49\dot{7}.$

$\frac{1}{6} = .2.$

$\frac{1}{7} = .\dot{1}86\,t\,3\dot{5}$*.

$\frac{1}{8} = .16.$

$\frac{1}{9} = .14.$

$\frac{1}{10} = .1\dot{2}49\dot{7}.$

$\frac{1}{11} = .\dot{1}.$

$\frac{1}{12} = .1.$

In a scale whose radix is 10,

$\frac{1}{2} = .5$

$\frac{1}{3} = .3$

$\frac{1}{4} = .25$

$\frac{1}{5} = .2$

$\frac{1}{6} = .1\dot{6}$

$\frac{1}{7} = .\dot{1}4285\dot{7}$

$\frac{1}{8} = .125$

$\frac{1}{9} = .\dot{1}$

$\frac{1}{10} = .1$

$\frac{1}{11} = .\dot{0}\dot{9}$

$\frac{1}{12} = .08\dot{3}$

An examination of the above results seems to indicate that 12 is a more convenient basis for a system of notation than 10.

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* *t* stands for 10.

CASTING OUT NINES

417. I. The remainder obtained by dividing any integer by 9 is the same as the remainder obtained by dividing the sum of the digits of the number by 9.

Proof. Denote the digits of the number by a, b, c, d, e.

$$\therefore \text{ The number} = a \times 10^4 + b \times 10^3 + c \times 10^2 + d \times 10 + e$$

$$= 10{,}000\,a + 1000\,b + 100\,c + 10\,d + e.$$

$$9999\,a + 999\,b + 99\,c + 9\,d + a + b + c + d + e.$$

9 is a measure of each of the first four terms of the last expression and therefore of their sum.

$$\therefore \text{ The number} \div 9 = \text{an integer} + \frac{a+b+c+d+e}{9}.$$

$\therefore$ The proposition is proved.

II. Let $A = 9\,m + a$ (A, m, a, integers, and a less than 9).

Let $B = 9\,n + b$ (B, n, b, integers, and b less than 9).

$$\therefore AB = 81\,mn + 9\,na + 9\,mb + ab.$$

$$\therefore \frac{AB}{9} = \text{an integer} + \frac{ab}{9}.$$

Therefore, in an exercise in multiplication of integers the remainder obtained by casting the nines out of the product is the same as the remainder obtained by casting the nines out of the multiplier and multiplicand and then out of the product of these remainders.

TABLES

APOTHECARIES' WEIGHT

20 grains (gr.) = 1 scruple (℈)
3 scruples = 1 dram (ʒ)
8 drams = 1 ounce (℥)
12 ounces = 1 pound (℔)

LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)
31½ gallons = 1 barrel (bbl.)
2 barrels = 1 hogshead (hhd.)

LONG MEASURE

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
5½ yards = 1 rod (rd.) or pole
40 rods = 1 furlong
8 furlongs = 1 mile (mi.)

TROY WEIGHT

24 grains (gr.) = 1 pennyweight (pwt.)
20 pennyweights = 1 ounce (oz.)
12 ounces = 1 pound (lb.)

DRY MEASURE

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

NUMERICAL MEASURE

12 articles = 1 dozen
12 dozen = 1 gross
12 gross = 1 great gross
20 articles = 1 score

AVOIRDUPOIS WEIGHT

16 drams (dr.) = 1 ounce (oz.)
16 ounces = 1 pound (lb.)
25 pounds = 1 quarter
100 pounds = 1 hundredweight (cwt.)
20 cwt. = 1 ton (T.)
2240 pounds = 1 long ton

PAPER MEASURE

24 sheets = 1 quire
20 quires = 1 ream
2 reams = 1 bundle
5 bundles = 1 bale

CIRCULAR MEASURE

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°)
30 degrees = 1 sign (S.)
12 signs = 1 circle (C.) or circumference
360 degrees = 1 circumference

NAUTICAL MEASURE

6 feet = 1 fathom
608 feet = 1 cable length
10 cable lengths = 1 nautical mile (6080 feet)

The following denominations are also used:

1.152 statute miles = 1 geographic mile, or knot
3 geographic miles = 1 league
60 geographic miles, or 69.1 statute miles = 1 degree of latitude on a meridian
360 degrees = the circumference of the earth

4 inches = 1 hand
9 inches = 1 span
21.888 inches = 1 sacred cubit
3 feet = 1 pace

TIME MEASURE

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
4 weeks = 1 lunar month
30 days = 1 commercial month
12 months = 1 year
365 days = 1 common year
366 days = 1 leap year

SURVEYORS' AND LAND MEASURE

7.92 inches = 1 link (l.)
25 links = 1 rod
4 rods = 1 chain (ch.)
10 square chains = 1 acre
640 acres = 1 square mile

625 square links (sq. l.) = 1 pole (P.)
16 poles = 1 square chain

APOTHECARIES' FLUID MEASURE

60 minims (m.) = 1 fluidrachm (fʒ)
8 fluidrachms = 1 fluidounce (f℥)
16 fluidounces = 1 pint (O)
8 pints = 1 gallon (Cong.)

Cubic Measure

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
16 cubic feet, or
8 cord feet } = 1 cord of wood (cd.)
24¾ cubic feet = 1 perch of stone or masonry (pch.)

Square Measure

144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
30¼ square yards = 1 square rod or perch (sq. rd. or sq. pch.)
40 square rods = 1 square rood (sq. R.)
4 roods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Spanish Land Measure

In Texas, California, New Mexico, and other parts of this country which were formerly parts of the Spanish empire, the vara, the unit of linear measure, is still used in connection with original grants of land. In Texas, the value of the vara is 33⅓ in. In California and New Mexico it is usually considered 33 in.

1,000,000 square varas = 1 labor = 177.136 acres
25,000,000 square varas = 1 league = 4428.4 acres
3,612,800 square varas = 1 square mile = 640 acres
1,806,400 square varas = ½ square mile = 320 acres
903,200 square varas = ¼ square mile = 160 acres
451,600 square varas = ⅛ square mile = 80 acres
225,800 square varas = $\frac{1}{16}$ square mile = 40 acres
5645 square varas = 1 acre

ANSWERS

Exercise 7. — **1.** 153. **2.** 275. **3.** 239. **4.** 298. **5.** 348. **6.** 308. **7.** 2470. **8.** 2628. **9.** 3028. **10.** 2799. **11.** 3137. **12.** 2302. **13.** 32,953. **14.** 31,620. **15.** 32,384. **16.** 36,795. **17.** 21,726. **18.** 484,537. **19.** 447,919. **20.** 438,924. **21.** 437,417. **22.** 3,456,643. **23.** 4,708,821. **24.** 4,814,755. **25.** 5,473,279. **26.** 6,849,601 **27.** 6,381,790. **28.** 74,801,826. **29.** 75,453,652. **30.** 74,944,016. **31.** 73,781,838. **32.** 80,963,637. **33.** 80,903,198.

Exercise 8. — **1.** $909. **2.** 732. **3.** 273 da. **4.** 156. **5.** $12,479. **6.** 248,012. **7.** 55,191 bales. **8.** 202,655 bales. **9.** 339,650 sq. mi. **10.** 21,579,890.

Exercise 10. — **1.** 2350. **2.** 1214. **3.** 2225. **4.** 5137. **5.** 3232. **6.** 5002. **7.** 5102. **8.** 2312. **9.** 3058. **10.** 13,634. **11.** 25,002. **12.** 71,400. **13.** 20,009. **14.** 35,000. **15.** 22,111.

Exercise 11. — **1.** 18. **2.** 19. **3.** 35. **4.** 37. **5.** 24. **6.** 39. **7.** 118. **8.** 38. **9.** 357. **10.** 467. **11.** 1137. **12.** 2488. **13.** 974. **14.** 4988. **15.** 4289. **16.** 5908. **17.** 497. **18.** 3491. **19.** 3999. **20.** 2799. **21.** 1799. **22.** 3999. **23.** 2990. **24.** 2998. **25.** 877. **26.** 1989. **27.** 6544. **28.** 7276. **29.** 3699. **30.** 2989. **31.** 48,989. **32.** 13,997. **33.** 2097. **34.** 19,773. **35.** 4896. **36.** 72,159. **37.** 98,997. **38.** 298,897. **39.** 9899. **40.** 298,998. **41.** 200,207. **42.** 500,088. **43.** 92,686. **44.** 395,998. **45.** 893,909. **46.** 690,505.

Exercise 12. — **1.** $8592. **2.** 144,805 sq. mi. **3.** 5,620,187 T.; 102,212,004 T. **4.** 23,721,000; 126,621,000; 145,401,000. **5.** 529,417 bales; 6,000,183 bales. **6.** 572,747 T. **7.** 478,455 T.; 192,784 T.; 89,397 T.; 136,476 T. **8.** 91,895 sq. mi. **9.** 7522. **10.** 990,000. **11.** $973. **12.** $29,100. **13.** 7 bills; $17. **14.** $10,084. **15.** 574 mi.

Exercise 15. — **1.** 28,224. **2.** 82,944. **3.** 153,664. **4.** 164,025. **5.** 321,489. **6.** 390,625. **7.** 419,904. **8.** 200,704. **9.** 112,896. **10.** 30,625. **11.** 254,016. **12.** 194,481. **13.** 379,456. **14.** 853,776. **15.** 746,496. **16.** 451,584. **17.** 234,256. **18.** 73,402. **19.** 6,364,248. **20.** 4,543,056. **21.** 2,251,375.

Exercise 16. — 1. $558. 2. $1470. 3. $378. 4. $275. 5. 84 ft.
6. 9504 in. 7. $900. 8. 2080¢ or $20.80. 9. $188. 10. $546.
11. 468¢ or $4.68. 12. 8640 rd. 13. 972¢ or $9.72. 14. 52,800 ft.
15. 26 mi. 16. 1428¢ or $14.28. 17. 28¢.

Exercise 17. — 1. 1292. 2. 4998. 3. 2475. 4. 3332.
5. 4988. 6. 14,504. 7. 25,024. 8. 30,566. 9. 94,374.
10. 268,025. 11. 396,891. 12. 362,880. 13. 969,969.
14. 841,338. 15. 318,168. 16. 477,252. 17. 1,014,664.
18. 954,504. 19. 1,340,780. 20. 3,932,695. 21. 3,064,682.
22. 3,064,682. 23. 1,055,808. 24. 2,111,616. 25. 841,455.
26. 458,800. 27. 3,211,600. 28. 898,432. 29. 2,271,528.
30. 973,452. 31. 1,121,400. 32. 350,854. 33. 2,806,832.
34. 209,088. 35. 248,832. 36. 350,464. 37. 705,894. 38. 927,472.
39. 1,021,468. 40. 1,254,528. 41. 1,533,168. 42. 1,434,726.
43. 861,056. 44. 1,778,112. 45. 3,172,059. 46. 2,109,807.
47. 8,910,225. 48. 4,262,592. 49. 4,477,484. 50. 1,891,308.
51. 5,105,212. 52. 9,590,832. 53. 10,478,336. 54. 5,904,384.
55. 1,385,670. 56. 39,304. 57. 1,521,500.

Exercise 18. — 1. 9000 sq. yd. 2. 7056 sq. yd. 3. 102,400 sq. rd.
4. 11,760 sq. yd. 5. 22,848 sq. yd. 6. 167,200 sq. yd. 7. 4725 sq. rd.
8. 20,160 ft. 9. 252 sq. in. 10. 36 sq. mi. 11. 432 sq. mi.
12. 40,480 sq. yd. 13. 2200 sq. yd. 14. 5332 sq. ft. 15. 2880 sq. in.
16. 1296 sq. in. 17. 3,097,600 sq. yd. 18. 4212 sq. yd.

Exercise 19. — 1. 1173 mi. 2. 690 mi. 3. 1035 mi. 4. 273 da.
5. 44,640 min. 6. 100,440 cars. 7. 108,675 lb. 8. 725,040 lb.
9. $1764. 10. $6084. 11. 24 hr., 2136 hr. 12. 60 min., 1440 min.,
44,640 min. 13. 294,400 rd. 14. 765,440 rd., 1,159,680 rd. 15. 1,740,800 rd.
16. 62,720 A., 250,880 sheep. 17. 2,220,800 rd. 18. 329,280 lb. 19. $1170.
20. 1344 mi. 21. 1598 mi. 22. $33,250. 23. $178. 24. 739,200 ft.
25. 5,702,400 in. 26. 487,956 lb. 27. 5,955,200 A. 28. 26,278,400 A.;
4,147,060. 29. 31,468,800 A. 30. 11,520 bu. 31. $13,695.
32. New York, 7,277,160; Massachusetts, 2,802,155; Illinois, 4,815,250;
Louisiana, 1,412,880; Japan, 41,934,020; Germany, 52,207,500; France,
37,961,112; England, 29,160,000. 33. $4485. 34. $972. 35. 76,680 lb.
36. $7095. 37. $780. 38. 42,504 cu. in. 39. 118 mi. 40. $725.
41. 103,950 words. 42. 8736 hr. 43. 102,400 sq. rd. 44. 9,046,400 yd.

Exercise 22. — 1. 239. 2. 158, rem. 1. 3. 116. 4. 199. 5. 116.
6. 494. 7. 623, rem. 4. 8. 557, rem. 6. 9. 1309. 10. 1158, rem. 6.
11. 9039, rem. 5. 12. 8978, rem. 3. 13. 8305, rem. 8. 14. 13,077.
15. 16,359, rem. 3. 16. 5858, rem. 3. 17. 1769, rem. 4. 18. 32,994,

rem. 2. **19.** 23,183, rem. 3. **20.** 4176, rem. 6. **21.** 11,750, rem. 1. **22.** 3326, rem. 1. **23.** 8138, rem. 3. **24.** 49,939, rem. 1. **25.** 266,560. **26.** 114,240. **27.** 6429, rem. 2. **28.** 6142. **29.** 12,947, rem. 4. **30.** 16,658. **31.** 14,285, rem. 5. **32.** 12,502, rem. 7. **33.** 39,426, rem. 5. **34.** 114,463, rem. 2. **35.** 180,780, rem. 4. **36.** 130,442, rem. 1. **37.** 289,169, rem. 1. **38.** 79,284, rem. 6. **39.** 67,358, rem. 4. **40.** 48,415, rem. 8. **41.** 44,872, rem 2. **42.** 78,986, rem. 3. **43.** 74,802, rem. 7. **44.** 54,782, rem. 10. **45.** 32,829. **46.** 76,961, rem. 9. **47.** 8333, rem. 4. **48.** 28,806, rem. 6.

Exercise 23. — **1.** 4834, rem. 10. **2.** 4408, rem. 14. **3.** 4892, rem. 14. **4.** 4416, rem. 2. **5.** 4166, rem. 16. **6.** 2957, rem. 23. **7.** 2485, rem. 13. **8.** 2668. **9.** 2760, rem. 32. **10.** 1706, rem. 28. **11.** 1234, rem. 39. **12.** 2380, rem. 28. **13.** 1431, rem. 10. **14.** 2519, rem. 25. **15.** 17,530, rem. 43. **16.** 14,616, rem. 31. **17.** 14,792, rem. 22. **18.** 8261, rem. 35. **19.** 8741, rem. 42. **20.** 3377, rem. 42. **21.** 2133, rem. 74. **22.** 8214, rem. 4. **23.** 9701, rem. 37. **24.** 8217, rem. 3.

Exercise 24. — **1.** 14 lb. **2.** 29 hr. **3.** 375 pk. **4.** 29 hr., 4 mi. rem.; 94 mi. **5.** 384 T. **6.** 179 da. **7.** 225 da. **8.** 2083, 4 in. rem. **9.** 52 wk. **10.** 14 doz. **11.** 16. **12.** 37 boxes. **13.** 417 bbl. **14.** 24 hr. **15.** $125. **16.** 161 sheep. **17.** 159 coins. **18.** 45 hogs. **19.** 245 boxes. **20.** 167 bbl. **21.** 19 lb. **22.** 36 horses. **23.** $262\frac{1}{2}$ A. **24.** 5,280 ft.; 36,960 ft. **25.** $1250. **26.** 48 cd.

Exercise 25. — **1.** 18. **2.** 18. **3.** 21. **4.** 22. **5.** 21. **6.** 25. **7.** 102. **8.** 117. **9.** 128. **10.** 102. **11.** 148. **12.** 114. **13.** 129. **14.** 184. **15.** $196\frac{4}{49}$. **16.** $159\frac{13}{53}$. **17.** 342. **18.** $335\frac{15}{67}$. **19.** $483\frac{3}{69}$. **20.** $584\frac{18}{78}$. **21.** $518\frac{18}{74}$. **22.** $532\frac{9}{76}$. **23.** $747\frac{56}{83}$. **24.** $516\frac{74}{86}$. **25.** $696\frac{48}{87}$. **26.** $846\frac{17}{94}$. **27.** $475\frac{15}{95}$. **28.** $873\frac{19}{97}$. **29.** 439. **30.** 1288. **31.** 8784. **32.** 3527. **33.** 2839. **34.** 2196. **35.** $4512\frac{66}{78}$. **36.** $65,206\frac{19}{47}$. **37.** 9080. **38.** 755. **39.** 3003. **40.** 153. **41.** 5325. **42.** $6789\frac{100}{234}$. **43.** $6782\frac{12}{534}$. **44.** 1728. **45.** 1456. **46.** $3952\frac{6}{247}$. **47.** $4675\frac{15}{187}$. **48.** 1622. **49.** 1233. **50.** 954. **51.** 1573. **52.** 1102. **53.** 1242. **54.** 1238. **55.** 1340. **56.** 1117. **57.** 702. **58.** 1181. **59.** $1121\frac{380}{865}$. **60.** 1018.

Exercise 26. — **1.** 42 bags. **2.** 39 bbl. **3.** 79 bbl. **4.** 2777 yd., 28 in. rem. **5.** 241 bu. **6.** 125 da. **7.** 36°. **8.** 364 sq. yd. **9.** 36 gal. **10.** 33 chests. **11.** 325 bbl. **12.** 114 hr. **13.** 625 A. **14.** 1250 sq. mi. **15.** 31,250 sq. mi. **16.** $23\frac{7}{15}$. **17.** $75\frac{1}{2}$ nearly. **18.** 28 sec., 880 ft. rem. **19.** Nearly 47 times. **20.** 71 times nearly. **21.** More than 24 times. **22.** Between 182 and 183. **23.** $63\frac{16}{29}$ hr. **24.** $29\frac{44}{64}$ mi. per hr. **25.** $26\frac{24}{39}$ mi. per hr. **26.** $173\frac{4}{15}$ hr. **27.** $59\frac{5600}{6080}$ knots. **28.** 75.

Exercise 27. — **1.** (*a*) 36,793 A.; (*b*) 1,025,513 bu.; (*c*) $533,313; (*d*) Maine, 27 bu.; New Hampshire, $23\frac{24567}{49915}$ bu.; Vermont, 30 bu.; Massachusetts, $24\frac{906}{1712}$ bu.; Rhode Island, 28 bu.; (*e*) Maine, $ $15\frac{1453}{12109}$; New Hampshire, $ $13\frac{32096}{49915}$; Vermont, $ $14\frac{1774}{17739}$; Massachusetts, $ $16\frac{291}{1712}$; Rhode Island, $ $17\frac{25}{315}$. **2.** 21,252,370 T. **3.** (*a*) 9843, (*b*) 798. **4.** If the vertical and the horizontal sums are the same the answer is correct. **5.** (*a*) 570,900 bags, (*b*) 2,878,034 bags. **6.** (*a*) 4,449,342 bags, (*b*) 3,878,442 bags, (*c*) 1,000,408 bags. **7.** (*a*) $62,197,479. (*b*) $67,335,579. (*c*) $93,237,885. (*d*) $102,241,557. **8.** $36,070,661. **9.** (*a*) $16\frac{160}{320}$ bu., (*b*) $ $6\frac{192}{321}$. **10.** $56,084,938. **11.** 94 nearly. **12.** (*a*) 57,633,098 A.; (*b*) 1,466,520,834 bu., (*c*) $389,953,739. **13.** 55,040,040 bu. **14.** 2,508,261 bu. **15.** Than Nebraska, 742,882 A.; than Iowa, 1,016,918 A.; than Illinois, 1,637,301 A.; than Missouri, 2,351,417 A.; than Texas, 4,089,160 A.; than Indiana, 4,715,001 A.; than Georgia, 5,348,556 A.; than Tennessee, 5,361,561 A.; than Kentucky, 5,554,975 A.; than Alabama, 5,657,186 A.; than Ohio, 5,523,481 A. **16.** $4,013,587. **17.** Between 33 and 34. **18.** Between 263 and 264. **19.** $211,098,199, $75,120,755. **20.** 9,522,023 A. **21.** 952,602,031. **22.** $1,453,701. **23.** Died, 107 officers, 2803 enlisted men; 271,914 enlisted men returned.

Exercise 32. — **1.** $661.03. **2.** $878.20. **3.** $402.67. **4.** $1442.49. **5.** $386.74. **6.** $522.63. **7.** $554.34. **8.** $999.24. **9.** $1454.72. **10.** $172.925. **11.** $6052.13. **12.** $6200.34. **13.** $7034.36. **14.** $6699.34. **15.** $7561.32. **16.** $8165.21. **17.** $5466.78. **18.** $5641.

Exercise 33. — **1.** $64.95. **2.** $82.30. **3.** $215.16. **4.** $99.16. **5.** $267.30. **6.** $60.54. **7.** $9949.50. **8.** $7899.25. **9.** $243.29. **10.** $697.58. **11.** $236.37. **12.** $243.56. **13.** $191.97. **14.** $3999.02. **15.** $5228.81. **16.** $3999.43. **17.** $99.85. **18.** $400.96. **19.** $399.48. **20.** $900.08.

Exercise 34. — **1.** $639.52. **2.** $225.40. **3.** $476.94. **4.** $898.03. **5.** $3594.51. **6.** $5455.08. **7.** $8023.18. **8.** $2136.42. **9.** $5046.93. **10.** $2285.44. **11.** $359.31. **12.** $555.03. **13.** $239.76. **14.** $1076.28. **15.** $1866.95. **16.** 6399.52. **17.** $7402.50. **18.** $10,986.58. **19.** $9345. **20.** $6594.75. **21.** $6498.38. **22.** $3269.10. **23.** $93.60 **24.** $159.60. **25.** $187.65. **26.** $322.56. **27.** $195.44. **28.** $7.02. **29.** $19.50. **30.** $6075. **31.** $78,030.

Exercise 35. — **1.** $12.14. **2.** $111.41. **3.** $107.75. **4.** $36.37. **5.** $236.24. **6.** $99.72. **7.** $94.99. **8.** $36.08. **9.** $19.84.

10. $97.60. **11.** $12.18. **12.** $12.05. **13.** $11.85. **14.** $11.71. **15.** $14.35. **16.** $14.24. **17.** $13.79. **18.** $14.01. **19.** $95.03. **20.** $97.06.

Exercise 36. — **1.** 68. **2.** 69. **3.** 427. **4.** 5320. **5.** 245. **6.** 385. **7.** 198. **8.** 17. **9.** 2125. **10.** 717 sacks. **11.** 810. **12.** 563 bbl. **13.** 17,503. **14.** 396. **15.** 39. **16.** 25,000. **17.** 90,922 oz. nearly. **18.** 3025. **19.** 2625. **20.** 3246.

Exercise 37. — **1.** $3.75. **2.** 117 bills, change $75. **3.** $116.03. **4.** $4410. **5.** 36 doz. **6.** $819. **7.** 26 qt. **8.** $28. **9.** $5525. **10.** $344. **11.** $75. **12.** $30,810. **13.** $493.20. **14.** $2925. **15.** $8.25 nearly. **16.** $20.60. **17.** 931,500,000. **18.** $81.25. **19.** $311.04 **20.** $1450.

Exercise 38. — **1.** $1. **2.** 42 ft. **3.** 5 da. **4.** 10 da. **5.** 60 lb. **6.** $24, 5 sheep. **7.** $1.80, 47 oranges. **8.** 60¢ on each kind.

Exercise 39. — **1.** 1. **2.** 162. **3.** 4. **4.** 35. **5.** 27. **6.** 6. **7.** 27. **8.** 55. **9.** 3. **10.** 5. **11.** 2. **12.** 4. **13.** 20. **14.** 2. **16.** 24 cows. **17.** 42 doz. **18.** 15 da. **19.** 22 hr. **20.** 200.

Exercise 44. — **1.** $\frac{7}{2}$. **2.** $\frac{11}{3}$. **3.** $\frac{38}{7}$. **4.** $\frac{75}{8}$. **5.** $\frac{65}{7}$. **6.** $\frac{118}{11}$. **7.** $\frac{71}{8}$. **8.** $\frac{35}{6}$. **9.** $\frac{118}{12}$. **10.** $\frac{120}{11}$. **11.** $\frac{111}{10}$. **12.** $\frac{41}{4}$. **13.** $\frac{135}{7}$. **14.** $\frac{166}{9}$. **15.** $\frac{302}{11}$. **16.** $\frac{461}{12}$. **17.** $\frac{199}{5}$. **18.** $\frac{59}{8}$. **19.** $\frac{230}{11}$. **20.** $\frac{271}{16}$. **21.** $\frac{115}{8}$. **22.** $\frac{107}{7}$. **23.** $\frac{175}{9}$. **24.** $\frac{262}{9}$. **25.** $\frac{329}{12}$. **26.** $\frac{181}{13}$. **27.** $\frac{100}{7}$. **28.** $\frac{299}{8}$. **29.** $\frac{488}{11}$. **30.** $\frac{303}{10}$.

Exercise 47. — **1.** 2. **2.** 3. **3.** 2. **4.** 10. **5.** 34. **6.** $22\frac{1}{6}$. **7.** $13\frac{3}{4}$. **8.** 26. **9.** $19\frac{1}{3}$. **10.** $8\frac{2}{3}$. **11.** $44\frac{1}{3}$. **12.** 11. **13.** $10\frac{4}{5}$. **14.** $8\frac{3}{4}$. **15.** 12. **16.** 15.

Exercise 48. — **1.** $\frac{6}{9}, \frac{10}{15}, \frac{16}{24}, \frac{20}{30}, \frac{24}{36}$. **2.** $\frac{6}{8}, \frac{12}{16}, \frac{18}{24}, \frac{24}{32}, \frac{30}{40}$. **3.** $\frac{6}{10}, \frac{12}{20}, \frac{15}{25}, \frac{21}{35}, \frac{24}{40}$. **4.** $\frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \frac{20}{35}, \frac{28}{49}$. **5.** $\frac{10}{16}, \frac{15}{24}, \frac{25}{40}, \frac{30}{48}, \frac{40}{64}$. **6.** $\frac{12}{32}, \frac{18}{48}, \frac{21}{56}, \frac{27}{72}, \frac{30}{80}$. **7.** $\frac{10}{18}, \frac{15}{27}, \frac{25}{45}, \frac{35}{63}, \frac{40}{72}$. **8.** $\frac{18}{20}, \frac{45}{50}, \frac{63}{70}, \frac{72}{80}, \frac{81}{90}$. **9.** $\frac{14}{24}, \frac{28}{48}, \frac{42}{72}, \frac{49}{84}, \frac{56}{96}$. **10.** $\frac{21}{7}, \frac{30}{10}, \frac{36}{12}, \frac{60}{20}$. **11.** $\frac{20}{4}, \frac{45}{9}, \frac{70}{14}, \frac{125}{25}$. **12.** $\frac{21}{3}, \frac{56}{8}, \frac{70}{10}, \frac{84}{12}$.

Exercise 49. — **1.** $\frac{1}{2}, \frac{4}{9}, \frac{2}{3}, \frac{2}{3}, \frac{3}{8}, \frac{4}{7}, \frac{4}{5}, \frac{5}{6}, \frac{2}{3}$. **2.** $\frac{2}{3}, \frac{2}{5}, \frac{2}{3}, \frac{4}{5}, \frac{5}{12}, \frac{5}{8}, \frac{5}{6}, \frac{7}{8}$. **3.** $\frac{2}{3}, \frac{2}{3}, \frac{3}{4}, \frac{3}{4}, \frac{3}{4}, \frac{3}{5}, \frac{3}{5}, \frac{3}{5}$. **4.** $\frac{2}{9}, \frac{4}{9}, \frac{4}{9}, \frac{7}{12}, \frac{7}{12}, \frac{7}{12}, \frac{7}{12}, \frac{7}{12}$. **5.** $\frac{2}{3}, \frac{2}{3}, \frac{2}{3}, \frac{3}{4}, \frac{9}{16}, \frac{7}{9}, \frac{7}{9}, \frac{5}{12}$.

Exercise 50. — **1.** $1\frac{1}{12}$. **2.** $1\frac{11}{12}$. **3.** $1\frac{5}{12}$. **4.** $1\frac{7}{8}$. **5.** $1\frac{3}{4}$. **6.** $1\frac{1}{4}$. **7.** $\frac{5}{6}$. **8.** $1\frac{1}{15}$. **9.** $1\frac{5}{8}$. **10.** $1\frac{4}{5}$. **11.** $1\frac{4}{5}$. **12.** $1\frac{1}{3}$. **13.** $1\frac{2}{9}$. **14.** $1\frac{17}{24}$. **15.** $1\frac{37}{48}$. **16.** $2\frac{11}{16}$. **17.** $1\frac{1}{12}$. **18.** $2\frac{1}{18}$. **19.** $2\frac{1}{24}$. **20.** $1\frac{19}{24}$. **21.** $2\frac{3}{10}$. **22.** $1\frac{19}{20}$. **23.** $1\frac{9}{10}$. **24.** $1\frac{17}{20}$. **25.** $\frac{23}{24}$. **26.** $2\frac{7}{24}$. **27.** $1\frac{13}{24}$. **28.** $1\frac{11}{24}$. **29.** $2\frac{1}{6}$. **30.** $1\frac{19}{24}$. **31.** $7\frac{7}{12}$. **32.** $15\frac{1}{4}$. **33.** $8\frac{5}{8}$. **34.** $25\frac{5}{12}$. **35.** $26\frac{11}{12}$. **36.** $22\frac{5}{6}$. **37.** $17\frac{3}{5}$. **38.** $12\frac{19}{24}$. **39.** $19\frac{7}{24}$.

40. $17\frac{1}{3}$. 41. \$$25\frac{1}{20}$. 42. $14\frac{19}{32}$ hr. 43. \$$\frac{7}{10}$. 44. $1\frac{1}{12}$ hr. 45. $\frac{5}{6}$ is largest; $\frac{2}{3}$ is smallest. 46. $1\frac{1}{12}$ ft. 47. \$$1\frac{3}{20}$. 48. $1\frac{2}{3}$ qt. 49. \$$2\frac{2}{3}$. 50. $3\frac{11}{12}$ hr. 51. \$$1\frac{1}{10}$. 52. 21¢.

Exercise 51. — 1. $\frac{1}{4}$. 2. $\frac{1}{6}$. 3. $\frac{1}{2}$. 4. $\frac{1}{6}$. 5. $\frac{2}{5}$. 6. $\frac{1}{3}$. 7. $\frac{1}{2}$. 8. $\frac{1}{6}$. 9. $\frac{1}{4}$. 10. $\frac{1}{12}$. 11. $\frac{1}{3}$. 12. $\frac{1}{2}$. 13. $1\frac{1}{2}$. 14. $2\frac{1}{4}$. 15. $1\frac{5}{8}$. 16. $1\frac{3}{10}$. 17. $10\frac{1}{3}$. 18. $12\frac{1}{3}$. 19. $8\frac{5}{6}$. 20. $11\frac{5}{12}$. 21. $6\frac{1}{10}$. 22. $7\frac{1}{4}$. 23. $7\frac{5}{12}$. 24. $3\frac{1}{2}$. 25. $5\frac{1}{3}$. 26. $16\frac{1}{2}$. 27. $6\frac{1}{6}$. 28. $2\frac{7}{24}$. 29. $11\frac{11}{24}$. 30. $10\frac{5}{24}$. 31. $2\frac{5}{6}$. 32. $3\frac{11}{12}$. 33. $3\frac{11}{12}$. 34. $\frac{7}{12}$. 35. $6\frac{7}{12}$. 36. $2\frac{7}{10}$. 37. $6\frac{7}{10}$. 38. $4\frac{1}{3}$. 39. $4\frac{5}{6}$. 40. $8\frac{11}{15}$. 41. $19\frac{5}{9}$. 42. $7\frac{9}{10}$. 43. $5\frac{5}{8}$. 44. $2\frac{1}{4}$. 45. $2\frac{11}{12}$. 46. \$1.15.

Exercise 52. — 1. 12. 2. $10\frac{1}{2}$. 3. $5\frac{5}{8}$. 4. $16\frac{1}{3}$. 5. $11\frac{1}{3}$. 6. $4\frac{1}{2}$. 7. $11\frac{7}{8}$. 8. $24\frac{1}{3}$. 9. $5\frac{1}{7}$. 10. $12\frac{1}{3}$. 11. $19\frac{1}{3}$. 12. $38\frac{1}{2}$. 13. $35\frac{3}{4}$. 14. $47\frac{1}{3}$. 15. $38\frac{1}{4}$. 16. $15\frac{3}{4}$. 17. $16\frac{2}{3}$. 18. $22\frac{1}{3}$. 19. $33\frac{3}{4}$. 20. $301\frac{1}{2}$. 21. $247\frac{1}{2}$. 22. 152. 23. $335\frac{1}{3}$. 24. $247\frac{1}{2}$. 25. 243. 26. $160\frac{7}{11}$. 27. $753\frac{1}{3}$. 28. $705\frac{1}{2}$. 29. $446\frac{1}{4}$. 30. $166\frac{1}{2}$. 31. $221\frac{2}{3}$. 32. $132\frac{2}{3}$. 33. \$$4\frac{1}{2}$. 34. \$$42\frac{1}{2}$. 35. \$$15.23\frac{7}{16}$. 36. $52\frac{1}{2}$¢. 37. \$$129.76\frac{1}{2}$. 38. \$$31.87\frac{1}{2}$. 39. \$$3.84\frac{3}{4}$. 40. \$35.40. 41. \$$7.81\frac{1}{4}$. 42. 4840 sq. yd. 43. 792 in. 44. \$11.90. 45. $1036\frac{1}{2}$ mi.; 2764 mi. 46. \$7350. 47. 1792 lb. copper, 448 lb. lead. 48. Gain \$805, investment at end of year \$3105. 49. \$787.50. 50. \$135. 51. 1008 lb.

Exercise 53. — 1. $\frac{1}{3}$. 2. $\frac{3}{8}$. 3. $\frac{8}{11}$. 4. $\frac{63}{80}$. 5. $\frac{51}{200}$. 6. $\frac{19}{48}$. 7. $\frac{24}{37}$. 8. $\frac{3}{7}$. 9. $\frac{3}{8}$. 10. $\frac{5}{16}$. 11. $\frac{3}{7}$. 12. $\frac{3}{7}$. 13. $\frac{1}{14}$. 14. $\frac{1}{4}$. 15. $\frac{1}{10}$. 16. $\frac{3}{8}$. 17. $\frac{3}{10}$. 18. $\frac{1}{17}$. 19. $\frac{14}{101}$. 20. $\frac{1}{27}$. 21. $2\frac{7}{9}$. 22. $3\frac{3}{8}$. 23. $\frac{3}{4}$. 24. $\frac{3}{4}$. 25. $\frac{5}{9}$. 26. 7. 27. 9. 28. $10\frac{2}{3}$. 29. 15. 30. $3\frac{1}{9}$. 34. 22. 32. $26\frac{11}{16}$. 33. $\frac{1}{3}$. 34. $14\frac{1}{16}$. 35. $64\frac{2}{9}$. 36. 1. 37. $420\frac{1}{4}$. 38. $31\frac{1}{2}$. 39. 7. 40. 2. 41. $272\frac{44}{465}$. 42. \$$398\frac{35}{54}$. 43. \$$3524\frac{1}{4}$. 44. \$$1.93\frac{3}{4}$. 45. \$$3.65\frac{5}{8}$. 46. \$162.50. 47. \$$52\frac{1}{4}$. 48. 11 mi. 49. 135 mi. 50. \$2625. 51. \$631.25. 52. \$$45.56\frac{1}{4}$.

Exercise 54. — 1. $\frac{3}{16}$. 2. $\frac{3}{35}$. 3. $\frac{5}{48}$. 4. $\frac{5}{63}$. 5. $\frac{3}{110}$. 6. $\frac{5}{72}$. 7. $\frac{3}{28}$. 8. $\frac{3}{76}$. 9. $\frac{2}{33}$. 10. $\frac{2}{63}$. 11. $\frac{2}{75}$. 12. $\frac{5}{108}$. 13. $\frac{3}{56}$. 14. $\frac{3}{52}$. 15. $\frac{2}{39}$. 16. $\frac{2}{55}$. 17. $\frac{2}{45}$. 18. $\frac{1}{42}$. 19. 18. 20. 49. 21. 28. 22. 21. 23. 28. 24. 30. 25. 35. 26. 28. 27. $60\frac{1}{2}$. 28. 96. 29. $67\frac{1}{2}$. 30. $49\frac{1}{2}$. 31. 2. 32. $1\frac{1}{3}$. 33. $5\frac{1}{2}$. 34. $2\frac{3}{5}$. 35. $2\frac{5}{6}$. 36. $9\frac{1}{9}$. 37. $\frac{3}{8}$. 38. $\frac{17}{24}$. 39. $\frac{4}{9}$. 40. $\frac{7}{22}$. 41. $\frac{10}{39}$. 42. $\frac{5}{24}$. 43. $\frac{9}{70}$. 44. $\frac{8}{39}$. 45. $\frac{4}{15}$. 46. $\frac{7}{10}$. 47. $\frac{3}{35}$. 48. $\frac{3}{14}$. 49. $\frac{5}{28}$. 50. $\frac{4}{33}$. 51. $\frac{5}{9}$. 52. $\frac{5}{52}$. 53. $\frac{2}{17}$. 54. $\frac{3}{16}$. 55. $\frac{5}{6}$. 56. $\frac{4}{5}$. 57. $1\frac{19}{35}$. 58. $\frac{63}{64}$. 59. $\frac{9}{16}$. 60. $1\frac{4}{7}$. 61. $1\frac{4}{5}$. 62. $\frac{11}{20}$. 63. $\frac{5}{8}$. 64. $1\frac{1}{9}$. 65. $1\frac{5}{49}$. 66. $3\frac{1}{3}$. 67. $\frac{5}{8}$. 68. $2\frac{11}{13}$. 69. $\frac{8}{15}$. 70. $2\frac{2}{3}$. 71. $1\frac{29}{76}$. 72. $\frac{49}{104}$. 73. $2\frac{2}{35}$. 74. $\frac{4}{17}$. 75. $4\frac{2}{7}$. 76. $\frac{4}{19}$. 77. $\frac{7}{50}$. 78. $\frac{3}{29}$. 79. 4. 80. $7\frac{15}{23}$. 81. $\frac{1}{3}$. 82. $\frac{11}{15}$. 83. $\frac{9}{11}$. 84. $1\frac{5}{7}$.

Exercise 55. — **9.** 4. **10.** 9. **11.** 16. **12.** $5\frac{1}{3}$. **13.** 25. **14.** $8\frac{1}{8}$. **15.** 9. **16.** $\frac{2}{3}$. **17.** $\frac{1}{3}$. **18.** $1\frac{1}{3}$. **19.** $1\frac{1}{3}$. **20.** $\frac{135}{224}$. **21.** $1\frac{2}{9}$. **22.** $\frac{128}{441}$. **23.** $\frac{2}{3}$. **24.** $2\frac{8}{39}$. **25.** $4\frac{1}{2}$. **26.** $1\frac{31}{32}$. **27.** $1\frac{37}{80}$. **28.** $1\frac{8}{13}$. **29.** $\frac{7}{12}$. **30.** $1\frac{1}{9}$. **31.** 6. **32.** $1\frac{151}{275}$.

Exercise 56. — **1.** $4290. **2.** $\$2\frac{1}{2}$. **3.** $216.75. **4.** $\$137\frac{1}{3}$. **5.** $630. **6.** $575. **7.** $24.70. **8.** $68.88. **9.** $\$881\frac{2}{3}$. **10.** $7. **11.** $4.25. **12.** $8.41. **13.** $\$7.72\frac{1}{2}$. **14.** $5.75. **15.** $\$13\frac{1}{2}$. **16.** $120. **17.** 108 ft. **18.** 72. **19.** 255 T. **20.** 60. **21.** $456. **22.** $72. **23.** $4800; $1800, A's share; $1600, B's share. **24.** $4000. **25.** $4380. **26.** $164\frac{1}{2}$ A. **27.** $1680. **28.** $600. **29.** 36 marbles. **30.** $\$13\frac{5}{7}$.

Exercise 57. — **1.** $\frac{1}{12}$. **2.** $\frac{562}{6}$. **3.** $\frac{7}{8}$ greatest; $\frac{4}{5}$ least. **4.** $\frac{1}{3}$. **5.** $325. **6.** 190. **7.** 3. **8.** $1\frac{3}{7}$. **9.** 16. **10.** $3\frac{1}{20}$. **11.** $2\frac{1}{2}$. **12.** $\frac{5}{7}$ yd. **13.** $2\frac{2}{7}$. **14.** $27\frac{8}{21}$. **15.** $\frac{18}{24}$, $\frac{16}{24}$, $\frac{7}{24}$. **16.** $19\frac{9}{20}$. **17.** $840. **18.** 27 coats. **19.** A, $2025; B, $1350. **20.** $630. **21.** $960. **22.** $1260, $945. **23.** 95 sheep. **24.** $99. **25.** $6.40. **26.** $3552. **27.** 15 lb. **28.** $16.50. **29.** $\frac{7}{32}$, $\frac{1}{16}$. **30.** $3\frac{1}{2}$ hr. **31.** $12\frac{1}{2}$ hr. **32.** 38,500 gr. **33.** 2800 gr. **34.** 800,000 min. **35.** $18\frac{6}{7}$ min. **36.** $7899\frac{307}{528}$ mi. **37.** $42\frac{1}{2}$¢. **38.** 5¢. **39.** $2\frac{3}{8}$ ft. **40.** $1.20. **41.** 72¢. **42.** $4.13. **43.** $1\frac{1}{2}$ min. **44.** 720. **45.** $2. **46.** 75 T. **47.** $14\frac{2}{5}$ lb. **48.** $3200. **49.** $660. **50.** 140 T. **51.** 1120 yd. **52.** 436 min. **53.** $.60. **54.** $3.90. **55.** $57\frac{1}{3}$¢. **56.** $92.96. **57.** $61.36. **58.** $57.05. **59.** $\frac{5}{16}$. **60.** 60 sec.

Exercise 61. — **1.** 666.973. **2.** 151.987. **3.** 261.304. **4.** 1943.232. **5.** 135.148. **6.** 160.687. **7.** 282.238. **8.** 317.187. **9.** 176.483. **10.** 212.728. **11.** 281.308. **12.** 377.077.

Exercise 63. — **1.** $4.32, $13.70, $43.20, $10.08. **2.** $31.05, $22.59, $60.64, $59.29. **3.** $11.90, $34.74, $40.992, $32.85. **4.** $10.10, $40.50, $180.30, $24.71. **5.** $430, $693.70, $337.80. **6.** 31.5 A., 4 A., 30 A. **7.** .01, .12, .09, .002, .012. **8.** 502.2. **9.** 2123.2. **10.** 1806.42. **11.** 482.036. **12.** .425. **13.** 2.625. **14.** 28.56. **15.** 271.9846. **16.** .56056. **17.** 249.048. **18.** .1702. **19.** .4182. **20.** .6. **21.** 3.3642. **22.** 17.328. **23.** .20001. **24.** .9409. **25.** .4624. **26.** .139129. **27.** .811801. **28.** .644809. **29.** .480249. **30.** .001. **31.** .027. **32.** .064. **33.** .343. **34.** 1.124864. **35.** 1.191016. **36.** 1.259712. **37.** .015625. **42.** 153.9384. **43.** 19,7061,258. **44.** 15,205,344. **45.** 1110 mi., 66,600 mi. **46.** 8796 ft., 527,760 ft. **47.** .2944 in.

Exercise 64. — **1.** 9.15125. **2.** .8625. **3.** 3.227. **4.** .199. **5.** .9128. **6.** .490875. **7.** .003. **8.** 1.47433. **9.** 1.174. **10.** .657. **11.** .925. **12.** 1.142875. **13.** .02468. **14.** .12201. **15.** .1638. **16.** .7843875.

17. .912. **18**. .9172. **19**. .03866. **20**. .07484. **21**. .142857. **22**. 1.0535. **23**. 1.0446. **24**. .2478.

Exercise 65. — **1**. 2.925. **2**. .024. **3**. 8.6875. **4**. .2058. **5**. .3596. **6**. 2.62. **7**. 3.725. **8**. 2.531. **9**. 1.35. **10**. 3.36. **11**. 6.3234. **12**. .5. **13**. 40. **14**. 62.5. **15**. 125. **16**. 50. **17**. 15. **18**. 50. **19**. 160. **20**. 12,500. **21**. 8000. **22**. 500. **23**. 25. **24**. .5. **25**. 1000. **26**. 1250. **27**. 1000. **28**. 50. **29**. 250. **30**. 250. **31**. 25. **32**. 2.5. **33**. .0125. **34**. .00125. **35**. .0004. **36**. .00666+. **37**. .125. **38**. .15. **39**. 32. **40**. .0027. **41**. 48.7. **42**. 8.03. **43**. .0904. **44**. .708. **45**. 1.012. **46**. .01014. **47**. .517. **48**. .01054. **49**. .00377. **50**. .0365.

Exercise 66. — **1**. 1.1318. **2**. 1.1038. **3**. 1.1597. **4**. .1189. **5**. .1217. **6**. .0122. **7**. .1539. **8**. 1.5394. **9**. .0155. **10**. .1629. **11**. .2079. **12**. .1551. **13**. .1629. **14**. .198. **15**. .9801. **16**. .7056. **17**. .6889. **18**. .0089. **19**. .049. **20**. .2879. **21**. .3869. **22**. .3694. **23**. .0826. **24**. .8295. **25**. .4377. **26**. 1975. **27**. .0339. **28**. .0074. **29**. .0906. **30**. .0213. **31**. .0558. **32**. 12.566. **33**. 5.7737. **34**. 60.1047. **35**. .3715.

Exercise 67. — **1**. .5, .75, .8, .3125, .24, .21875, .171875, .6171875. **2**. .85, .38, .1025, .415, .536, .348. **3**. .3, .17, .019, .0011, .00101, .001279. **4**. .2875, .425, .45, .74, .206, .02725. **5**. .25, .025, .0025, .0065, .00775, .01875. **6**. .021875, .36875, .1234375, .10375, .0872. **7**. .3333, .1429, .1111, .0909, .0833, .0769, .0714, .0667, .0588. **8**. .1852, .4783, .3214, .1290, .4571, .0959. **9**. .2424, .0795, .3077, .4286, .4737, .4762. **10**. .3846, .3529, .0808, .1311, .0232, .0522. **11**. .5556, .1313, .0141, .0211, .0320, .0082.

Exercise 68. — **1**. $\frac{3}{10}$, $\frac{4}{5}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{16}$. **2**. $\frac{7}{100}$, $\frac{1}{80}$, $\frac{7}{800}$, $\frac{1}{16}$, $\frac{3}{40}$. **3**. $\frac{9}{1000}$, $\frac{9}{400}$, $\frac{9}{80}$, $\frac{11}{400}$. **4**. $\frac{9}{125}$, $\frac{13}{1250}$, $\frac{7}{200}$, $\frac{119}{10000}$, $\frac{3}{80}$. **5**. $\frac{18}{125}$, $\frac{63}{1250}$, $\frac{48}{625}$, $\frac{81}{500}$, $\frac{7}{625}$. **6**. $\frac{36}{125}$, $\frac{11}{625}$, $\frac{13}{400}$, $\frac{7}{400}$, $\frac{17}{40}$. **7**. $\frac{23}{80}$, $\frac{27}{80}$, $\frac{41}{80}$, $\frac{59}{80}$.

Exercise 69. — **1**. 4 hr. **2**. 12 hr. **3**. 6.6667 hr. **5**. 14 hr. **6**. 50.1875 mi. per hr. **8**. 53.33 mi. per hr. **9**. 54.13 mi. per hr. **10**. 29.13 mi. per hr. **11**. 546.45 mi. per da., 22.77 mi. per hr. **12**. 1233.3 times. **13**. \$2400. **14**. \$280. **15**. 850 cu. ft. **16**. 420 cu. ft. **17**. 7.48 gal. **18**. \$38.28. **19**. \$26.25. **20**. \$17.55. **21**. \$422.50. **22**. \$8.00. **23**. \$4320. **24**. \$1.75. **25**. 682.5 bu. **26**. 186,325.3 mi. per sec., 16.7 sec. **27**. 14.484 kilometers.

Exercise 71. — **1**. $\frac{2}{3}$, $\frac{7}{22}$, $\frac{9}{11}$, $\frac{15}{22}$. **2**. $\frac{2}{5}$, $\frac{1}{3}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$. **3**. $\frac{1}{7}$, $\frac{333}{700}$, $\frac{6}{7}$, $\frac{4}{9}$, $\frac{1}{15}$, $\frac{7}{15}$, $\frac{11}{15}$. **4**. $\frac{1}{18}$, $\frac{5}{18}$, $\frac{7}{18}$, $\frac{9}{18}$, $\frac{11}{18}$, $\frac{13}{18}$, $\frac{15}{18}$. **5**. $\frac{9}{200}$, $\frac{5}{24}$, $\frac{11}{24}$, $\frac{17}{24}$, $\frac{23}{24}$, **6**. $\frac{1}{11}$, $\frac{1}{13}$, $\frac{6}{7}$, $\frac{17}{18}$, $\frac{23}{60}$, $\frac{29}{36}$. **7**. $\frac{7}{33}$, $\frac{2}{3}$, $\frac{3}{7}$, $\frac{4}{7}$. **8**. $\frac{113}{1650}$, $\frac{1}{45}$, $\frac{1}{65}$, $\frac{4}{55}$. **9**. $\frac{7}{20}$,

$\frac{3}{8}$, $\frac{5}{8}$. **10.** $\frac{1}{20}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{21}{110}$, $\frac{21}{40}$, $\frac{147}{440}$. **11.** $\frac{33}{38}$. **12.** $\frac{7}{8}$, $\frac{7}{10}$, $\frac{9}{10}$, $\frac{1}{70}$, $\frac{1}{80}$, $\frac{2}{25}$, .03. **13.** 14 da. **14.** 10 da. **15.** 4 da. **16.** 84 ¢. **17.** 93¾ ¢. **18.** $3.75. **19.** 9 lb. **20.** 19¼ lb. **21.** 21 cans. **22.** 11¼ lb. **23.** $\frac{2}{3}$. **24.** $\frac{7}{8}$. **25.** $\frac{1}{3}$. **26.** $\frac{3}{20}$. **27.** $\frac{12}{35}$. **28.** $16. **29.** $\frac{7}{20}$. **30.** A's gain, $2030; B's gain, $1470.

Exercise 72. — **1.** 12,500 sq. ft., 1388$\frac{8}{9}$ sq. yd. **2.** 4840 sq. yd. **3.** 4356 sq. ft., 484 sq. yd. **4.** 1296 sq. in. **5.** 3,097,600. **6.** 16 A. **7.** 288 bu., $242.88. **8.** 4096 bu., $1996.80. **9.** 71.6 A. **10.** 1425.6 thousand. **11.** 8640 tiles. **12.** 62.2223 sq. yd., 83.5556 sq. yd. **13.** 413.33 sq. yd. **14.** 149.333 sq. yd. **15.** 1966$\frac{2}{3}$ sq. yd. **16.** 6442$\frac{2}{3}$ sq. yd., 2512 ft. **17.** 14,400. **18.** (*a*) 15,762.22 sq. ft. (*b*) 21,112.5 sq. ft. (*c*) 4430.54 sq. ft. (*d*) 1291.79 ft. (*e*) 16,032.99 sq. ft. (*f*) 10,911.25 sq. ft. (*g*) 17,459 sq. ft. (*h*) 80,711.38 sq. ft. **19.** (*a*) 720.6 sq. yd. (*b*) 849.58 sq. yd. (*c*) 912.88 sq. yd. (*d*) 1626.31 sq. yd. (*e*) 1146.23 sq. yd.

Exercise 73. — **1.** $58.48. **2.** $136.15. **3.** $16.35. **4.** $31.42. **5.** $1666.86.

Exercise 75. — **1.** 4 cwt. 87 lb. 8 oz. **2.** 6 cwt. 10 lb. 3 oz. **3.** 2 T. 7 cwt. 18 lb. 9 oz. **4.** 31 T. 5 cwt. **5.** 3 T. 19 cwt. 87 lb. **6.** 16 T. 721 lb. **7.** 112 T. **8.** 4$\frac{13}{28}$ T. **9.** 41$\frac{83}{112}$ T.

Exercise 76. — **1.** 608,000 oz. **2.** 28,400 lb. **3.** 34,300 lb. **4.** 803,200 oz. **5.** 7502 lb. **6.** 9158 lb. **7.** 16,273 lb. **8.** 56 short tons. **9.** $81. **10.** 700 long tons.

Exercise 78. — **1.** 168 in. **2.** 3972 in. **3.** 9680 yd. **4.** 12,980 yd. **5.** 18,209 yd. **6.** 784,520 yd. **7.** 13,622.4 yd. **8.** 2672 rd. **9.** 3927 ft. **10.** 63,360 in. **11.** 42,768 in. **12.** 1056.

Exercise 79. — **1.** 1 rd. 1 yd. 1 ft. 4 in. **2.** 23 rd. 1 yd. 1 ft. 6 in. **3.** 1 mi. 3 rd. 1 yd. 10 in. **4.** 7611.111 yd. **5.** 32 mi. **6.** 132 knots. **7.** 190 mi. **8.** 16.95 mi. **9.** 616 ch. **10.** 16,500 ft. **11.** 22 ch. **12.** 7.92 in. 134.64 in. **13.** Everest, 5.493 mi.; Aconcagua, 4.330 mi.; Ararat, 3.281 mi.; Blanc, 2.989 mi.; Fremont, 2.612 mi.; McKinley, 3.876 mi.; Logan, 3.701 mi.; Orizaba, 3.469 mi.; Pikes Peak, 2.672 mi.

Exercise 81. — **1.** 8 pt. **2.** 8 gi. **3.** 32 gi. **4.** $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$. **5.** 16 pt., 32 pt., 48 pt. **6.** 32 qt. **7.** 64 pt. **8.** $\frac{1}{64}$. **9.** 33.6 cu. in. **10.** 28.875 cu. in. **11.** .86. **12.** 9.45 cu. in. **13.** 6 qt. **14.** 88 qt. **15.** 17 qt. **16.** 98.1 nearly.

Exercise 85. — 1. 102,400 sq. rd. **2.** 1600 A. **3.** 522,720 sq. ft. **4.** 7350¼ sq ft. **5.** 16,665 rd. **6.** 42,900 ft. **7.** 4840 yd. **8.** 6600 yd. **9.** 11,220 ft. **10.** 675 cu. ft. **11.** 1046 cu. ft. **12.** 816,480 cu. in. **13.** 506¼ cu. ft. **14.** 944,784 cu. in. **15.** 10 qt. **16.** 23 qt. **17.** 25 pt. **18.** 57 pt. **19.** 154 pt. **20.** 118 pt. **21.** 680 gi. **22.** 86 gal. **23.** 131 pt. **24.** 119 gi. **25.** 35,380 lb. **26.** 34,500 lb. **27.** 66,250 lb. **28.** 28,750 lb. Troy. **29.** 350,000 lb. Troy. **30.** 128 qt. **31.** 124 qt. **32.** 116 qt. **33.** 56 qt. **34.** 59 qt. **35.** 1208 pt. **36.** 838 pt. **37.** 215,042 cu. in. **38.** 2079 cu. in. **39.** 9.31 gal. nearly. **40.** 13,728 ft. **41.** 22.110 ft. **42.** 6270 yd. **43.** 2970 ft. **44.** 8250 ft. **45.** 9504 in.

Exercise 86. — 1. 66,020″. **2.** 324,000″. **3.** 43,510″. **4.** 855′. **5.** 27,900″. **6.** 433,080.″ **7.** 4545′. **8.** 15,247.5′. **9.** 163,820″. **10.** 280′, 1130′. **11**. 525,600 min. **12.** 1,578,240 min. **13.** 41,760′. **14.** 129,600 min.; 131,040 min. **15.** 31,556,927 sec. **16.** 36,892,800 ti.; 147,268,800 ti. **17.** 311,760 min. **18.** 584 da. **19.** 960 hr. **20.** 178,826.4 hr. **21.** 2922 da. **22.** 48°. **23.** 174°. **24.** 18,262 da. **25.** 18,262 da.

Exercise 87. — 1. 135 sq. rd. 12 sq. yd. 1 sq. ft. 62 sq. in. **2.** 53 sq. rd. 17 sq. yd. 3 sq. ft. 17 sq. in. **3.** 2 A. 125 sq. rd. 13 sq. yd. 2 sq. ft. 84 sq. in. **4.** 29 gal. 1 qt. **5.** 31 bu. 1 pk. **6.** 432 gal. **7.** 17 cwt. 50 lb. **8.** 1 cu. yd. 729 cu. in. **9.** 1566⅔ bu. **10.** 1031¼ bu. of 32 lb. **11.** 2 wk. 20 hr. 31 min. 36 sec. **12.** 2 wk. 12 hr. 53 min. 20 sec. **13.** 3 wk. 2 da. 3 hr. 33 min. 20 sec. **14.** 51 mi. 245 rd. 3 yd. 1 ft. 10 in. **15.** 85 mi. 77 rd. 4 yd. 0 ft. 10 in. **16.** 8 cu. yd. 11 cu. ft. 744 cu. in. **17.** 100 T. 31 lb. 4 oz. **18.** 302 rd. 2 ft. **19.** 32 mi. **20.** 31,250 sq. mi. **21.** 1875 A. 100 sq. rd. **22.** 70 lb. **23.** 7926.59 mi. **24.** 7899.58 mi. **25.** 27.01 mi. **26.** 24,902.18 mi. ; 69.407 mi.; 1.157 mi. **27.** 186,325 mi. **28.** 5.183 mi.; 5.682 mi.; 4.773 mi.; 3.523 mi.

Exercise 88. — 1. 43 yd. 2 ft. 4 in. **2.** 1232 rd. 3 yd. 1 ft. 6 in. **3.** 379 mi. 184 rd. **4.** 383 yd. 1 ft. 9 in. **5.** 63 yd. 2 ft. 9 in. **6.** 1132 rd. 2 yd. 1 ft. 6 in. **7.** 366 A. 140 sq. rd. **8.** 770 A. 151 sq. rd. **9.** 1191 A. 56 sq. rd. **10.** 177 sq. yd. 4 sq. ft. 59 sq. in. **11.** 172 sq. yd. 1 sq. ft. 60 sq. in. **12.** 77 sq. rd. 26 sq. yd. 108 sq. in. **13.** 16 gal. 3 qt. **14.** 44 gal. 3 qt. 1 pt. **15.** 42 bu. 3 pk. 4 qt. **16.** 38 bu. 2 pk. 3 qt. **17.** 77 gal. 3 qt. **18.** 153 bu. 3 pk. 5 qt. **19.** 117 bu. 3 pk. 3 qt. **20.** 183 bu. 1 pk. 7 qt. **21.** 147 T. 1379 lb. **22.** 667 lb. 4 oz. **23.** 546 lb. 13 oz. **24.** 77 T. 628 lb. **25.** 272 lb. 7 oz. **26.** 225 T. 1908 lb. **27.** 218 da. 18 hr. 30 min. **28.** 241 wk. 6 da. 13 hr. **29.** 85 hr. 14 min. 52 sec. **30.** 141 da. 13 hr. 47 min. **31.** 46 wk. 5 da. 2 hr. **32.** 54 hr. 20 min. 3 sec. **33.** 64 cu. ft. 1032 cu. in. **34.** 307 cu. yd. 9 cu. ft.

35. 362 cu. yd. **36.** 77 cu. ft. 1022 cu. in. **37.** 197 cu. yd. **38.** 67.2 cu. in., 57.75 cu. in., 537.605 cu. in. **39.** 997.14 oz.

Exercise 89. — **1.** 79° 2′ 17″. **2.** 8° 0′ 56″. **3.** 51° 54′ 25″. **4.** 117° 45′ 25″. **5.** 5° 54′ 45″. **6.** 66° 46′ 6″. **7.** 85° 6′ 10″. **8.** 55° 57′. **9.** 60° 5′ 43″. **10.** 78° 59′ 50″. **11.** 43° 5′ 48″. **12.** 1 bu. 3 pk. 7 qt. **13.** 8 gal. 0 qt. 1 pt. **14.** 6 gal. 0 qt. 1 pt. **15.** 5 gal. 2 qt. 1 pt. **16.** 19 bu. 2 pk. 5 qt. **17.** 9 bu. 1 pk. 5 qt. **18.** 19 gal. 2 qt. 1 pt. **19.** 21 gal. 0 qt. 1 pt. **20.** 3 qt. **21.** 28 qt. **22.** 120 qt. **23.** 1 da. 20 hr. 57 min. **24.** 6 da. 17 hr. 51 min. **25.** 11 hr. 40 min. **26.** 15 hr. 5 min., also 14 hr. 5 min. **27.** 7 da. 13 hr. 44 min., 8 da. 3 hr. 56 min., 15 da. 3 hr. 3 min. **28.** 6 min. 19 sec. **29.** 16 min. 18 sec. **30.** 33 min. **31.** 2 hr.

Exercise 90. — **3.** 4 yr. 6 mo. 28 da. **4.** 6 yr. 4 mo. 29 da. **5.** Milton, 65 yr. 10 mo. 29 da.; Pope, 56 yr. 0 mo. 9 da.; Shakespeare, 52 yr.; Burke, 67 yr. 5 mo. 27 da.; Lee, 63 yr. 8 mo. 23 da.; Grant, 63 yr. 2 mo. 26 da.; Goldsmith, 45 yr. 4 mo. 24 da; Franklin, 84 yr. 3 mo.; Hamilton, 47 yr. 6 mo. 1 da.; Longfellow, 75 yr. 0 mo. 27 da.; Newman, 89 yr. 5 mo. 20 da.; Gladstone, 88 yr. 5 mo. 10 da.

Exercise 91. — **1.** 42 yd. 2 ft. 3 in. **2.** 72 yd. 1 ft. 3 in. **3.** 79 yd. 2 ft. 4 in. **4.** 26 bu. 1 qt. **5.** 53 bu. 1 pk. **6.** 59 gal. 1 qt. **7.** 79 gal. 2 qt. **8.** 27 bu. 3 pk. 1 qt. **9.** 146 T. 800 lb. **10.** 118 T. 1483 lb. **11.** 171 T. 540 lb. **12.** 87° 51′ 28″. **13.** 249° 38′. **14.** 138 da. 20 hr. 6 min. **15.** 99 da. 2 hr. 24 min. **16.** 118 sq. yd. 8 sq. ft. 81 sq. in. **17.** 1527 ch. 30 l. **18.** 399 ch. 36 l. **19.** 779 A. 40 sq. rd. **20.** 166 A. 137 sq. rd. **21.** 151 sq. yd. 5 sq. ft. 36 sq. in. **22.** 129 T. **23.** 254° 18′ 36″. **24.** 11,250 lb. **25.** 75,240 ft.

Exercise 92. — **1.** 2 T. 877 lb. **2.** 10 T. 699⅔ lb. **3.** 2 yd. 1 ft. 3 in. **4.** 13° 19′ 19$\frac{2}{7}$″. **5.** 7° 1′ 11⅙″. **6.** 10 ch. 94 l. **7.** 5 ch. 73 l. **8.** 5 yd. 2 ft. 5⅓ in. **9.** 11 yd. 0 ft. 4 in. **10.** 2 bu. 1 pk. 2 qt. **11.** 1875 lb. **12.** 10 times. **13.** 240 times. **14.** 8000 l. **15.** 6400. **16.** 1609.35. **17.** 3.1416. **18.** 4° 5′ 27$\frac{3}{11}$. **19.** 264.

Exercise 93. — **1.** 1750 lb. **2.** 9′. **3.** 18 hr. 40 min. **4.** 469 lb. **5.** 22 hr. 48 min. **6.** 12 qt. **7.** 97,152 ft. **8.** 1120 yd. **9.** 60 rd. **10.** 945′. **11.** 243 da. 8 hr. **12.** 114 da. 1 hr. 30 min. **13.** 1 pk. 6 qt. 3.2 gi. **14.** 2 qt. 1 pt. 1⅓ gal. **15.** 136 sq. rd. **16.** 2 sq. ft. 65.808 sq. in. **17.** 104 rd. 5 yd. 10.08 in. **18.** 60 ch. 48 l., or 241 rd. 5 yd. 2.16 in. **19.** 205 da. 7 hr. 30 min. **20.** ⅗. **21.** .8. **22.** 210 da. 10 hr. 48 min. **23.** 1821.6 ft. **24.** 3740 sq. yd.

25. 4290 sq. yd. **26.** 26 qt. **27.** $3\frac{1}{2}$ pt. **28.** 4180 ft. **29.** 31 lb. 4 oz. **30.** 20 hr. 30 min. **31.** 6 hr. 21 min. 36 sec. **32.** 67° 30′. **33.** 100° 48′.

Exercise 95. — **1.** .8333. **2.** .16667 nearly. **3.** .06667. **4.** .375. **5.** .4375. **6.** .5625. **7.** 796875. **8.** .09375. **9.** .195. **10.** $\frac{13}{360}$. **11.** 4.275. **12.** $\frac{25}{1584}$. **13.** $\frac{1}{8}$. **14.** $\frac{1}{16}$. **15.** $\frac{7}{32}$. **16.** .656. **17.** .15625. **18.** .1111. **19.** .25. **20.** 1.12. **21.** .893 nearly. **22.** .165. **23.** .03, .024, .016. **24.** $\frac{3}{8}$. **25.** $\frac{3}{32}$. **26.** .2. **27.** .0666+. **28.** .25. **29.** .16667, $\frac{1}{6}$. **30.** .25. **31.** .2. **32.** .6. **33.** .05. **34.** $\frac{1}{5}$. **35.** .241, .616, 1.882, 11.87.

Exercise 96. — **1.** 1 A. **2.** 22 A. **3.** 2.492 A. **4.** 77 A. **5.** $1493\frac{1}{3}$ A.; \$ $37,333\frac{1}{3}$. **6.** 3.6 A. **7.** 1200 A.

Exercise 97. — **3.** \$1200. **4.** \$10,800. **5.** \$6720. **6.** \$3600. **7.** \$5000.

Exercise 98. — **1.** 504 cu. ft. **2.** 72 cu. yd. **3.** 9216 cu. ft. **4.** 3150 cu. yd. **5.** 13,824 gal. **6.** 115.71 bu. **7.** 560 cu. yd. **8.** 3520 cu. yd. **9.** 270 boxes. **10.** $1,251,555\frac{5}{9}$ cu. yd. **11.** 24.48 T. **12.** $1481\frac{2}{3}$ lb. **13.** 120 T. **14.** 212 lb. $\frac{1}{2}$ oz. **15.** 1728 boxes. **16.** 45 cd. **17.** 128.75 T.

Exercise 100. — **1.** 8. **2.** 8. **3.** 9. **4.** 12. **5.** 15. **6.** 15. **7.** 18. **8.** 36. **9.** 12. **10.** 12. **11.** 7. **12.** 13. **13.** 19. **14.** 23. **15.** 24. **16.** 28. **17.** 10. **18.** 15. **19.** 12. **20.** 13. **21.** 12. **22.** 18. **23.** 16. **24.** 21. **25.** 32. **26.** 36. **27.** 44. **28.** 63. **29.** 48. **30.** 54. **31.** 56. **32.** 84.

Exercise 101. — **1.** 2, 3, 4, 6, 12. **2.** 2, 4, 8. **3.** 3, 9. **4.** 3, 9. **5.** 2, 3, 5, 6, 10, 15, 30.

Exercise 102. — **1.** 7. **2.** 20 ft. **3.** 2 lb. **4.** 120 lb. **5.** 80 lb. **6.** 33 mi. **7.** \$40. **8.** 13 ft.

Exercise 103. — **1.** 111. **2.** 39. **3.** 19. **4.** 23. **5.** 13. **6.** 101. **7.** 131. **8.** 137. **9.** 151. **10.** 161. **11.** 37. **12.** 59. **15.** 7. **16.** 3. **17.** 3. **18.** 3.

Exercise 105. — **1.** 80. **2.** 42. **3.** 180. **4.** 90. **5.** 147. **6.** 140. **7.** 168. **8.** 108. **9.** 102. **10.** 144. **11.** 480. **12.** 420. **13.** 160. **14.** 78. **15.** 336. **16.** 480. **17.** 756. **18.** 385. **19.** 270. **20.** 216. **21.** 375. **22.** 216. **23.** 180. **24.** 576. **25.** 180. **26.** 210. **27.** 210. **28.** 420. **29.** 16,800. **30.** 720. **31.** 300. **32.** 180. **33.** 630. **34.** 280. **35.** 144. **36.** 840. **37.** 600. **38.** 2002. **39.** 180. **40.** 210. **41.** 240. **42.** 420. **43.** 108. **44.** 720. **45.** 360.

Exercise 106. — **1.** 124. **5.** 240. **6.** 12. **7.** 90. **8.** 4 wk. **9.** 24. **10.** In 1 hr. **11.** 8, 24, 40, 120. **13.** L. C. M. 2,425,500. **15.** 66 mi. **19.** 3300 ft.

Exercise 107. — 1. $\frac{2}{3}$. 2. $\frac{5}{7}$. 3. $\frac{5}{7}$. 4. $\frac{11}{18}$. 5. $\frac{3}{4}$. 6. $\frac{2}{5}$. 7. $\frac{3}{7}$. 8. $\frac{4}{11}$. 9. $\frac{4}{13}$. 10. $\frac{5}{8}$. 11. $\frac{9}{10}$. 12. $\frac{3}{5}$. 13. $\frac{2}{3}$. 14. $\frac{7}{8}$. 15. $\frac{7}{9}$. 16. $\frac{5}{9}$. 17. $\frac{5}{12}$. 18. $\frac{6}{13}$. 19. $\frac{11}{14}$. 20. $\frac{3}{4}$. 21. $\frac{4}{5}$. 22. $\frac{7}{13}$. 23. $\frac{3}{4}$. 24. $\frac{2}{3}$. 25. $\frac{5}{8}$. 26. $\frac{11}{14}$. 27. $\frac{2}{9}$. 28. $\frac{2}{5}$. 29. $\frac{3}{7}$. 30. $\frac{2}{11}$. 31. $\frac{5}{12}$. 32. $\frac{7}{13}$.

Exercise 108. — 1. $\frac{1}{4}$. 2. $\frac{3}{4}$. 3. $\frac{17}{20}$. 4. $\frac{3}{8}$. 5. $\frac{5}{8}$. 6. $\frac{7}{20}$. 7. $\frac{13}{20}$. 8. $\frac{17}{20}$. 9. $\frac{5}{16}$. 10. $\frac{3}{16}$. 11. $\frac{7}{16}$. 12. $\frac{9}{16}$. 13. $\frac{11}{16}$. 14. $\frac{13}{16}$. 15. $\frac{15}{16}$. 16. $\frac{1}{40}$. 17. $\frac{3}{4}$. 18. $\frac{1}{2}$. 19. $\frac{1}{4}$. 20. $\frac{1}{11}$. 21. $\frac{1}{11}$. 22. $\frac{1}{8}$. 23. $\frac{1}{48}$. 24. $\frac{1}{20}$. 25. $\frac{1}{8}$. 26. $\frac{9}{16}$. 27. $\frac{5}{16}$.

Exercise 113. — 1. $\frac{10}{20}$, $\frac{5}{20}$, $\frac{4}{20}$. 2. $\frac{21}{42}$, $\frac{14}{42}$, $\frac{6}{42}$. 3. $\frac{20}{24}$, $\frac{18}{24}$, $\frac{21}{24}$. 4. $\frac{30}{75}$, $\frac{25}{75}$, $\frac{12}{75}$. 5. $\frac{12}{18}$, $\frac{9}{18}$, $\frac{16}{18}$. 6. $\frac{15}{120}$, $\frac{10}{120}$, $\frac{18}{120}$. 7. $\frac{100}{180}$, $\frac{135}{180}$, $\frac{36}{180}$. 8. $\frac{63}{252}$, $\frac{144}{252}$, $\frac{224}{252}$. 9. $\frac{176}{264}$, $\frac{165}{264}$, $\frac{216}{264}$. 10. $\frac{273}{429}$, $\frac{286}{429}$, $\frac{264}{429}$. 11. $\frac{18}{24}$, $\frac{2}{24}$, $\frac{9}{24}$. 12. $\frac{24}{36}$, $\frac{27}{36}$, $\frac{32}{36}$. 13. $\frac{225}{360}$, $\frac{200}{360}$, $\frac{252}{360}$. 14. $\frac{39}{468}$, $\frac{36}{468}$, $\frac{130}{468}$. 15. $\frac{30}{120}$, $\frac{72}{120}$, $\frac{100}{120}$, $\frac{105}{120}$.

Exercise 115. — 1. $\frac{18}{25}$. 2. $\frac{5}{27}$. 3. $\frac{44}{63}$. 4. $\frac{2}{55}$ 5. $\frac{7}{22}$. 6. $1\frac{7}{45}$. 7. $\frac{29}{72}$. 8. $\frac{59}{120}$. 9. $1\frac{7}{15}$. 10. $\frac{41}{48}$. 11. $\frac{577}{630}$. 12. $1\frac{317}{360}$. 13. $\frac{173}{240}$. 14. $\frac{193}{280}$. 15. $8\frac{47}{54}$. 16. $5\frac{17}{144}$. 17. $20\frac{1}{20}$. 18. $7\frac{11}{12}$. 19. $5\frac{11}{48}$. 20. $\frac{47}{48}$. 21. $6\frac{139}{840}$. 22. $\frac{3}{8}$. 23. $\frac{241}{250}$. 24. $\frac{59}{60}$. 25. $11\frac{149}{360}$. 26. $3\frac{121}{216}$. 27. $9\frac{115}{216}$. 28. $13\frac{53}{144}$. 29. $10\frac{25}{36}$. 30. $9\frac{95}{336}$.

Exercise 117. — 1. $2\frac{3}{4}$. 2. $5\frac{1}{12}$. 3. $4\frac{17}{22}$. 4. $5\frac{13}{24}$. 5. $3\frac{1}{44}$. 6. $2\frac{17}{30}$. 7. $10\frac{13}{18}$. 8. $9\frac{17}{18}$. 9. $2\frac{23}{30}$. 10. $2\frac{13}{18}$. 11. $11\frac{1}{24}$. 12. $11\frac{1}{42}$. 13. $2\frac{17}{28}$. 14. $\frac{11}{20}$. 15. $\frac{57}{100}$. 16. $\frac{1}{132}$. 17. $\frac{1}{36}$. 18. $\frac{5}{72}$. 19. $\frac{1}{90}$. 20. $\frac{1}{30}$. 21. $\frac{1}{132}$. 22. $\frac{7}{120}$. 23. $\frac{1}{90}$. 24. $3\frac{13}{14}$. 25. $\frac{23}{36}$. 26. $2\frac{25}{36}$. 27. $2\frac{35}{48}$. 28. $\frac{67}{72}$. 29. $1\frac{53}{60}$. 30. $3\frac{5}{6}$. 31. $1\frac{11}{12}$. 32. \$ $1272\frac{1}{2}$. 33. $16\frac{5}{24}$. 34. $16\frac{5}{12}$. 35. $9\frac{7}{10}$. 36. $\frac{769}{1000}$.

Exercise 118. — 1. $1\frac{3}{7}$. 2. 3. 3. $\frac{1}{2}$. 4. 1. 5. $\frac{1}{2}$. 6. $1\frac{17}{40}$. 7. $1\frac{2}{7}$. 8. 8. 9. $\frac{7}{8}$. 10. 20. 11. 1. 12. $8\frac{1}{15}$. 13. $2\frac{1}{4}$. 14. $17\frac{1}{2}$. 15. 5. 16. $\frac{5}{16}$. 17. $\frac{7}{12}$. 18. $\frac{299}{400}$. 19. $\frac{11}{16}$. 20. $1\frac{7}{10}$. 21. $12\frac{3}{11}$. 22. $15\frac{15}{16}$. 23. $1\frac{1}{12}$. 24. 0. 25. 0. 26. $3\frac{3}{8}$.

Exercise 119. — 1. \$2360. 2. 380. 3. 3750 mi. 4. 3540 mi. 5. 1899 mi. 6. 900 mi. 7. 9112 mi. 8. 8995 mi. 9. 500 mi. 10. 2355 mi. 11. 2030 mi. 12. 605 mi. 13. \$51,937,925. 14. \$730,377. 15. \$2,944,492. 16. \$.238. 17. \$.498. 18. 19.3¢. 19. 1.09375. 20. 305,000 T.

Exercise 121. — 1. $16\frac{1}{2}$. 2. 21. 3. 32. 4. $21\frac{1}{3}$. 5. $17\frac{1}{2}$. 6. $1\frac{1}{2}$. 7. $1\frac{4}{5}$. 8. $\frac{22}{35}$. 9. $1\frac{7}{8}$. 10. $\frac{7}{8}$. 11. 14. 12. $1\frac{1}{3}$. 13. $6\frac{6}{11}$. 14. $\frac{1}{40}$. 15. $\frac{2}{5}$. 16. $2\frac{4}{5}$. 17. $\frac{3}{5}$. 18. $\frac{7}{15}$. 19. $1\frac{12}{25}$. 20. $1\frac{151}{272}$. 21. $3\frac{1}{9}$. 22. $\frac{459}{15386}$. 23. $\frac{1}{4}$. 24. $\frac{3}{10}$. 25. $\frac{1}{4}$. 26. $3\frac{1}{3}$.

Exercise 122. — **1.** $\frac{3}{25}$. **2.** $\frac{1}{6}$. **3.** $\frac{2}{5}$. **4.** $\frac{3}{16}$. **5.** $\frac{9}{16}$. **6.** $\frac{5}{6}$. **7.** $1\frac{23}{32}$. **8.** $1\frac{3}{7}$. **9.** $\frac{5}{9}$. **10.** $\frac{5}{12}$. **11.** $\frac{5}{8}$. **12.** $\frac{11}{24}$. **13.** $\frac{14}{15}$. **14.** $\frac{13}{15}$. **15.** $\frac{2}{5}$. **16.** $1\frac{1}{2}$ **17.** $1\frac{1}{4}$. **18.** $1\frac{5}{7}$. **19.** $\frac{1}{40}$. **20.** $1\frac{1}{2}$.

Exercise 124. — **1.** 24 da. **2.** 114 T. **3.** $10\frac{1}{2}$ hr. **4.** $49\frac{1}{2}$ yr. **5.** \$$4233\frac{1}{3}$, \$$1693\frac{1}{3}$. **6.** \$200. **7.** \$300. **8.** $1\frac{1}{14}$. **9.** 10 and 15. **10.** $\frac{1}{3}$, 81 cattle. **11.** 75 T. **12.** 165. **13.** \$340. **14.** \$6300. **15.** 1989. **16.** 1600. **17.** $1\frac{3}{8}$ hr. **18.** 30 mi., 6 hr. 56 min. **19.** $37\frac{1}{2}$ ft. **20.** $\frac{1}{3}$. **21.** \$2. **22.** 48 oranges. **23.** $41\frac{1}{4}$ yd. **24.** 60 ¢ per lb. **25.** 91 ¢ per yd. **26.** \$201.18. **27.** 42 pencils.

Exercise 126. — **1.** 40. **2.** 100,000. **3.** 224. **4.** £.2055, 4.1098*s.* **5.** .25 ch., 92.12 ch. **6.** 6400 sq. ch. **7.** 400 sq. ch. **8.** 40.406. **9.** 5,451,776,000 cu. yd. **10.** 547 cars. **11.** 218.4 T. **12.** 46.24 T. **13.** 92.28 T. **14.** 10 yd. **15.** 2 in. **16.** 57° 17′ 45″ nearly. **17.** 7.92 in. **18.** 94,459.2. **19.** 27,000 cu. cm. **20.** 188 sq. yd., 240 cu. yd. **21.** 33.57 A. nearly, .9301 mi. **22.** 442.361 sq. yd.

Exercise 127. — **1.** $\frac{1}{6}$, 16, 13. **2.** $\frac{1}{10}$, 45, 4. **3.** $\frac{1}{8}$, 63, 116. **4.** $\frac{13}{28}$, 7, 8. **5.** $\frac{1}{15}$, 36, 55. **6.** $\frac{8}{45}$, 27, 20. **7.** $\frac{5}{72}$, 81, 40. **8.** $\frac{20}{143}$, 13, 22. **9.** $\frac{1}{40}$, 224, 135. **10.** $\frac{5}{132}$, 77, 108. **11.** $\frac{7}{12}$, 8, 9. **12.** $\frac{8}{99}$, 22, 27.

Exercise 128. — **1.** .15. **2.** .067083. **3.** .009166+. **4.** .03083+. **5.** .005694+. **6.** .0200277+. **7.** .02822+. **8.** .0371+. **9.** .06416+. **10.** .10816+. **11.** .0916+. **12.** .173616+. **13.** 81.722. **14.** .309629. **15.** .003694. **16.** .18833+. **17.** 37.24875. **18.** 20.015625. **19.** 1.33875. **20.** 28.090625. **21.** 15.7675. **22.** 6.37875. **23.** .9. **24.** 1.64583.

Exercise 129. — **1.** $1\frac{7}{8}$. **2.** $15\frac{5}{6}$. **3.** $6\frac{1}{3}$. **4.** The first. **5.** $\frac{21}{320}$. **6.** 0. **7.** 50 lb. **8.** $\frac{9}{48}$. **9.** 4950 ft. **10.** .1212+ rd. **11.** .456. **12.** .013875 T. **13.** 1400 lb. **14.** 45°. **15.** 12°. **16.** $\frac{1}{4}$. **17.** .3, \$1500, A's \$300, B's \$750. **18.** \$1875. **19.** 7.9. **20.** 11.3984. **21.** .0524. **22.** \$1178.85. **23.** 112.5. **24.** 228. **25.** \$150. **26.** $\frac{11}{18}$. **27.** 3.9. **28.** \$18.70. **29.** \$1500. **30.** $13\frac{1}{2}$ da. **31.** 6 da. **32.** 9 da. **33.** 14 hr. **34.** 20 da. **35.** 1.7875. **36.** $\frac{4}{15}$. **37.** 325 lb. **38.** \$326.67. **39.** \$64, \$240. **40.** \$75. **41.** \$89.25. **42.** \$6.06. **43.** 126 lb. $15\frac{1}{4}$ oz. **44.** \$18.20. **45.** \$121.67. **46.** $11\frac{1}{4}$ ¢, \$11.25. **47.** 11 ¢, \$11. **48.** 13 ¢, \$13. **49.** \$34,588, \$32,858.60. **50.** \$133,016. **51.** \$62.675. **52.** \$117.60. **53.** 5 mills, 50 ¢. **54.** 21.82 knots per hr., 25.126 mi. per hr. **55.** 20.584 knots per hr., 23.703 mi. per hr. **56.** 24.13 knots per hr. **57.** 65.5 mi. nearly. **58.** 63.617 mi. per hr. **59.** 50.77 mi. per hr. **60.** Wheat \$$1.31\frac{1}{4}$, corn $81\frac{1}{3}$¢ nearly, oats \$$1.06\frac{1}{4}$ nearly.

Exercise 131. — 1. 24° 10′. 2. 14° 16′. 3. 11° 17′ 40″. 4. 29° 17′ 46″. 5. 108° 53′ 48″. 6. 73° 28′. 7. 229° 22′ 32″. 8. 94° 50′ 10″. 9. 116° 5′ 40″. 10. 88° 10′ 54′. 11. 114° 8′ 45″. 12. 78° 37′ 37″. 13. 25° 33′ 35″. 14. 35° 23′ 54″. 15. 120° 35′ 40″. 16. 74° 44′ 55″. 17. 116° 4′ 2″. 18. 206° 32′ 28″. 19. 234° 31′ 30″. 20. 218° 42′ 3″.

Exercise 132 (answers correct to the second). — 1. 1 hr. 42 min. 14 sec. 2. 3 hr. 9 min. 2 sec. 3. 4 hr. 35 min. 14 sec. 4. 12 hr. 53 min. 56 sec. 5. 15 hr. 9 min. 38 sec. 6. 6 hr. 50 min. 59 sec. 7. 8 hr. 32 min. 15 sec. 8. 5 hr. 33 min. 43 sec. 9. 15 hr. 20 min. 10 sec. 10. 13 hr. 41 min. 28 sec. 11. 13 min. 38 sec. 12. 7 hr. 13 min. 58 sec. 13. 2 hr. 10 min. 26 sec. 14. 6 hr. 46 min. 56 sec. 15. 14 hr. 13 min. 10 sec. 16. 14 hr. 15 min. 2 sec. 17. 1 hr. 12 min. 59 sec. 18. 7 hr. 15 min. 35 sec. 19. 1 hr. 46 min. 41 sec. 20. 14 hr. 5 min. 4 sec.

Exercise 133. — 1. 13° 22′ 6″ E. 2. 4° 20′ 36″ E. 3. 88° 17′ 36″ E. 4. 3° 12′ 24″ W. 5. 9° 56′ 36″ E. 6. 1′ 54″ W. 7. 3° 43′ 54″ W. 8. 16° 17′ 36″ E. 9. 5° 45′ 3″ W. 10. 11 hr. 27 min. A.M. Feb. 23; 9 hr. 56 min. 49 sec. A.M. Feb. 23; 2 hr. 57 min. 57 sec. A.M. Feb. 23; 4 hr. 9 min. 21 sec. A.M. Feb. 23; 11 hr. 47 min. 57 sec. A.M. Feb. 23.

Exercise 134. — 1. Charleston, 19 min. 32 sec. nearly; New York, 3 min. 58 sec. nearly; Norfolk, 5 min. 9 sec. nearly; Providence, 14 min. 23 sec. nearly; Philadelphia, 36 sec. nearly; Washington, 8 min. 2 sec. nearly. 2. Austin, 30 min. 56 sec.; Bismarck, 43 min. 8 sec.; Chicago, 9 min. 20 sec.; Cincinnati, 22 min. 24 sec.; Detroit, 27 min. 48 sec.; Galveston, 19 min. 8 sec.; Louisville, 16 min. 56 sec.; New Orleans, 14 sec.; Pensacola, 10 min. 56 sec.; Savannah, 35 min. 38 sec.; St. Louis, 1 min. 4 sec. 3. 9 min. 38 sec., 10 min. 40 sec. 7. 96° W. 8. 86° W. 9. 94° 45′ W.

Exercise 135. — 1. 72° W. 2. 11 o'clock A.M. March 3, 1 o'clock P.M. March 3, 4 o'clock A.M. March 3. 3. 9 o'clock A.M. in London, Manchester, Glasgow; 5 o'clock P.M. in Tien-Tsin; 11 o'clock A.M. in Constantinople. 4. 15 min. past 3 o'clock A.M. 5. 165° W. 6. 6 o'clock A.M. the following day.

Exercise 136. — 1. 68.702 mi., 68.784 mi., 68.992 mi., 69.231 mi., 69.409 mi. 2. 65.027 mi., 59.956 mi., 53.064 mi., 44.553 mi. 6. 6045.767 ft., 6071.333 ft., 6092.333 ft., 6106.082 ft., 6108 ft. 7. 77.83 ft., 328.39 ft. 8. 209.848 mi.

Exercise 138. — 1. $264. 2. 367.50. 3. 77 bd. ft. 4. $87\frac{1}{2}$ bd. ft. 5. 96 bd. ft. 6. $364\frac{1}{2}$ bd. ft. 7. 10 cu. ft. 8. 396 bd. ft. 9. $20\frac{25}{36}$ bd. ft. 10. $141\frac{3}{4}$ bd. ft.

Exercise 139. — 1. 81.45 perches, 44,352 bricks. 2. 1984 cu. ft., 1856 cu. ft. 3. 13,200 bricks, $118.80. 4. 1000 cu. ft., 198,000 bricks. 5. $1909.84.

Exercise 140. — 1. $42\frac{2}{3}$ yd. 2. 64 yd. 3. 24 yd. 4. $66\frac{2}{3}$ yd. 5. $33.60.

Exercise 142. — 1. .675. 2. .625. 3. .88646. 4. .93125. 5. $27.37. 6. 41.77. 7. $43.92. 8. £23 12*s*. 9. £35 19*s*. 10*d*. 10. .2055. 11. 4.1098. 12. .243325. 13. $.006674. 14. 8 da. 15. 15. 16. £5 5*s*. $1\frac{1}{2}$*d*. 17. £15 13*s*. 10*d*. 18. 25.

Exercise 143. — 1. 25.222 francs, 20.43 marks, 24.014 krones. 2. (*a*) $86.83, (*b*) $106.12, (*c*) $75.04, (*d*) $214.39, (*e*) $93.07, (*f*) $145.91, (*g*) $120.60. 3. $.05. 4. (*a*) 2148.92 colons, (*b*) 4848.48 krones, (*c*) 5182.69 pesetas, (*d*) 3731.10 krones. 5. $38,590. 6. $9839. 7. 2.0549 sols. 8. 3.731 krones. 9. $16. 10. $10,804.

Exercise 144. — 1. 7830 oz. 2. .579 oz., 8.355 lb. 3. 77.78 lb. nearly. 4. 2.7. 5. 1.036. 6. 1.04. 7. 14.724 lb. 8. 62.5 T. 9. 4.17 T. 10. 11.146 oz., 6.059 oz., 12.442 oz., 6.586 oz. 11. 345 lb. 12. 1.0125 T. 13. $39\frac{1}{6}$ lb. 14. 54 T., 139.715 lb. 15. 810 lb. 16. 1.3 17. 2.402 cu. in. 18. $166\frac{1}{4}$ lb. 19. 280 oz. 20. 4.6955 times. 21. 108.194 lb. 22. 212.5 lb. 23. Steel, $23\frac{1}{3}$ lb.

Exercise 145. — 1. 30° C. 2. 25° C. 3. 95° C. 4. 120° C. 5. 20° C. 6. $12\frac{2}{9}$° C. 7. $3\frac{1}{3}$° C. 8. − 5° C. 9. − 10° C. 10. − 25° C. 11. − 40° C. 12. − $67\frac{7}{9}$° C.

Exercise 146. — 1. 95° Fahr. 2. 131° Fahr. 3. 77° Fahr. 4. 68° Fahr. 5. 64.4° Fahr. 6. 46.4° Fahr. 7. 14° Fahr. 8. − 4° Fahr. 9. 6.8° Fahr. 10. − 0.4° Fahr. 11. − 11.2° Fahr. 12. − 459.4° Fahr. 13. Mercury, − 40°; sulphur, 235.4°; lead, 618.8°; zinc, 779°; gold, 1895°; cast iron, 2012° to 2192°. 14. − 40°. 15. 39.2°.

Exercise 147. — 1. $80, $50. 2. $591, $394. 3. $1440, $1800. 4. $1188, $2376, $3564. 5. $1700, $2550, $2975. 6. 756, 972, 1188. 7. 912.5, 1095, 1277.5, 1460. 8. 1508, 1624, 1740, 1856. 9. 1176, 1232, 1288, 1344. 10. 540, 612, 684, 756. 11. 57, 38. 12. 992, 744. 13. 140, 225. 14. 975, 546. 15. 1170, 1300, 1755. 16. 396, 770, 1023. 17. $490, $700, $800. 18. $720, $900, $1080. 19. $403\frac{7}{11}$ gr. 20. A's share, $810; B's share, $360; C's share, $450. 21. $3000. 22. $30,000. 23. A, $8000; B, $2000. 24. A, $550; B, $200. 25. 355,230. 26. $40, $24. 27. $1.75, $.50. 28. $13\frac{1}{2}$ mi. 29. 24 mi.

Exercise 148. — 1. $8.25. 2. $11\frac{1}{5}$ da. 3. $110.50. 4. $7\frac{1}{3}$ da. 5. 5 wk. 6. 400 men. 7. $21\frac{1}{3}$ da. 8. 4 men. 9. 12 da. 10. $490.

11. 8 da. **12.** 20 men. **13.** $44. **14.** 477 lb. **15.** $26.44. **16.** $147. **17.** $148.50. **18.** $81. **19.** 40 men. **20.** $198. **21.** 8 men. **22.** 177 mi. **23.** 138 mi. **24.** 20 da.

Exercise 149. — **1.** 75¢. **2.** 6¢. **3.** $4.07. **4.** 60¢. **5.** 5 : 11, 90 lb., 198 lb. **6.** 64 lb. **7.** 80 lb. **8.** 29¢. **9.** $2.40. **10.** $\frac{5}{7}$, $\frac{2}{7}$.

Exercise 150. — **3.** $2\frac{1}{5}$ hr. **4.** $1\frac{13}{47}$ hr. **5.** $4\frac{8}{13}$ hr. **6.** $\frac{13}{60}$. **7.** $\frac{8}{9}$ hr. **8.** 1 hr. **9.** $10\frac{1}{2}$ hr. **10.** 100 da., 20 da. **11.** 1 hr. **12.** $6\frac{18}{37}$ da.; A, $14\frac{12}{17}$; B, $18\frac{6}{13}$; C, $34\frac{2}{7}$. **13.** 25 hr., $37\frac{1}{2}$ hr. **14.** 48 da., 28.8 da.

Exercise 151. — **1.** 1260 mi. **2.** 5 hr., 150 mi. **3.** 10.38 A.M. **4.** 22 mi. per hr. **5.** 105.6 yd. **6.** 180 mi. **7.** 30 mi. **8.** 360.

Exercise 152. — **2.** 48, 72, etc. **3.** 6 min. spaces. **4.** $10\frac{10}{11}$ min. past 2 o'clock, $16\frac{4}{11}$ min. past 3 o'clock, $27\frac{3}{11}$ min. past 5 o'clock, $38\frac{2}{11}$ min. past 7 o'clock, $49\frac{1}{11}$ min. past 9 o'clock, $54\frac{6}{11}$ min. past 10 o'clock, At no time. **5.** (*a*) $38\frac{2}{11}$ min. past 1 o'clock, (*b*) $49\frac{1}{11}$ min. past 3 o'clock, (*c*) $10\frac{10}{11}$ min. past 8 o'clock, (*d*) $16\frac{1}{11}$ min. past 9 o'clock, (*e*) $27\frac{3}{11}$ min. past 11 o'clock, (*f*) $32\frac{8}{11}$ min. past 12 o'clock. **6.** $49\frac{1}{11}$ min. past 3 o'clock, $5\frac{5}{11}$ min. past 7 o'clock. **7.** (*a*) $27\frac{3}{11}$ min. past 2 o'clock, (*b*) $32\frac{8}{11}$ min. past 3 o'clock, (*c*) $5\frac{5}{11}$ min. past 4 o'clock, $38\frac{2}{11}$ min. past 4 o'clock, (*d*) $16\frac{4}{11}$ min. past 6 o'clock, $49\frac{1}{11}$ min. past 6 o'clock, (*e*) $27\frac{3}{11}$ min. past 8 o'clock, (*f*) $32\frac{8}{11}$ min. past 9 o'clock, (*g*) $43\frac{7}{11}$ min. past 11 o'clock, (*h*) $16\frac{4}{11}$ min. past 12 o'clock. **8.** $13\frac{1}{11}$ min. past 4 o'clock, 5 o'clock, 48 min. past 4 o'clock. **9.** $38\frac{2}{11}$ min. past 5 o'clock.

Exercise 153. — **1.** 115.96 da. **2.** (*a*) 586.6 da., (*b*) 398.57 da., (*c*) 377.8 da., (*d*) 369.4 da., (*e*) 367.23 da. **3.** (*a*) 334.6 da., (*b*) 62,615 da.

Exercise 154. — **1.** $2\frac{2}{3}$ mi. **2.** 2 mi. **3.** 5 mi. **4.** 2 hr.

Exercise 155. — **1.** 97.5. **2.** 544. **3.** $4\frac{37}{72}$. **4.** 24 mi., 96 mi. **5.** 16 da. **6.** $\frac{7}{9}$.

Exercise 158. — **1.** $7.56. **2.** $$34\frac{1}{2}$. **3.** $48. **4.** $100. **5.** $260. **6.** $315. **7.** $225. **8.** $440. **9.** $870. **10.** $144. **11.** $49.15. **12.** $60. **13.** $51. **14.** $87.50. **15.** $1210. **16.** $987. **17.** $1800. **18.** $1625. **19.** $3000. **20.** $2450. **21.** $1960. **22.** $7800. **23.** $512. **24.** $1656. **25.** $5000. **26.** $12,150.

Exercise 159. — **1.** $3200. **2.** $2160. **3.** $1230. **4.** $900. **5.** $106. **6.** $660. **7.** $21. **8.** $82. **9.** $390. **10.** $720. **11.** $410. **12.** $1740. **13.** $3575. **14.** $1109. **15.** $8320. **16.** $6595.56. **17.** $470. **18.** $3769.60. **19.** $4410. **20.** $301. **21.** $646. **22.** $1040. **23.** $1120. **24.** $200. **25.** $1505. **26.** $784. **30.** $1125. **31.** 11.2 oz. **32.** $27\frac{1}{2}$ lb.

33. $2050. **34.** N.Y., $476,850,000; Boston, $132,982,500; Philadelphia, $64,387,500; Baltimore, $113,475,000; New Orleans, $81,600,000; Galveston, $91,417,500. **35.** $5,087,250; $18,387,250. **36.** $8,400,000; $20,900,000. **37.** $851,378,043; $39,037,872; $403,933,540.64. **39.** 457.2 lb.

Exercise 160. — **1.** 3700. **2.** $1244. **3.** $864. **4.** 7200, 6000, 4500, 4000. **5.** 4000, 3000, 2400, 2000, 1500, 1000. **6.** $12,500; $11,250; $10,000; $6,000. **7.** $360, $432, $324, $243. **8.** $7200, $6750, $5400, $3600, $2700. **9.** $1120, $672, $480. **10.** 4200 oz. **11.** 76,300,712. **12.** 39,007,793. **13.** 37,220,331. **14.** 6,166,710. **15.** $213,293,651. **16.** $1,960,233. **17.** 3500. **18.** 3500. **19.** 304,800. **20.** 4000 lb.

Exercise 161. — **1.** 80%. **2.** 20%, 35%, 55%, 65%. **3.** 5%, 20%, 30%, 42½%. **4.** 10%, 13⅓%, 30%, 12½%. **5.** 12½%, 15%, 37½%, 66⅔%. **6.** 6%, 7%, 8⅓%, 18¾%, 104⅙%. **7.** 20%, 9%, 16⅔%, 6¼%, 200%. **8.** 9 1/11%, 33⅓%, 15%. **9.** 1⅔%, 8⅓%, 6¼%, 2 1/12%. **10.** 3⅛%. **11.** 2½%. **12.** ⅝%. **13.** 4 6/11%. **14.** 6¼%. **15.** 12½%. **16.** 25%. **17.** 33⅓%. **18.** 115.7% nearly. **19.** 88.44% **20.** 1¼%, 10%. **21.** 6¼%. **22.** 20%, 60%, 80%. **23.** 91.44% nearly, 109.36% nearly. **24.** 62.14% nearly. **25.** 82 4/7%. **26.** 109 5/7%. **27.** (*a*) 1.43%, (*b*) 2.42%, (*c*) .65%, (*d*) 1.02%, (*e*) .71%, (*f*) .63%, (*g*) 1.16%, (*h*) .3%, (*i*) .97%. **28.** Mobile, 23.79%; Little Rock, 48.05%; Los Angeles, 103.35%; Denver, 25.44%; Pensacola, 51.04%, Savannah, 25.6%; Springfield, 36.84%; Evansville, 16.26%; Dubuque, 19.75%; Kansas City, Kan., 34.19%; Lexington, 22.27%; Kansas City, Mo., 23.38%; Minneapolis, 23.05%. **29.** 1.12% nearly. **30.** 9.07% nearly. **31.** 64.69%, 13.12%, 22.19%. **32.** 11 2/9%.

Exercise 162. — **1.** $3250. **2.** $300. **3.** $3640. **4.** 12½%. **5.** 8⅓%. **6.** 20%. **7.** 10%. **8.** 9%. **9.** $97. **10.** $86. **11.** $75. **12.** $105. **13.** $80, $15. **14.** $120. **15.** $4.20. **16.** 25%. **17.** $982. **18.** 12,080. **19.** $16.67 nearly. **20.** $200. **21.** $10,240. **22.** $11,520. **23.** $85,700. **24.** 475 lb. **25.** 12%. **26.** 14 2/7%. **27.** 22 2/9%. **28.** $165. **29.** 177 7/9%. **30.** 5¢. **31.** $2.10 per yd. **32.** 24¢. **33.** 16⅔%. **34.** 38 8/9%. **35.** 84¢ per lb. **36.** 11¼%. **37.** $98. **38.** $98.70. **39.** 96½¢. **40.** 440. **41.** 200. **42.** 33⅓%. **43.** $148, 30⅚%. **44.** 66⅔%. **45.** (*a*) 40% above cost, (*b*) 30% above cost, (*c*) 60% above cost, (*d*) 22 2/9% above cost, (*e*) 42 6/7% above cost. **46.** $8.40. **47.** 12%. **48.** 12½%.

Exercise 164. — **1.** $47.25. **2.** $612, 32%. **3.** 30%. **4.** $898.88. **5.** $283.50. **6.** $232.56. **7.** $10. **8.** 32½%. **9.** 24 4/9%. **10.** 25%. **11.** $2, $30. **12.** (*a*) $241.92, (*b*) $547.20, (*c*) $465, (*d*) $739.20, (*e*) $403.20, (*f*) $739.20, (*g*) $943.92, (*h*) $962.50. **13.** 38 8/9%. **14.** $320, 45 5/7%. **15.** (*a*) $700, (*b*) $7000, (*c*) $3000, (*d*) $9690, (*e*) $5000, (*f*) $3000, (*g*) $4000, (*h*) $6000.

Exercise 165. — 1. $320. 2. $1050; $16,450. 3. $45, $705. 4. $4. 5. 96,000 bu. 6. $56.16, $1815.84. 7. $45. 8. $56.70, $1563.30. 9. $1125; $21,375; 95%. 10. 900 T. 11. 844 bales, 8¢ per pound. 12. $17.28. 13. 3¼%. 14. 227,200 bu. 15. $4200. 16. $7800, $7449. 17. $25. 18. $235.50, 78½¢. 19. 4000 bu. 20. 3750 lb. 21. 150 A.

Exercise 166. — 1. $24, $30, $36, $48. 2. $59.50, $68, $76.50. 3. $28.50, $38, $76. 4. $78.56, $147.30, $88.38. 5. $75, $85. 6. $128, $119. 7. $213.75, $200. 8. $180, $152. 9. $120, $175. 10. $204.17, $163.33. 11. $7.04. 12. $8.35. 13. $9.39. 14. $12.38. 15. $12.87. 16. $16. 17. $28.14. 18. $9.19. 19. $12.85. 20. $56.07. 21. $203.52. 22. $243.83. 23. $164.79. 24. $175.50. 25. $205. 26. $191.25. 27. $16.11. 28. $47.14. 29. $39.36. 30. $53.36.

Exercise 167. — 1. $7.20. 2. $12.00. 3. $99.37. 4. $102.04. 5. $75.14. 6. $101.26. 7. $64.87. 8. $43.63. 9. $16.78. 10. $7.21.

Exercise 168. — 1. $838.67. 2. $693.10. 3. $802.50. 4. $355.61. 5. $337.79. 6. $753.13. 7. $880.28. 8. $336.46. 9. $1016.92. 10. $389.00. 11. $1332.62. 12. $3751.84. 13. $1532.50. 14. $4455.73. 15. $1243.87.

Exercise 169. — 1. $3.88. 2. $1.96. 3. $1.18. 4. $24.07. 5. $13.96. 6. $26.02. 7. $35.16. 8. $49.79. 9. $39.47. 10. $39.52.

Exercise 170. — 1. $485.04, Mar. 16. 2. $654.77, Feb. 6. 3. $908.40, Mar. 19. 4. $1006.67, Mar. 13. 5. $542.70, Apr. 6. 6. $837.79, July 7. 7. $930.73, July 10. 8. $537.95, Sept. 8. 9. $964.25, Oct. 4. 10. $727.20, Oct. 13. 11. $848.40, Jan. 9, 1903. 12. $735.84, Dec. 31. 13. $329.23, Mar. 30. 14. $623.75. 15. $826.40. 16. $562.83. 17. $667.55. 18. $880.10. 19. $1014.30.

Exercise 171. — 1. $2835.13. 2. $1776.30. 3. $233.67. 4. $480.85. 5. $1619.41. 6. $1158.16. 7. $136.02. 8. $445.02.

Exercise 172. — 1. $62.84. 2. $215.47. 3. $325.31.

Exercise 173. — 1. $3.78. 2. $2.12. 3. $2.98.

Exercise 174. — 1. $1156.43. 2. $622.91. 3. $1650. 4. $1278.12. 5. $983.59.

Exercise 175. — 1. $500. 2. $1200. 3. $1000. 4. $240. 5. $2400. 6. $300. 7. $600. 8. $800. 9. $195. 10. $450. 11. $1020. 12. $324. 13. $300. 14. $937.50. 15. $11,300.

Exercise 176. — 1. 1 yr. 2. 3 yr. 3. 1 yr. 4. 4 yr. 5. 1 yr. 6. 1 yr. 9 mo. 7. 3 yr. 3 mo. 8. 1 yr. 1 mo. 15 da. 9. 1 yr. 10 mo. 15 da. 10. 1 yr. 1 mo. 15 da. 11. 1 yr. 10 mo. 12. 8 mo. 15 da. 13. 2 yr. 11 mo. 14. 1 yr. 2 mo. 15 da. 15. 6 mo. 9 da.

Exercise 177. — 1. 6%. 2. 7%. 3. 6%. 4. 8%. 5. 7%. 6. 6%. 7. 5%. 8. 4%. 9. 5%. 10. 5%. 11. 8%. 12. 9%. 13. 4½%. 14. 6%. 15. 6%. 16. 7% 17. 5% 18. 6%. 19. 5½%. 20. 4½%.

Exercise 178. — 1. $800. 2. $700. 3. $600. 4. $840. 5. $360. 6. $230. 7. $11,505. 8. $162.50. 9. $580. 10. $630. 11. $740. 12. $450. 13. $403. 14. $240. 15. $1917.50. 16. $1800. 17. $245. 18. $240. 19. $22,530. 20. $803.

Exercise 179. — 1. $392. 2. $6.43. 3. $1.44. 4. $5.51. 5. $24.57. 6. $858.67. 7. $548.91. 8. $999.84. 9. $14,000. 10. 7%. 11. 7%. 12. $24,000. 13. $6000. 14. 1 mo. 18 da. 15. 4 mo. 16. 2 yr. 6 mo. 17. 1 yr. 8 mo. 18. $48. 19. $204.40. 20. 219 da. 21. 2 mo. 12 da. 22. $500. 23. $1072.50. 24. $13,000.

Exercise 180. — 1. Discount $4.13. 2. Discount $5.87. 3. Discount $7.50. 4. Discount $7.00. 5. Discount $9.50. 6. Discount $1.00. 7. Discount $7.25. 8. Discount $2.25. 9. Discount $4.00. 10. Discount $2.92. 11. Discount $13.33. 12. Discount $5.00. 13. Discount $10.40. 14. Discount $3.37. 15. Discount $1.90. 16. Discount $6.16. 17. Discount $10.92. 18. Discount $8.55.

Exercise 181. — 1. Discount $1.65, Proceeds $352.72. 2. Discount $9.25, Proceeds $391.87. 3. Discount $1.61, Proceeds $450.86. 4. Discount $6.08, Proceeds $601.92. 5. Discount $5.05, Proceeds $499.95. 6. Discount $6.64, Proceeds $898.31. 7. Discount $16.89, Proceeds $996.44. 8. Discount $7.58, Proceeds $749.92. 9. Discount $3.22, Proceeds $801.18. 10. Discount $4.24, Proceeds $399.96. 11. Discount $21.38, Proceeds $843.49. 12. Discount $1.96, Proceeds $904.79. 13. Discount $17.60, Proceeds $1201. 14. Discount $4.23, Proceeds $1518.27.

Exercise 182. — 1. $734.95. 2. $817.85. 3. $892.68. 4. $577.79. 5. $945.38. 6. $527.84. 7. $993.54. 8. $536.67. 9. $444.14. 10. $743.97. 11. $898.18. 12. $828.13. 13. 3 mo. 14. 1 mo. 15. 6%.

Exercise 184. — 1. $2726.80. 2. $3497.40. 3. $5086.35. 4. $6305.72. 5. $8279.64. 6. $9955.05. 7. $5484.76. 8. $5287.92. 9. $6027.92. 10. $6097.61. 11. $5393.25. 12. $9771.30. 13. $4509. 14. $6913.08. 15. $7789.72. 16. $1232.46. 17. $4.20. 18. $9987.50. 19. $598.50. 20. $993.33. 21. $891.

Exercise 185. — 1. $4872. 2. $2669.92. 3. $1116.50. 4. $17,064.12. 5. $352.42. 6. $1717. 7. $289.50. 8. (*a*) $4503.22, (*b*) $4134.75, (*c*) $600, (*d*) $1798.45, (*e*) $1636.25, (*f*) $2840.02.

Exercise 186. — 1. £ 1200. **2.** £ 250. **3.** 38,080 marks. **4.** 19,300 francs. **5.** 2680 krones. **6.** £ 486 10s. **7.** 5376.85 francs. **8.** 23,801.68 marks.

Exercise 187. — **2.** $ 1620, $ 1640. **3.** $ 3720. **4.** $ 29,075. **5.** $ 2965.62. **6.** $ 5337.50. **7.** $ 8200; $ 9750; $ 9300; $ 12,962.50; $ 14,287.50; $ 13,412.50; $ 10,850; $ 14,412.50; $ 16,837.50; $ 6300.

Exercise 188. — 1. $ 14,262.50; $ 141,250. **2.** $ 9737.50. **3.** 100 shares. **4.** $ 125. **5.** $ 125. **6.** $ 400. **7.** 200 shares. **8.** 100 shares. **9.** 1000 shares. **10.** 200 shares.

Exercise 189. — 1. $159\frac{7}{8}$. **2.** $149\frac{7}{8}$. **3.** $ 18,550. **4.** $ 32,718.75. **5.** $4\frac{29}{49}$%. **6.** $62\frac{3}{8}$. **7.** $30,918.75. **9.** $ 875; no brokerage. **10.** $5\frac{5}{9}$%. **11.** $ 200. **12.** $ 5093.75. **13.** Central. **14.** 438 shares.

Exercise 190. — 1. $ 118.25. **2.** $ 113.85. **3.** $ 129.48. **4.** $ 312.90. **5.** $ 435.20. **6.** $ 589.68. **7.** $ 31,500. **8.** $ 16,400. **9.** $ 27.10, 35 yr. **10.** $ 39.60, 46 yr. **11.** $ 2500. **12.** $ 5250. **13.** $ 23,300. **14.** $ 4500.

Exercise 191. — 1. $ 38.50. **2.** $ 54. **3.** $ 69.60. **4.** $ 115. **5.** $ 109.37. **6.** $ 104.50. **7.** $ 157.50. **8.** $ 152. **9.** $ 320. **10.** $ 225.

Exercise 192. — 1. $ 325. **2.** $ 230. **3.** $ 67.50. **4.** $ 37.50. **5.** $ 47.50. **6.** $ 93.50. **7.** $ 97.50. **8.** $ 72. **9.** $ 91.80. **10.** $ 59.40. **11.** $ 120. **12.** $ 101.50. **13.** $ 136.50. **14.** $ 240.62. **15.** $ 273.20. **16.** $ 106.65. **17.** $ 247.50. **18.** $ 219.

Exercise 193. — 1. $ 43.20. **2.** $ 139.50. **3.** $ 136. **4.** $ 141.75. **5.** $ 123.50. **6.** $ 86.40. **7.** $ 180.50. **8.** $ 210. **9.** $ 315. **10.** $ 2500. **11.** $ 248. **12.** $ 259.20. **13.** $ 34.20. **14.** $ 30.62.

Exercise 194. — 1. $ 79.20. **2.** $ 22.50. **3.** $ 157.50. **4.** $ 259.37. **5.** $ 430. **6.** $ 138.60. **7.** $ 166.50. **8.** $ 403.20. **9.** $ 184.80.

Exercise 195. — 1. $ 2800. **2.** $ 8500. **3.** $ 12,000. **4.** $ 19,000. **5.** $ 17,000. **6.** $ 22,500. **7.** $ 4900. **8.** $ 34,500. **9.** $ 8400. **10.** $ 49,500. **11.** $ 72,500. **12.** $ 46,500. **13.** $ 3500. **14.** $ 17,800. **15.** $ 3700. **16.** $ 3100. **17.** $ 2090. **18.** $ 3900. **19.** $ 3300. **20.** $ 4500. **21.** $ 9000. **22.** $ 4100. **23.** $ 9880. **24.** $ 800. **25.** $1\frac{1}{8}$%. **26.** 85¢ on $ 100.

Exercise 196. — 1. $ 31,144. **2.** $ 19. **3.** $ 42. **4.** $ 166,750. **5.** $ 551,100. **6.** $ 7860. **7.** $ 130.50. **8.** $ 42.50. **9.** $ 164.10. **10.** $ 25. **11.** $ 3,009,600. **12.** $\frac{17}{18}$¢. **13.** $ 5,215,500. **14.** $ .003077 nearly. **15.** 50¢ on $ 100. **16.** $ 4,256,000.

17. $350,000,000. **18.** $2,112,000. **19.** $72,000. **20.** $133,449.87. **21.** $8,404,000. **22.** $342.25. **23.** $380, $127.50. **24.** $23.94. **25.** $697,190.16; $669,302.55.

Exercise 197. — **1.** $62.40. **2.** $500. **3.** $2.16. **4.** $26.28, $25.31. **5.** £43 4*s*., or $210.23. **6.** $1.81. **7.** $1.46. **8.** $256.80. **9.** $58. **10.** $72, $8.19 each. **11.** $91.80. **12.** $17.50, $81. **13.** $65. **14.** $552, $2.208. **15.** $750. **16.** $28.80. **17.** $94.50. **18.** $125. **19.** £5. **20.** $82.12, 29.4¢.

Exercise 201. — **1.** 4. **2.** $3a$. **3.** $3a$. **4.** 4. **5.** $3a^2$. **6.** $3a$. **7.** $2a^2$. **8.** $6a$. **9.** $3a^2$. **10.** $11a^3$. **11.** $11a^3$. **12.** $4a^2$. **13.** $2a^2$. **14.** $3a^2$. **15.** $5a^2$. **16.** $7x$. **17.** $3x^4$. **18.** $7x$. **19.** $5x^3$. **20.** $5x$. **21.** 6. **22.** $2x^4$. **23.** $3b$. **24.** $3b^3$. **25.** $2b^3$.

Exercise 203. — **1.** 9. **2.** 8. **3.** 20. **4.** 30. **5.** 18. **6.** 35. **7.** 30. **8.** 40. **9.** 22. **10.** 45. **11.** 55. **12.** 55. **13.** 54. **14.** 60. **15.** 45. **16.** 49. **17.** 90. **18.** 55. **19.** 63. **20.** 40. **21.** 56. **22.** 18. **23.** 22. **24.** 30. **25.** 35.

Exercise 204. — **1.** $675. **2.** $23.04. **3.** $609.20. **4.** 10 da. **5.** $2175. **6.** 16 hr. 48 min. **7.** 20 da. **8.** $18\frac{3}{4}$ bu. **9.** 9 da. **10.** 25 da. **11.** 400 men. **12.** 21 da. **13.** 18 turkeys. **14.** 66¢. **15.** 280,176 ft. **16.** 75 ft.

Exercise 205. — **1.** 84 A. **2.** 150 ft. **3.** $260. **4.** 2057.5 T. **5.** 20 ft. **6.** 7 da. **7.** 12 da. **8.** 6 da. **9.** 7 hr. 12 min. **10.** $113.40. **11.** $365.625. **12.** $2\frac{1}{4}$ da. **13.** 8 da. **14.** 17.85 T. **15.** $17\frac{1}{7}$ hr.

Exercise 206. — **1.** A's share, $800; B's share, $1000; C's share, $1200. **2.** A's, $600; B's, $1500. **3.** A's, $90; B's, $81. **4.** A's, $900; B's, $700. **5.** A's, $780; B's, $954; C's, $420. **6.** A, $300; B, $300. **7.** $700, $750.

Exercise 210. — **1.** .01, .04, .09, .16, .25, .36, .49, .64, .81. **2.** 1, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{16}$, $\frac{1}{25}$, $\frac{1}{36}$, $\frac{1}{49}$, $\frac{1}{64}$, $\frac{1}{81}$, $\frac{1}{100}$, $\frac{1}{121}$, $\frac{1}{144}$, $\frac{1}{169}$, $\frac{1}{196}$, $\frac{1}{225}$, $\frac{1}{256}$, $\frac{1}{289}$, $\frac{1}{324}$, $\frac{1}{361}$, $\frac{1}{400}$, **3.** $\frac{4}{9}$, $\frac{9}{16}$, $\frac{25}{36}$, $\frac{4}{49}$, $\frac{81}{100}$, $\frac{121}{144}$, $\frac{100}{169}$, $\frac{81}{256}$. **4.** $2\frac{7}{9}$, $5\frac{4}{9}$, $1\frac{32}{49}$, $1\frac{9}{16}$, $3\frac{1}{16}$, $3\frac{33}{64}$, $4\frac{21}{25}$, $39\frac{1}{16}$. **6.** 1, $\frac{1}{8}$, $\frac{1}{27}$, $\frac{1}{64}$, $\frac{1}{125}$, $\frac{1}{216}$, $\frac{1}{343}$, $\frac{1}{512}$, $\frac{1}{729}$, $\frac{1}{1000}$, $\frac{1}{1331}$, $\frac{1}{1728}$, $\frac{1}{2197}$, $\frac{1}{2744}$, $\frac{1}{3375}$, $\frac{1}{4096}$, $\frac{1}{4913}$, $\frac{1}{5832}$, $\frac{1}{6859}$, $\frac{1}{8000}$. **7.** 3025. **8.** 25,333. **9.** (*a*) 3, (*b*) 85.6735, (*c*) 29.1708, (*d*) 66.0806, (*e*) .4670, (*f*) .5790, (*g*) .8991, (*h*) .5, (*i*) .6, (*j*) .9. **10.** 385. **11.** .001, .008, .027, .064, .125, .216, .343, .512, .729. **12.** 1.169859, 1.215506, 1.262477, 1.310796.

Exercise 211. — **1.** 13. **2.** 21. **3.** 25. **4.** 31. **5.** 32. **6.** 43. **7.** 53. **8.** 62. **9.** 65. **10.** 73. **11.** 76. **12.** 82. **13.** 84. **14.** 87. **15.** 92. **16.** 96. **17.** 98. **18.** 99. **19.** 89. **20.** 78. **21.** 69. **22.** 59. **23.** 49. **24.** 58.

Exercise 212. — **1.** 317. **2.** 332. **3.** 347. **4.** 414. **5.** 436. **6.** 447. **7.** 479. **8.** 527. **9.** 557. **10.** 595. **11.** 626. **12.** 676. **13.** 689. **14.** 708. **15.** 809. **16.** 879. **17.** 905. **18.** 909.

Exercise 213. — **1.** .388. **2.** .496. **3.** .587. **4.** .539. **5.** .513. **6.** .679. **7.** .729. **8.** .785. **9.** .885. **10.** .288. **11.** .249. **12.** .0608.

Exercise 214. — **1.** 1.095. **2.** 2.062. **3.** 1.049. **4.** 2.28. **5.** 1.817. **6.** 2.291. **7.** 1.768. **8.** 3.028. **9.** 1.173. **10.** 2.121. **11.** 1.696. **12.** .764. **13.** .816. **14.** .632. **15.** .797.

Exercise 215. — **1.** 69.57 yd. **2.** 241 yd. **3.** 238 yd. **4.** 1.025 mi. **5.** 739 rd. **6.** 311.13 yd., 155.56 yd. **7.** 538.89 yd., 179.63 yd. **8.** 19.41 rd.

Exercise 216. — **1.** 10. **2.** 13. **3.** 17. **4.** 29. **5.** 106. **6.** 101. **7.** 145. **8.** 89. **9.** 149. **10.** 68.5. **11.** 425. **12.** 305. **13.** 433. **14.** 305. **15.** 50 ft. **16.** 14.14 rd. **17.** 30.232 rd.

Exercise 217. — **1.** 152. **2.** 184. **3.** 280. **4.** 2.17. **5.** .261. **6.** .319. **7.** .2. **8.** .748. **9.** .445.

Exercise 218. — **1.** 126. **2.** 180. **3.** 264. **4.** 840. **5.** 522. **6.** 9240. **7.** 150,769. **8.** 86.25 A. **9.** 5.775 A. **10.** 1.638 A. **11.** 1.68 A. **12.** 43.301 sq. rd. **13.** 1082.53 sq. rd.

Exercise 219. — **1.** 69.12. **2.** 144.51. **3.** 471.24. **4.** 515.22. **5.** 615.75. **6.** 420.97. **7.** 540.35. **8.** 22.62. **9.** 37.07. **10.** 45.87. **11.** 102. **12.** 152. **13.** 7.8. **14.** 13.2. **15.** 60. **16.** 8.5. **17.** 9.6. **18.** 6.7. **19.** 480 times. **20.** 192 times.

Exercise 220. — **1.** 615.8. **2.** 1520.5. **3.** 4071.5. **4.** 69.4. **5.** 132.73. **6.** 232.35. **7.** 295.59. **8.** 4778.4. **9.** 3217. **10.** 7238.2. **11.** 4300.8. **12.** 6647.6. **13.** 795.8. **14.** 484.15. **15.** 548.2. **16.** 688.3. **17.** 6.023. **18.** 3.789. **19.** 7.643. **20.** 9.282.

Exercise 221. — **1.** 31. **2.** 17. **3.** 33. **4.** 42. **5.** 2.6. **6.** 7.2. **7.** 9.2. **8.** 9.8. **9.** 11.8. **10.** 13.4. **11.** 7.6. **12.** 9.3. **13.** 99. **14.** 850. **15.** 650. **16.** 39.25 yd. **17.** 288,576,452.4. **18.** 329.9 sq. in. **19.** 1661.9 sq. in. **20.** 259.8. **21.** 139.1 sq. in. **22.** 259.8 sq. in., 225 sq. in. **23.** Circle.

Exercise 222. — **1.** 46.5 in. **2.** 49.22 in. **3.** 1.01 ft. **4.** 36°. **5.** $3° 36'$. **6.** $31'.2$. **7.** $11''$. **8.** $32' 4''$.

Exercise 223. — **1.** 20 ft. **2.** 176 ft. **3.** 110 mi. **4.** $7\frac{1}{8}$ mi. **5.** 22.36 ft. **6.** 7.07 ft. **7.** 16 : 121. **8.** 4 : 9. **9.** 186.96 sq. mi. **10.** $\sqrt{2} : 1$.

Exercise 224. — **1.** 576 sq. ft. **2.** 680 ft. **3.** $10\frac{5}{8}$ ft. **4.** $7\frac{1}{2}$ ft. **5.** 1280 ft. **6.** 630 sq. ft. **7.** $26.40. **8.** 147 sq. yd. **9.** 53.45 sq. ft. **10.** 4021 sq. in. **11.** 67 yd. **12.** 2984.5 sq. ft. **13.** 15456.6 sq. in. **14.** $392.70. **15.** 120,687 sq. in. **16.** 5541.7 sq. in. **17.** 28,842,700 sq. mi.

18. 186,265,000 sq. mi. **19.** (1) Jupiter, 23,235 million sq. mi.; (2) Uranus, 3217 million sq. mi.; (3) Neptune, 3421 million sq. mi.; (4) Saturn, 16,741 million sq. mi. **20.** 58 in. **21.** 1257 sq. in. each. **22.** 55 in. **23.** 50 yd.

Exercise 225.—**1.** 5280 cu. in. **2.** 700 cu. in. **3.** 4071.5 cu. in. **4.** 15,708 cu. in. **5.** 2598 cu. in. **6.** 33,510 cu. in. **7.** 2144.7 cu. ft. **8.** 4094 gal. **9.** 4562 gal. **10.** 3 : 2. **11.** 1 : 2. **12.** 52 cu. ft. 1269 cu. in. **13.** 3 ft. **14.** 4189 cu. in. **15.** 40 ft. **16.** 1,367,631. **17.** 48 times. **18.** 760 times. **19.** 1321 times.

Exercise 229.—**1.** 37. **2.** 58. **3.** 119. **4.** 66. **5.** 63.

Exercise 230.—**1.** 130. **2.** 336. **3.** 630. **4.** 1008. **5.** 625. **6.** 650. **7.** 570.

Exercise 231.—**1.** 126. **2.** 13,120. **3.** $105\frac{1}{2}$. **4.** $76\frac{7}{27}$. **5.** $162\frac{3}{32}$.

Exercise 232.—**1.** 3. **2.** 27. **3.** $\frac{2}{3}$. **4.** $\frac{4}{11}$. **5.** $\frac{7}{11}$. **6.** $\frac{10}{11}$. **7.** $\frac{80}{99}$. **8.** $\frac{40}{99}$. **9.** 1.

Exercise 233.—**1.** $.\dot{6}$, $.\dot{5}$, $.\dot{6}\dot{3}$, $.\dot{1}7\dot{1}$, $.236\dot{1}$, $.34\dot{0}\dot{9}$, $.\dot{1}5\dot{3}$, $.\dot{1}4285\dot{7}$, $.\dot{6}9230\dot{7}$, $.33\dot{7}$. **2.** $\frac{4}{11}$, $\frac{7}{11}$, $\frac{10}{11}$, $\frac{80}{99}$. **3.** $\frac{1177}{15000}$, $\frac{3}{7}$, $\frac{1}{150}$, $\frac{6}{275}$. **4.** $1\frac{1801}{9900}$, $1.18\dot{1}\dot{9}$. **5.** .841471. **6.** .540302. **7.** 2.718282. **8.** 1.414201. **9.** .09066. **10.** .049958.

Exercise 234.—**1.** $1735.96. **2.** $3096.92. **3.** $2429.73. **4.** $1419.20.

Exercise 235.—**1.** $1400. **2.** For $1550. **3.** $22.50. **4.** In 8 mo. **5.** $1200. **6.** $8\frac{1}{3}$%. **7.** $286.20. **8.** $2030. **9.** 10 mo. **10.** $2791.97. **11.** $2707.36. **12.** $20,000. **13.** $4151.85 **14.** $230.97. **15.** (*a*) $70.73, (*b*) $60.83. **16.** $1132.25. **17.** $1439.13.

Exercise 236.—**1.** 13. **2.** 22. **3.** 25. **4.** 34. **5.** 42. **6.** 45. **7.** 54. **8.** 63. **9.** 67. **10.** 76. **11.** 84. **12.** 88. **13.** 96. **14.** 99. **15.** 82.

Exercise 237.—**1.** 243. **2.** 258. **3.** 364. **4.** 465. **5.** 495. **6.** (*a*) .5848, (*b*) .8879, (*c*) .9655, (*d*) 1.035, (*e*) 1.186, (*f*) 1.032, (*g*) 1.94, (*h*) .63, (*i*) .8434.

MISCELLANEOUS EXAMPLES (A)

8. 8999.991. **9.** 274.999225. **10.** 8,447,537,940,492. **11.** 636,300,000. **12.** 75,170,080,000. **13.** 7.1407. **14.** 814,591.5346827284. **15.** 166.375 mi. **16.** $3828.12. **17.** $3570.12. **18.** .000504. **19.** 1.12550881. **20.** 278,500,000. **21.** (1) $.475, (2) $3.508, (3) $51.877, (4) $.243, (5) $1.427. **22.** 372.015. **23.** 28,127,000 nearly. **25.** 52,800 mi. **26.** .341. **27.** .8251. **28.** .1704. **29.** .000439625. **30.** 14.461°. **31.** 8° 39′ 6″. **32.** 80.7 ¢. **33.** 113,400. **34.** 250. **35.** 80,032,000 nearly. **36.** 77. **37.** $3.78. **38.** $1800. **39.** $384.45. **40.** $30.80. **41.** 12,290.

42. $20.25. **43.** .5774. **44.** .037. **45.** $\frac{5}{128}$, or .0390625. **46.** 144,864.
47. 2, 3, 6, 9, 18. **48.** 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72. **49.** $1110 =$
$2 \cdot 3 \cdot 37$; $777 = 3 \cdot 7 \cdot 37$; $1001 = 7 \cdot 11 \cdot 13$; L. C. M. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 37$.
50. 26. **51.** 65,520. **52.** 37. **53.** 9; 7,567,560. **54.** 2, 3, 5^2, 7^2, 11.
55. 2,784,873. **56.** $2^4 \cdot 3^3 \cdot 11 \cdot 13$. **57.** 25 A. **58.** 18,480 poles.
59. $\frac{7}{9}$, $\frac{3}{4}$, $\frac{11}{15}$. **60.** $\frac{31}{168}$. **61.** $16\frac{23}{24}$. **62.** $\frac{9}{10}$. **63.** 1.09375, .109375, .128.
64. $\frac{3}{80}$, $\frac{7}{400}$, $\frac{1}{32}$. **65.** $3\frac{3}{25}$. **66.** $1\frac{1}{20}$. **67.** .025. **68.** .027. **69.** .075.
70. .16583. **72.** Upwards of 30,000 yr. **73.** (1) 39.534,
(2) 48.321, (3) 54.197, (4) 63.25, (5) 63.615, (6) 68.234, (7) 72.932,
(8) 76.459, (9) 79.68, (10) 82.951, (11) 70.13. **74.** $11,687.50.
75. 20,000 bu. **76.** $3087.50. **77.** $42.40. **78.** $92.67. **79.** $2.174.
80. $728.42. **81.** 36.3 nearly. **82.** 5,201,300. **83.** 52,100,000 nearly.
84. 55,310,000. **85.** 12.44496 ft. **86.** 712.5 lb. **87.** 320. **88.** 1.292 sec.
89. $4888\frac{8}{9}$ cu. yd. **90.** .002055 nearly. **91.** $6256. **92.** $3344.
93. .2565. **94.** 19*s.* $2\frac{1}{4}$*d.* **95.** .15625. **96.** (*a*) 147,824; (*b*) 22,098;
(*c*) 463,180; (*d*) 823,750. **97.** (*a*) 166,488; (*b*) 526,380. **98.** 640 A.
99. 2640 revolutions. **100.** 112.5 bu. **101.** $\frac{8.25}{12}$. **102.** $75. **103.** $24,800.
104. 10 A. **105.** $$14\frac{2}{7}$. **106.** (i) $.00\frac{3}{8}$; (ii) .00375. **107.** (*a*) $\frac{12.5}{100}$;
(*b*) $\frac{20}{100}$; (*c*) $\frac{8\frac{1}{3}}{100}$; (*d*) $\frac{6.25}{100}$. **108.** 99.99. **109.** $\frac{268}{15}$. **110.** 2.28.
111. $107. **112.** $\frac{11}{20}$. **113.** 66,720. **114.** $101.02. **115.** 25 ft. **116.** 35.
117. 847.15625. **118.** $28\frac{1}{3}$. **119.** $7260. **120.** 390. **121.** 1092 eggs.
122. $\frac{119}{143}$. **123.** $23\frac{6}{7}$. **124.** 17.66. **125.** 5.25 ft., 212.0045 sq. ft.,
98.328 sq. ft., 59.427 sq. ft. **126.** $330. **127.** $67\frac{1}{2}$ mi. **128.** 7.15 P.M.,
Apr. 5. **129.** 9.55 A.M. **130.** 7 hr. 29 min. 46 sec. **131.** 63° 35 W.
132. 156° 15′ W. **133.** 135° 27′ E. **134.** (*a*) 70° 15′ W., (*b*) 85° W.,
(*c*) 81° 45′ W. **135.** 24.62%. **136.** (i) 4.16%, (ii) 14.6%, (iii) 28.79%,
(iv) 17.8%. **137.** 96,600. **138.** $150. **139.** $1.44. **140.** 20%, or 16¢ per
yard. **141.** $8. **142.** $24\frac{4}{9}$%. **143.** $\frac{1}{4}$. **144.** $150. **145.** $38\frac{8}{9}$%.
146. $12\frac{1}{2}$%. **147.** (i) $64\frac{7}{12}$%, (ii) $3\frac{2}{3}$ gal. **148.** (*a*) 59.99%, (*b*) 32.34%,
(*c*) 62.84%, (*d*) 49.02%, (*e*) 65.57%, (*f*) 36.75%, (*g*) 65.28%,
(*h*) 40.83%, (*i*) 41.94%, (*j*) 39.01%, (*k*) 42.02%. **149.** $1.61.
150. $1.20. **151.** $182.67. **152.** £7 17*s.* 8*d.* **153.** £41 16*s.* 1*d.*
154. $300. **155.** $14,400. **156.** $400. **157.** $1200. **158.** $1000.
159. 7%. **160.** $500. **161.** 62¢. **162.** $546.69. **163.** July 23, $597.14.
164. $149.15. **165.** $891.80. **166.** $1004.72. **167.** Mar. 11, 1903, $824.89.
168. $459.50. **169.** 7500 sq. yd. **170.** 3500 sq. yd. **171.** $62.80.
172. $301\frac{1}{3}$ sq. yd. **173.** 1 mi. **174.** 1 mi. long, $\frac{1}{2}$ mi. wide.
175. 4330 sq. ft. **176.** 6495 sq. ft. **177.** 355 in. **178.** 314.16 sq. in.
179. 1256.64 sq. in. **180.** 1.9531 cu. ft. **181.** 6.07 in.

MISCELLANEOUS EXAMPLES (B)

1. $\frac{3}{160}$. **2.** 38° 15′. **3.** 216. **4.** 2.77 ft. **5.** 2.0575 yr. **6.** 3 gal. 1.02 gi. **7.** $4176. **8.** $20.94. **9.** 111 bu. 1 pk. $2\frac{1}{2}$ qt. **10.** $17\frac{5}{7}$ lb. **11.** $1\frac{857}{868}$. **12.** $504. **13.** 1.2857142. **14.** 1.777+. **15.** 16 ft. **16.** 12.56 A.M. **17.** $500. **18.** $11.28. **19.** $3\frac{1}{3}$ da. **20.** $2\frac{119}{148}$. **21.** $45\frac{1}{2}$ bu. **22.** 82.28 yr. **23.** $23\frac{52}{103}$. **24.** 226.27+ rd. **25.** $6.66. **26.** 149.61+gal. **27.** 734 rd. **28.** 90 yd. **29.** $16.67. **30.** 137 da. **31.** $331.86. **32.** $453\frac{1}{3}$ mi. **33.** Answers will depend on date. **34.** 14. **35.** $9.60. **36.** 10 A. **37.** 30 min. 50 sec. **38.** 6 ch. 89.6+ l. **39.** $\$116.35\frac{5}{9}$. **40.** $37.50, $30, $52.50. **41.** 62.72 bu. **42.** $3403\frac{1}{3}$ T. **43.** Neither gained nor lost. **44.** $9\frac{27}{125}$. **45.** 2295. **46.** 140 A. **47.** 4844.996. **48.** $75.15, $83.50, $300.60. **49.** 6 hr. 1 min. 28 sec. A.M. **50.** Answer will depend on day calculation is made. **51.** $2800. **52.** $604.66. **53.** $50 loss. **54.** 25%. **55.** $460. **56.** 10.8%. **57.** 16 yr. 8 mo., 12 yr. 6 mo., 11 yr. **58.** $453.561. **59.** $352.35. **60.** $4\frac{1}{6}$%. **61.** 210,526.3 lb. **62.** $275.80. **63.** $35\frac{7}{8}$ %. **64.** 41¢ on the $100, $4.92. **65.** $6.67. **66.** $56.25. **67.** $30,563.38+. **68.** 80. **69.** $2\frac{8}{11}$ %. **70.** $33\frac{1}{3}$%. **71.** $855. **72.** $27,200. **73.** $32.32 loss. **74.** $1092.56. **78.** 30%. **79.** 1 yr. 7 mo. 12 da. **80.** $492.50. **81.** 300%. **82.** $3200. **83.** $33\frac{1}{3}$%. **84.** 35%. **85.** $37.525. **86.** $108.90. **87.** 25%. **88.** $753. **89.** $5.69. **90.** Dec. 13, 1902. **91.** $6644.63. **92.** $3515.63. **93.** $3\frac{3}{4}$%. **94.** $530. **95.** $15,000. **96.** $1533.75. **97.** $9\frac{3}{8}$ %. **98.** Yes, $20 better. **99.** $2042. **100.** 25%. **101.** $1000. **102.** $7045. **103.** $28.01, $42.069. **104.** $6\frac{278}{487}$.

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